OCR
Oxford Cambridge and RSA

GCE

Mathematics

Unit **4725**: Further Pure Mathematics 1

Advanced Subsidiary GCE

Mark Scheme for June 2015

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			$z^* = x - iy$ $\|z\| = \sqrt{x^2 + y^2}$ $2(x^2 + y^2)$	B1 B1 B1 **[3]**	Conjugate stated or used Modulus or it's square stated or used Obtain correct answer, a.e.f. but not involving i	
2			½ $n(n + 1)(2n + 1) - 5n$ ½ $n(2n - 3)(n + 3)$ or $n(n - \frac{3}{2})(n + 3)$	M1* A1 DM1 A1 **[4]**	Express as difference using standard result for $\sum r^2$ Correct unsimplified expression Obtain at least factor of n Obtain correct answer, only these versions	
3	(i)		$\frac{1}{2}\begin{pmatrix} 1 & -a \\ 0 & 2 \end{pmatrix}$ or equivalent	B1 B1 **[2]**	Both diagonals correct Divide by correct determinant	
3	(ii)	***Either***	$\mathbf{P} = \mathbf{BA}^{-1}$ $\begin{pmatrix} 1 & 0 \\ 2 & 1-2a \end{pmatrix}$	B1 M1 A1ft **[3]**	State or use correct expression for **P** Multiplication attempt, 2 elements correct for any pair of matrices Obtain correct answer a.e.f. ft for their **(i)**	
		Or	Using $\mathbf{PA} = \mathbf{B}$	B1 M1 A1	State or find correct 1st column of **P** Multiplication attempt to find "1 – 2a " Obtain completely correct answer	
4			$k(k+1)^2 + (k+1)(3k+4)$ $(k+1)(k+2)^2$	B1 M1* DM1 A1 B1 **[5]**	Show sufficient working to verify result true when $n = 1$ Add next term in series Attempt to factorise their expression Sufficient working to obtain this correct answer Clear statement of induction process, provided previous 4 marks earned	

Question			Answer	Marks	Guidance	
5	(i)			B1 B1 B1 B1 **[4]**	Circle centre (–2, 0) or circle centre (2, 0) Touching *y*-axis at origin Half line with negative slope upwards Completely correct diagram	
5	(ii)		$-2-\sqrt{3}+i$	B1ft B1ft **[2]**	Correct real part and correct imaginary part of a complex number, ft for their half line from centre of their circle, allow decimals (-3.73 or better) or trig expressions	
5	(iii)			B1ft B1 **[2]**	Shade inside their circle Completely correct diagram and shading **S.C. allow last B1 for radius or complete line**	
6	(i)		$A'(0,-1)\quad B'(2,-1)\quad C'(2,0)$	B1 B1 B1 **[3]**	Coordinates of any 2 images seen Coordinates of 3rd image seen Completely correct labelled diagram, must include indication of coordinates	Might be columns
6	(ii)		$\begin{pmatrix}0&1\\-1&0\end{pmatrix}$ and $\begin{pmatrix}2&0\\0&1\end{pmatrix}$ Or $\begin{pmatrix}1&0\\0&2\end{pmatrix}$ and $\begin{pmatrix}0&1\\-1&0\end{pmatrix}$	B1 B1 B1ft B1ft **[4]**	Rotation and stretch or vice versa Rotation 90° clockwise, then Stretch s.f. 2 parallel to *x*-axis Or Stretch s.f. 2 parallel to *y*-axis & Rotation 90° clockwise Correct matrix, Correct matrix **S.C. If 1 matrix correct, correct 2nd matrix can be found by matrix multiplication and not be necessarily consistent with their transformation, but not ft.**	Must be a correct pair in correct order Consistent with their pair of transformations Just a trig form for rotation not acceptable

