

2.

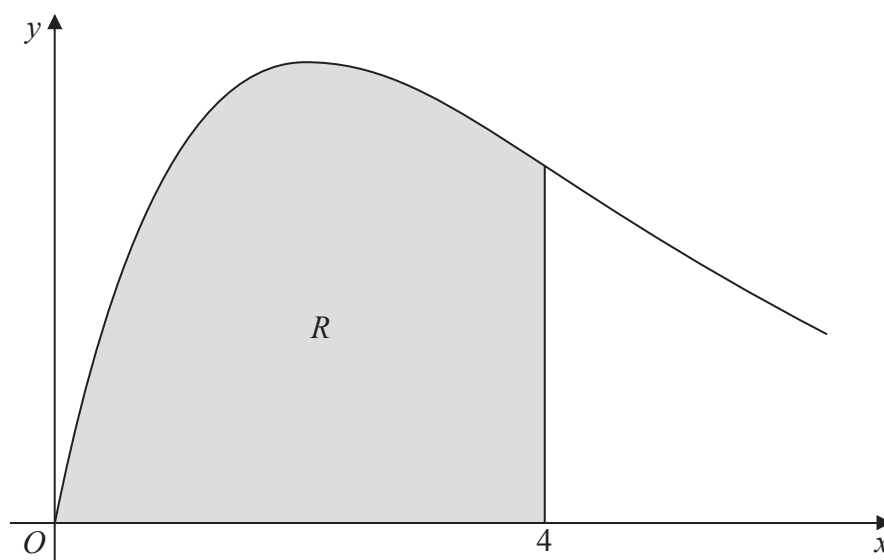


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = xe^{-\frac{1}{2}x}$, $x \geq 0$.

The finite region R , shown shaded in Figure 1, is bounded by the curve, the x -axis, and the line $x = 4$.

The table shows corresponding values of x and y for $y = xe^{-\frac{1}{2}x}$.

x	0	1	2	3	4
y	0	$e^{-\frac{1}{2}}$		$3e^{-\frac{3}{2}}$	$4e^{-2}$

- (a) Complete the table with the value of y corresponding to $x = 2$ (1)
- (b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R , giving your answer to 2 decimal places. (4)
- (c) (i) Find $\int xe^{-\frac{1}{2}x} dx$.
- (ii) Hence find the exact area of R , giving your answer in the form $a + be^{-2}$, where a and b are integers. (6)



7.

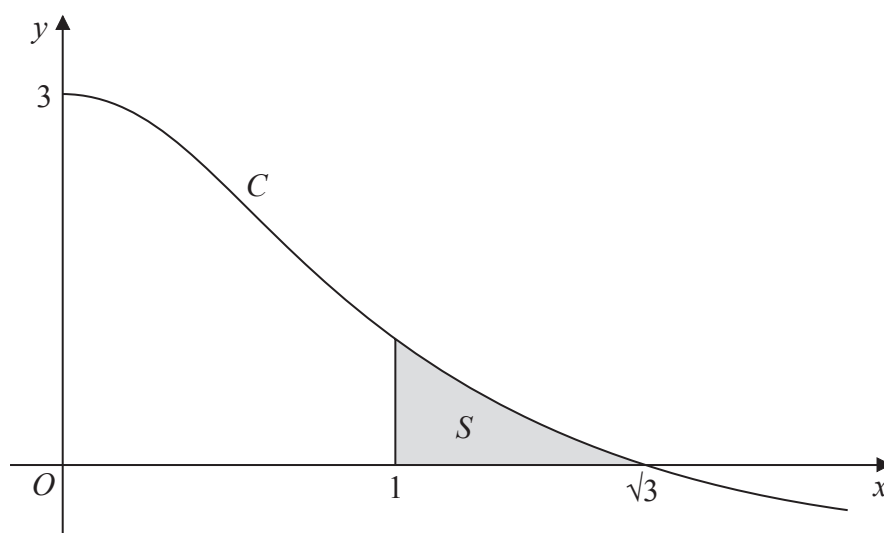


Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = \tan \theta, \quad y = 1 + 2 \cos 2\theta, \quad 0 \leq \theta < \frac{\pi}{2}$$

The curve C crosses the x -axis at $(\sqrt{3}, 0)$. The finite shaded region S shown in Figure 2 is bounded by C , the line $x=1$ and the x -axis. This shaded region is rotated through 2π radians about the x -axis to form a solid of revolution.

(a) Show that the volume of the solid of revolution formed is given by the integral

$$k \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (16 \cos^2 \theta - 8 + \sec^2 \theta) d\theta$$

where k is a constant.

(5)

(b) Hence, use integration to find the exact value for this volume.

(5)



