

Mark Scheme 4723
June 2005

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1	(i)	State $f(x) \leq 10$	B1	1 [Any equiv but must be or imply \leq]
	(ii)	Attempt correct process for composition of functions Obtain 6 or correct expression for $ff(x)$ Obtain -71	M1 A1 A1	[whether algebraic or numerical] 3
2		<u>Either</u> Obtain $x = 0$ Form linear equation with signs of $6x$ and x different State $6x - 1 = -x + 1$ Obtain $\frac{2}{7}$ and no other non-zero value	B1 M1 A1 A1	[ignoring errors in working] [ignoring other sign errors] [or correct equiv with or without brackets] 4 [or exact equiv]
	<u>Or</u>	Obtain $36x^2 - 12x + 1 = x^2 - 2x + 1$ Attempt to solve quadratic equation Obtain $\frac{2}{7}$ and no other non-zero value Obtain 0	B1 M1 A1 B1	[or equiv] [as far as factorisation or subn into formula] [or exact equiv] (4) [ignoring errors in working]
3	(i)	Attempt solution involving (natural) logarithm Obtain $-0.017t = \ln \frac{25}{180}$ Obtain 116	M1 A1 A1	[or equiv] 3 [or greater accuracy rounding to 116]
	(ii)	Differentiate to obtain $ke^{-0.017t}$ Obtain correct $-3.06e^{-0.017t}$ Obtain 1.2	M1 A1 A1	[any constant k different from 180; solution must involve differentiation] [or unsimplified equiv; accept + or -] 3 [or greater accuracy; accept + or - answer]
4	(a)	State or imply $\int \pi y^2 dx$ Integrate to obtain $k \ln x$ Obtain $4\pi \ln x$ or $4 \ln x$ Obtain $4\pi \ln 5$	B1 M1 A1 A1	[any constant k , involving π or not; or equiv such as $k \ln 4x$] [or equiv] 4 [or similarly simplified equiv]

	<p>(b) Attempt calculation involving attempts at y values</p> <p>Attempt $\frac{1}{3} \times 1(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4)$</p> <p>Obtain $\frac{1}{3}(\sqrt{2} + 4\sqrt{5} + 2\sqrt{10} + 4\sqrt{17} + \sqrt{26})$</p> <p>Obtain 12.758</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>[with each of 1, 4, 2 present at least once as coefficients]</p> <p>[with attempts at five y values]</p> <p>[or exact equiv or decimal equivs]</p> <p>4 [or greater accuracy]</p>
5	<p>(i) Obtain $R = \sqrt{13}$, or 3.6 or 3.61 or greater accuracy</p> <p>Attempt recognisable process for finding α</p> <p>Obtain $\alpha = 33.7$</p>	<p>B1</p> <p>M1</p> <p>A1</p>	<p>[allow sine/cosine muddles]</p> <p>3 [or greater accuracy]</p>
	<p>(ii) Attempt to find at least one value of $\theta + \alpha$</p> <p>Obtain value rounding to 76 or 104</p> <p>Subtract their α from at least one value</p> <p>Obtain one value rounding to 42 or 43, or to 70</p> <p>Obtain other value 42.4 or 70.2</p>	<p>*M1</p> <p>A1✓</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>[following their R]</p> <p>[dependent on *M]</p> <p>5 [or greater accuracy; no other answers between 0 and 360; ignore answers outside 0 to 360]</p>
6	<p>(a) Attempt use of product rule</p> <p>Obtain $\ln x + 1$</p> <p>Equate attempt at first derivative to zero and obtain value involving e</p> <p>Obtain e^{-1}</p>	<p>*M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>[or unsimplified equiv]</p> <p>[dependent on *M]</p> <p>4 [or exact equiv]</p>
	<p>(b) Attempt use of quotient rule</p> <p>Obtain $\frac{(4x-c)4 - 4(4x+c)}{(4x-c)^2}$</p> <p>Show that first derivative cannot be zero</p>	<p>M1</p> <p>A1</p> <p>A1</p>	<p>[or equiv using product rule or ...]</p> <p>[or equiv]</p> <p>3 [AG; derivative must be correct]</p>
7	<p>(i) State $2\cos^2 x - 1$</p>	<p>B1</p>	<p>1</p>
	<p>(ii) Attempt to express left hand side in terms of $\cos x$</p> <p>Identify $\frac{1}{\cos x}$ as $\sec x$</p>	<p>M1</p> <p>M1</p>	<p>[using expression of form $a\cos^2 x + b$]</p> <p>[maybe implied]</p>

