OCR
Oxford Cambridge and RSA

# Thursday 14 May 2015 – Morning

## AS GCE MATHEMATICS

**4725/01** Further Pure Mathematics 1

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**
- Printed Answer Book 4725/01
- List of Formulae (MF1)

**Other materials required:**
- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes

### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

### INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

**1** The complex number $x+\mathrm{i}y$ is denoted by $z$. Express $3zz^*-|z|^2$ in terms of $x$ and $y$. **[3]**

**2** Find $\sum_{r=1}^{n}(3r^2-5)$, expressing your answer in a fully factorised form. **[4]**

**3** The matrix **A** is given by $\mathbf{A}=\begin{pmatrix}2 & a\\ 0 & 1\end{pmatrix}$, where $a$ is a constant.

**(i)** Find $\mathbf{A}^{-1}$. **[2]**

The matrix **B** is given by $\mathbf{B}=\begin{pmatrix}2 & a\\ 4 & 1\end{pmatrix}$.

**(ii)** Given that $\mathbf{PA}=\mathbf{B}$, find the matrix **P**. **[3]**

**4** Prove by induction that, for $n\geqslant 1$, $\sum_{r=1}^{n}r(3r+1)=n(n+1)^2$. **[5]**

**5** The loci $C_1$ and $C_2$ are given by $|z+2|=2$ and $\arg(z+2)=\frac{5}{6}\pi$ respectively.

**(i)** Sketch, on a single Argand diagram, the loci $C_1$ and $C_2$. **[4]**

**(ii)** Find the complex number represented by the intersection of $C_1$ and $C_2$. **[2]**

**(iii)** Indicate, by shading, the region of the Argand diagram for which

$$|z+2|\leqslant 2 \text{ and } \tfrac{5}{6}\pi\leqslant\arg(z+2)\leqslant\pi. \quad \textbf{[2]}$$

**6** The matrix **M** is given by $\mathbf{M}=\begin{pmatrix}0 & 2\\ -1 & 0\end{pmatrix}$.

**(i)** The diagram in the Printed Answer Book shows the unit square $OABC$. The image of the unit square under the transformation represented by **M** is $OA'B'C'$. Draw and label $OA'B'C'$, indicating clearly the coordinates of $A'$, $B'$ and $C'$. **[3]**

**(ii)** The transformation represented by **M** is equivalent to a transformation P followed by a transformation Q. Give geometrical descriptions of a possible pair of transformations P and Q and state the matrices that represent them. **[4]**

**7** **(i)** Use an algebraic method to find the square roots of the complex number $5+12\mathrm{i}$. You must show sufficient working to justify your answers. **[5]**

**(ii)** Hence solve the quadratic equation $x^2-4x-1-12\mathrm{i}=0$. **[5]**

**8** **(i)** Show that $\frac{3}{r-1}-\frac{2}{r}-\frac{1}{r+1}\equiv\frac{4r+2}{r(r^2-1)}$. **[2]**

**(ii)** Hence find an expression, in terms of $n$, for $\sum_{r=2}^{n}\frac{4r+2}{r(r^2-1)}$. **[6]**

**(iii)** Hence find the value of $\sum_{r=4}^{\infty}\frac{4r+2}{r(r^2-1)}$. **[2]**

 

**9** The matrix **D** is given by $\mathbf{D} = \begin{pmatrix} 1 & 3 & 4 \\ 2 & a & 3 \\ 0 & 1 & a \end{pmatrix}$.

**(i)** Find the values of $a$ for which **D** is singular. **[6]**

**(ii)** Three simultaneous equations are shown below.

$$\begin{aligned} x + 3y + 4z &= 3 \\ 2x + ay + 3z &= 2 \\ y + az &= 0 \end{aligned}$$

For each of the following values of $a$, determine whether or not there is a unique solution. If a unique solution does not exist, determine whether the equations are consistent or inconsistent.

**(a)** $a = 3$

**(b)** $a = 1$

**[4]**

**10** The cubic equation $x^3 + 4x + 3 = 0$ has roots $\alpha$, $\beta$ and $\gamma$.

**(i)** Use the substitution $x = \sqrt{u}$ to obtain a cubic equation in $u$. **[3]**

**(ii)** Find the value of $\alpha^4 + \beta^4 + \gamma^4 + \alpha\beta\gamma$. **[7]**

**END OF QUESTION PAPER**

 