Question			Answer	Marks	Guidance	
7	(i)			M1	Attempt to equate real and imaginary parts of $(x + iy)$ and $5 + 12i$	
			$x^2 - y^2 = 5, 2xyi = 12i$	A1	Obtain both results or equivalent	
				M1	Obtain and solve a quadratic in x^2 or y^2 or solve by inspection	
			$3 + 2i$ and $-3 - 2i$ or $\pm(3 + 2i)$	A1 A1	Obtain correct answers as complex numbers	
				[5]	**S.C. $\pm(3 \pm 2i)$ scores A1**	
7	(ii)			M1	Solve using quadratic formula or complete square	
			$(4 \pm 2\sqrt{5 + 12i})/2$	A1	Obtain correct answers, or simpler version	
				M1	Use result(s) from **(i)**	
			$5 + 2i$ and $-1 - 2i$ or $2 \pm (3 + 2i)$	A1 A1	Obtain correct answers	If more than 2 roots A0 A0
				[5]		
8	(i)			M1	Use correct common denominator, numerator must be quadratic	
				A1	Obtain **given** result	
				[2]		
8	(ii)			M1	Express terms as differences using **(i)**	
				M1	Attempt this for at least first 3 terms	
				A1	First 3 terms all correct	
				A1	Last 2 terms correct	
			$\frac{7}{2} - \frac{3}{n} - \frac{1}{n+1}$	M1	Show terms cancelling	Need not be tidied up
				A1	Obtain correct answer, must be in terms of n	
				[6]		
8	(iii)		$\frac{5}{4}$	M1	Attempt to start summation at correct term	Could be $\Sigma_2^{\infty} - \Sigma_2^3$
				A1	Obtain correct answer from correct working	
				[2]		

Question			Answer	Marks	Guidance	
9	(i)			M1 M1 M1 A1 M1 A1 **[6]**	Attempt to find det **D** Show correct process for a 3 × 3, condone sign errors Show correct processes for a 2 × 2 Obtain correct answer Attempt to solve det **D** = 0 Obtain correct answers	Or Cramer's rule or similar
			$a^2 - 6a + 5$ $a = 5$ or 1			
9	(ii)	(a)(b)		B1 B1 M1 A1 **[4]**	State unique solution State non unique solutions Attempt to solve equations with $a = 1$ Explain inconsistency with correct working **S.C. Answer to (i) wrong, allow correct unique/non-unique B1ft, B1ft only**	

Question			Answer	Marks	Guidance	
10	(i)			B1	Use given substitution correctly in LHS of equation	
				M1	Rearrange and square to eliminate $\sqrt{u}$ or multiply by $u^{\frac{3}{2}} + 4u^{\frac{1}{2}} - 3$	
			$u^3 + 8u^2 + 16u - 9 = 0$	A1	Obtain correct answer, must be an equation = 0	
				[3]		
10	(ii)	***Either***	$\alpha\beta\gamma = -3$	B1	State or use correct result	
			$\sum\alpha^2 = -8 \qquad \sum\alpha^2\beta^2 = 16$	B1B1	Use correct result, using correct (i) or using an identity involving $\sum\alpha = 0, \sum\alpha\beta = 4$	
				M1*	Obtain an identity connecting $\sum\alpha^4$ and $(\sum\alpha^2)^2$	
			$(\sum\alpha^2)^2 = \sum\alpha^4 + 2\sum\alpha^2\beta^2$	A1	Obtain a correct answer	
				DM1	Use their values in their expression	
			29	A1	Obtain correct answer, c.w.o.	
				[7]		
		Or	$\alpha\beta\gamma = -3$	B1	State or use correct result	
			$\sum\alpha = 0, \sum\alpha\beta = 4, \sum\alpha^2 = -8,$ $\sum\alpha^2\beta^2 = 16$	B1 B1	Use any 2 correct B1, other 2 correct B1	
				M1	Expand $(\alpha+\beta+\gamma)^4$ and get expression involving symmetric functions only	
			$\sum\alpha^4 + 4\sum\alpha^2.\sum\alpha\beta + 6\sum\alpha^2\beta^2 + 8\alpha\beta\gamma\sum\alpha$	A1	Obtain correct expression	
				M1	Use their values in their expression	
				A1	Obtain correct answer, c.w.o.	

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