		Confirm result	A1	3 [AG; necessary detail required]
	(iii)	Use identity $\sec^2 x = 1 + \tan^2 x$ Attempt solution of quadratic equation in $\tan x$ Obtain $2 \tan^2 x + 3 \tan x - 9 = 0$ and hence $\tan x = -3, \frac{3}{2}$ Obtain at least two of 0.983, 4.12, 1.89, 5.03 (or of $0.313\pi, 1.31\pi, 0.602\pi, 1.60\pi$) Obtain all four solutions	B1 M1 A1 A1 A1	[or equiv] [allow answers with only 2 s.f.; allow greater accuracy; allow $0.983 + \pi, 1.89 + \pi$ allow degrees: 56, 236, 108, 288] 5 [now with at least 3 s.f.; must be radians; no other solutions in the range $0 - 2\pi$, ignore solutions outside range $0 - 2\pi$]
8	(i)	Attempt relevant calculations with 5.2 and 5.3 Obtain correct values Conclude appropriately	M1 A1 A1	x y_1 y_2 $y_1 - y_2$ 5.2 2.83 2.87 -0.04 5.3 2.89 2.88 0.006 3 [AG; comparing y values or noting sign change in difference in y values or equiv]
	(ii)	Equate expressions and attempt rearrangement to $x =$ Obtain $x = \frac{5}{3} \ln(3x + 8)$	M1 A1	2 [AG; necessary detail required]
	(iii)	Obtain correct first iterate Carry out correct process to find at least two iterates in all Obtain 5.29	B1 M1 A1	3 [must be exactly 2 decimal places; 5.2→5.2687→5.2832→5.2863→5.2869; 5.25→5.2793→5.2855→5.2868→5.2870; 5.3→5.2898→5.2877→5.2872→5.2871]
	(iv)	Obtain integral of form $k(3x + 8)^{\frac{4}{3}}$ Obtain integral of form $k e^{\frac{1}{5}x}$	M1 M1	

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		Obtain $\frac{1}{4}(3x+8)^{\frac{4}{3}} - 5e^{\frac{1}{5}x}$	A1	[or equiv]
		Apply limits 0 and their answer to (iii)	M1	[applied to difference of two integrals]
		Obtain 3.78	A1	5 [or greater accuracy]
9	(i)	Indicate stretch and (at least one) translation	M1	[... in general terms]
		State translation by 7 units in negative x direction	A1	[or equiv; using correct terminology]
		State stretch in x direction with factor $1/m$	A1	[must follow the translation by 7; or equiv; using correct terminology]
	(ii)	Indicate translation by 4 units in negative y direction	B1	4 [or equiv; at any stage; the two translations may be combined]
	(ii)	Refer to each y value being image of unique x value	B1	[or equiv]
		Attempt correct process for finding inverse	M1	
		Obtain expression involving $(x+4)^2$ or $(y+4)^2$	M1	
		Obtain $\frac{(x+4)^2 - 7}{m}$	A1	4 [or equiv]
	(iii)	Refer to fact that curves are reflections of each other in line $y = x$	B1	[or equiv]
		Attempt arrangement of either $f(x) = x$ or $f^{-1}(x) = x$	M1	
		Apply discriminant to resulting quadratic equation	M1	
		Obtain $(m-2)(m-14) < 0$	A1	[or equiv]
		Obtain $2 < m < 14$	A1	5