General Certificate of Education
January 2009
Advanced Subsidiary Examination

MATHEMATICS **MD01**
Unit Decision 1

Wednesday 21 January 2009 1.30 pm to 3.00 pm

For this paper you must have: • an 8-page answer book • the blue AQA booklet of formulae and statistical tables • an insert for use in Questions 3 and 4 (enclosed). You may use a graphics calculator.

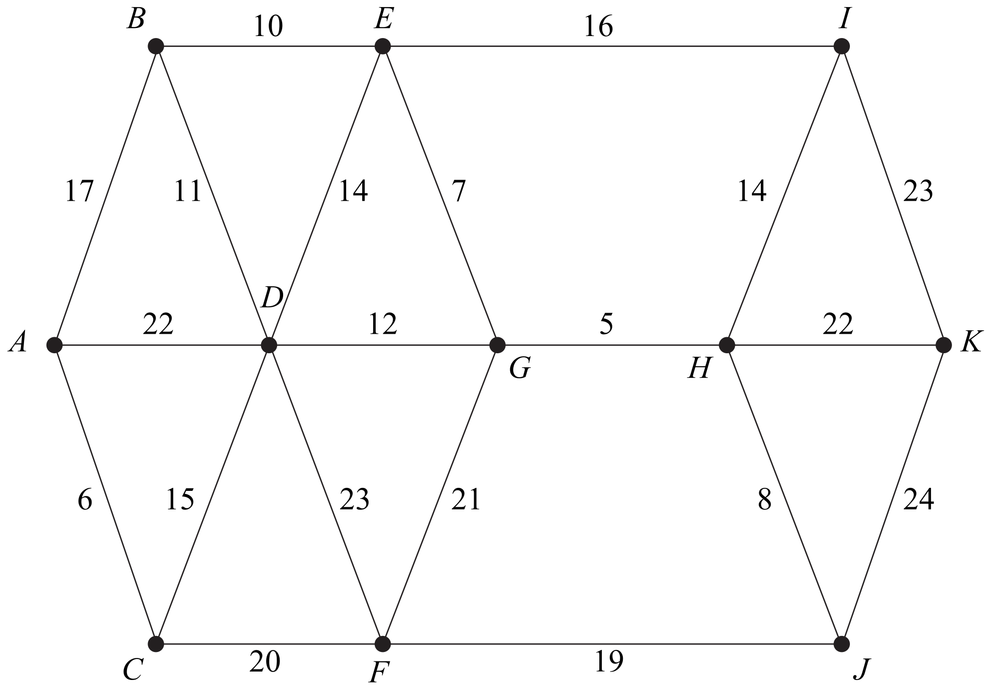
Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MD01.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Answer **all** questions.

1 The following network shows the lengths, in miles, of roads connecting 11 villages, A, B, ..., K.

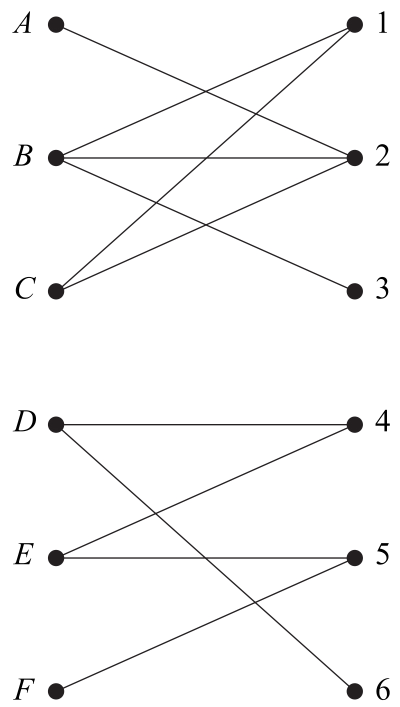


(a) Starting from G and showing your working at each stage, use Prim's algorithm to find a minimum spanning tree for the network. *(6 marks)*

(b) State the length of your minimum spanning tree. *(1 mark)*

(c) Draw your minimum spanning tree. *(3 marks)*

2 Six people, A, B, C, D, E and F, are to be allocated to six tasks, 1, 2, 3, 4, 5 and 6. The following bipartite graph shows the tasks that each of the people is able to undertake.



