4722 Mark Scheme January 2008

4722 Core Mathematics 2

		Mark	Total	
1	area of sector = $\frac{1}{2}$ x 11 ² x 0.7 = 42.35 area of triangle = $\frac{1}{2}$ x 11 ² x sin0.7 = 38.98 hence area of segment = 42.35 – 38.98 = 3.37	M1 A1 M1	4	Attempt sector area using $(1/2) r^2 \theta$ Obtain 42.35, or unsimplified equiv, soi Attempt triangle area using $1/2ab\sin C$ or equiv, and subtract from attempt at sector Obtain 3.37, or better
			4	
2	area $\approx \frac{1}{2} \times 2 \times \left\{2 + 2\left(\sqrt{12} + \sqrt{28}\right) + \sqrt{52}\right\}$ ≈ 26.7	M1 M1 M1 A1	4	Attempt <i>y</i> -values at $x = 1, 3, 5, 7$ only Correct trapezium rule, any h , for their y values to find area between $x = 1$ and $x = 7$ Correct h (soi) for their y values Obtain 26.7 or better (correct working only)
			4	
3	(i) $\log_a 6$	B1	1	State $\log_a 6$ cwo
	(ii) $2\log_{10} x - 3\log_{10} y = \log_{10} x^2 - \log_{10} y^3$ = $\log_{10} \frac{x^2}{y^3}$	M1* M1de	ep* 3	Use $b \log a = \log a^b$ at least once Use $\log a - \log b = \log^{a}/_{b}$ Obtain $\log_{10} \frac{x^2}{y^3}$ cwo
			4	
4	(i) $\frac{BD}{\sin 62} = \frac{16}{\sin 50}$ $BD = 18.4 \text{ cm}$	M1 A1	2	Attempt to use correct sine rule in ΔBCD , or equiv. Obtain 18.4 cm
	(ii) $18.4^{2} = 10^{2} + 20^{2} - 2 \times 10 \times 20 \times \cos \theta$ $\cos \theta = 0.3998$ $\theta = 66.4^{0}$	M1 M1	3	Attempt to use correct cosine rule in $\triangle ABD$ Attempt to rearrange equation to find $\cos BAD$ (from $a^2 = b^2 + c^2 \pm (2)bc \cos A$) Obtain 66.4^0
			5	
5	$\int 12x^{\frac{1}{2}} dx = 8x^{\frac{3}{2}}$	M1 A1√ A1		Attempt to integrate Obtain correct, unsimplified, integral following their $f(x)$. Obtain $8x^{\frac{3}{2}}$, with or without $+c$
	$y = 8x^{\frac{3}{2}} + c \Rightarrow 50 = 8 \times 4^{\frac{3}{2}} + c$ $\Rightarrow c = -14$ Hence $y = 8x^{\frac{3}{2}} - 14$	M1 A1√ A1	6	Use (4, 50) to find c Obtain $c = -14$, following $kx^{\frac{3}{2}}$ only State $y = 8x^{\frac{3}{2}} - 14$ aef, as long as single power of x
			6	