(a) Represent this information in an adjacency matrix. *(2 marks)*

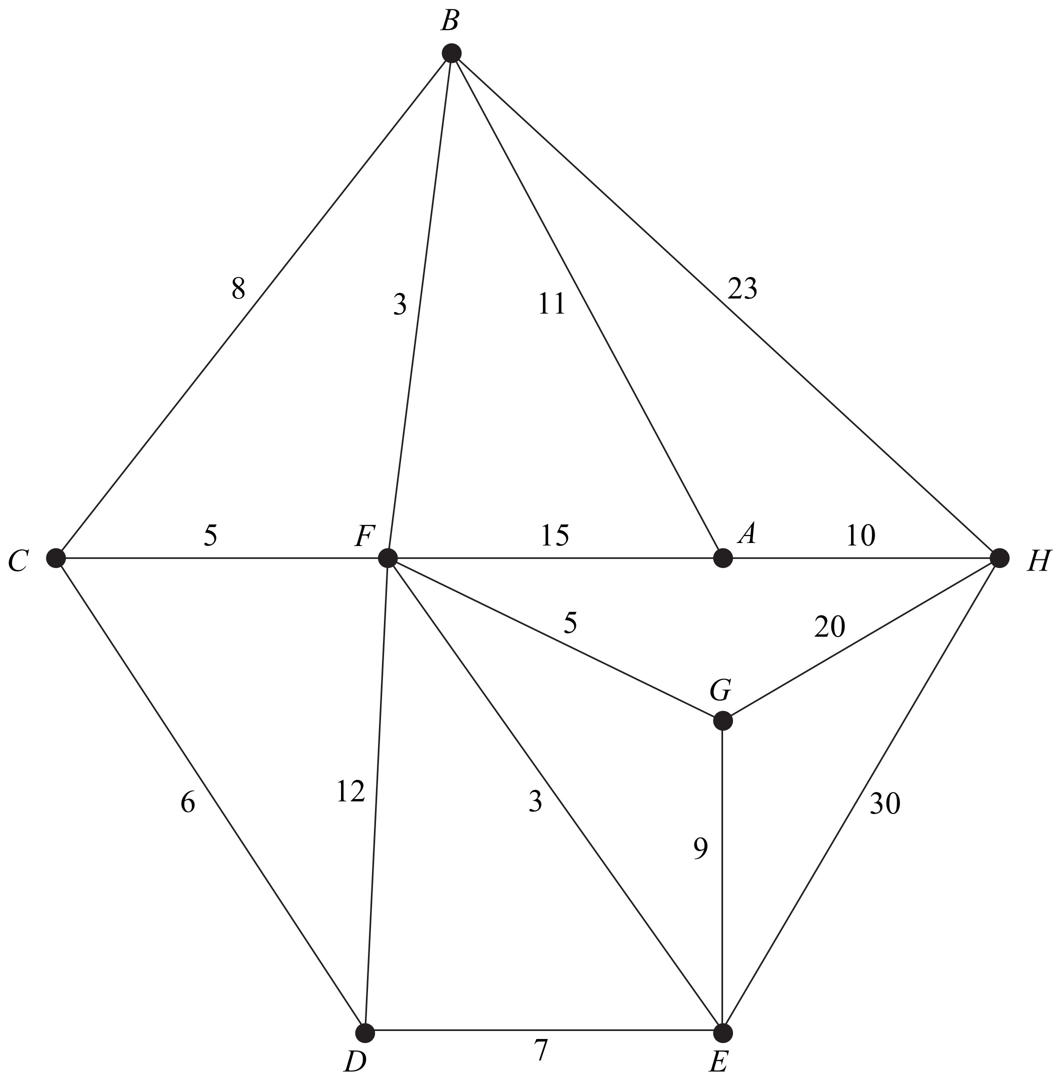
(b) Initially, B is assigned to task 1, C to task 2, D to task 4, and E to task 5.

Demonstrate, by using an algorithm from this initial matching, how each person can be allocated to a task. *(5 marks)*

Turn over for the next question

3 [**Figure 1**, printed on the insert, is provided for use in this question.]

The diagram shows roads connecting some places of interest in Berlin. The numbers represent the times taken, in minutes, to walk along the roads.



The total of all walking times is 167 minutes.

(a) Mia is staying at D and is to visit H.

(i) Use Dijkstra's algorithm on **Figure 1** to find the minimum time to walk from D to H. *(6 marks)*

(ii) Write down the corresponding route. *(1 mark)*

(b) Each day, Leon has to deliver leaflets along all of the roads. He must start and finish at A.

(i) Use your answer to part (a) to write down the shortest walking time from D to A. *(1 mark)*

(ii) Find the walking time of an optimum Chinese Postman route for Leon. *(6 marks)*

4 [**Figure 2**, printed on the insert, is provided for use in this question.]

Each year, farmer Giles buys some goats, pigs and sheep.

He must buy at least 110 animals.
He must buy at least as many pigs as goats.
The total of the number of pigs and the number of sheep that he buys must not be greater than 150.
Each goat costs £16, each pig costs £8 and each sheep costs £24.
He has £3120 to spend on the animals.

At the end of the year, Giles sells all of the animals. He makes a profit of £70 on each goat, £30 on each pig and £50 on each sheep. Giles wishes to maximize his total profit, £P.

Each year, Giles buys x goats, y pigs and z sheep.

(a) Formulate Giles's situation as a linear programming problem. *(5 marks)*

(b) One year, Giles buys 30 sheep.

(i) Show that the constraints for Giles's situation for this year can be modelled by

$$y \geqslant x, \qquad 2x + y \leqslant 300, \qquad x + y \geqslant 80, \qquad y \leqslant 120$$ *(2 marks)*

(ii) On **Figure 2**, draw a suitable diagram to enable the problem to be solved graphically, indicating the feasible region and the direction of the objective line. *(8 marks)*

(iii) Find Giles's maximum profit for this year and the number of each animal that he must buy to obtain this maximum profit. *(3 marks)*

Turn over for the next question

5 A student is using the algorithm below to find an approximate value of $\sqrt{2}$.

Line 10	Let $A = 1$, $B = 3$, $C = 0$
Line 20	Let $D = 1$, $E = 2$, $F = 0$
Line 30	Let $G = B/E$
Line 40	Let $H = G^2$
Line 50	If $(H - 2)^2 < 0.0001$ then go to Line 130
Line 60	Let $C = 2B + A$
Line 70	Let $A = B$
Line 80	Let $B = C$
Line 90	Let $F = 2E + D$
Line 100	Let $D = E$
Line 110	Let $E = F$
Line 120	Go to Line 30
Line 130	Print '$\sqrt{2}$ is approximately', B/E
Line 140	Stop

Trace the algorithm. *(6 marks)*

6 A connected graph G has five vertices and has eight edges with lengths 8, 10, 10, 11, 13, 17, 17 and 18.

(a) Find the minimum length of a minimum spanning tree for G. *(2 marks)*

(b) Find the maximum length of a minimum spanning tree for G. *(2 marks)*

(c) Draw a sketch to show a possible graph G when the length of the minimum spanning tree is 53. *(3 marks)*

7 Liam is taking part in a treasure hunt. There are five clues to be solved and they are at the points A, B, C, D and E. The table shows the distances between pairs of points. All of the distances are functions of x, **where x is an integer**.

Liam must travel to all five points, starting and finishing at A.

	A	***B***	***C***	***D***	***E***
A	–	$x+6$	$2x-4$	$3x-7$	$4x-14$
B	$x+6$	–	$3x-7$	$3x-9$	$x+9$
C	$2x-4$	$3x-7$	–	$2x-1$	$x+8$
D	$3x-7$	$3x-9$	$2x-1$	–	$2x-2$
E	$4x-14$	$x+9$	$x+8$	$2x-2$	–

(a) The nearest point to A is C.

(i) By considering AC and AB, show that $x < 10$. *(2 marks)*

(ii) Find two other inequalities in x. *(2 marks)*

(b) The nearest neighbour algorithm, starting from A, gives a **unique** minimum tour $ACDEBA$.

(i) By considering the fact that Liam's tour visits D immediately after C, find two further inequalities in x. *(3 marks)*

(ii) Find the value of the integer x. *(4 marks)*

(iii) Hence find the total distance travelled by Liam if he uses this tour. *(2 marks)*

END OF QUESTIONS

There are no questions printed on this page