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6	(i)	$u_1 = 7$	B1		Correct u_1
		$u_2 = 9$, $u_3 = 11$	B1	2	Correct u_2 and u_3
	(ii)	Arithmetic Progression	B1	1	Any mention of arithmetic
	(iii)	$\frac{1}{2}N(14 + (N-1) \times 2) = 2200$	B1		Correct interpretation of sigma notation
		2	M1		Attempt sum of AP, and equate to 2200
		$N^2 + 6N - 2200 = 0$	A1		Correct (unsimplified) equation
		(N-44)(N+50) = 0 hence $N = 44$	M1 A1	5	Attempt to solve 3 term quadratic in N Obtain $N = 44$ only ($N = 44$ www is full marks)
		Hence IV = 44	AI	3	Obtain 77 = 44 Only (17 = 44 www is full marks)
				8	
7	(i)	Some of the area is below the <i>x</i> -axis	B1	1	Refer to area / curve below x-axis or 'negative
	(ii)		M1		area' Attempt integration with any one term correct
	(H)		A1		Obtain $\frac{1}{3}x^3 - \frac{3}{2}x^2$
		$\left[\frac{1}{3}x^3 - \frac{3}{2}x^2\right]_0^3 = \left(9 - \frac{27}{2}\right) - \left(0 - 0\right)$	M1		Use limits 3 (and 0) – correct order / subtraction
		$=-4\frac{1}{2}$	A1		Obtain (-)4½
		$\left[\frac{1}{3}x^3 - \frac{3}{2}x^2\right]_3^5 = \left(\frac{125}{3} - \frac{75}{2}\right) - \left(9 - \frac{27}{2}\right)$	M1		Use limits 5 and 3 – correct order / subtraction
		$=8\frac{2}{3}$	A1		Obtain $8^2/_3$ (allow 8.7 or better)
		Hence total area is $13^{1}/_{6}$	A1	7	Obtain total area as $13^{1}/_{6}$, or exact equiv
					SR: if no longer $\int f(x)dx$, then B1 for using [0, 3] and [3, 5]
				8	
8	(i)	$u_4 = 10x0.8^3$	M1		Attempt u ₄ using ar^{n-1}
	. ,	= 5.12	A1	2	Obtain 5.12 aef
		()			
	(ii)	$S_{20} = \frac{10(1 - 0.8^{20})}{1 - 0.8}$	M1		Attempt use of correct sum formula for a GP
		1-0.8 = 49.4	A1	2.	Obtain 49.4
		- 17.1	711	-	Obtain 19.1
	(iii)	$\frac{10}{1 - 0.8} - \frac{10(1 - 0.8^{N})}{(1 - 0.8)} < 0.01$	M1		Attempt S_{∞} using $\frac{a}{1-r}$
		,	A1		Obtain $S_{\infty} = 50$, or unsimplified equiv
		$50 - 50(1 - 0.8^{N}) < 0.01$	M1		Link $S_{\infty} - S_N$ to 0.01 and attempt to rearrange
		$0.8^N < 0.0002$ A.G.	A1		Show given inequality convincingly
		$\log 0.8^{N} < \log 0.0002$	M1		Introduce logarithms on both sides
	λ1 ~	$N \log 0.8 < \log 0.0002$ 38.169, hence $N = 39$	M1	7	Use $\log a^b = b \log a$, and attempt to find N Obtain $N = 39$ only
	1 V >	30.107, Helice $N = 39$	A1	7	Obtain N = 39 only
				11	

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			Mark 7	otal	
9	(i)	(90°, 2), (-90°, -2)	B1 B1	2	State at least 2 correct values State all 4 correct values (radians is B1 B0)
	(ii)	(a) 180 - α	B1	1	·
	(11)	(b) $-\alpha$ or $\alpha - 180$	B1	1	State - α or $\alpha - 180$
					(radians or unsimplified is B1B0)
	(iii)	$2\sin x = 2 - 3\cos^2 x$ $2\sin x = 2 - 3(1 - \sin^2 x)$ $3\sin^2 x - 2\sin x - 1 = 0$ $(3\sin x + 1)(\sin x - 1) = 0$ $\sin x = -\frac{1}{3}, \sin x = 1$ $x = -19.5^\circ, -161^\circ, 90^\circ$	M1 A1 M1 A1		Attempt use of $\cos^2 x = 1 - \sin^2 x$ Obtain $3\sin^2 x - 2\sin x - 1 = 0$ aef with no bracket Attempt to solve 3 term quadratic in $\sin x$ Obtain $x = -19.5^\circ$ Obtain second correct answer in range, following
		<i>x</i> 15.5 , 161 , 50	111 (their x
			A 1	6	Obtain 90° (radians or extra answers is max 5 out of 6)
					SR: answer only (and no extras) is B1 B1√B1
			1	0	
10	(i)	$(2x+5)^4 = (2x)^4 + 4(2x)^3 + 6(2x)^2 + 4(2x)^3 + 5^4$	M1*		Attempt expansion involving powers of 2x and 5 (at least 4 terms)
		$= 16x^4 + 160x^3 + 600x^2 + 1000x + 625$	M1*		Attempt coefficients of 1, 4, 6, 4, 1
			A1dep		Obtain two correct terms
			A1	4	Obtain a fully correct expansion
	(ii)	$(2x+5)^4 - (2x-5)^4 = 320x^3 + 2000x$	M1		Identify relevant terms (and no others) by sign
			A1	2	change oe Obtain $320x^3 + 2000x$ cwo
	(iii)	$9^4 - (-1)^4 = 6560$ and $7360 - 800 = 6560$ A.G. $320x^3 - 1680x + 800 = 0$	B1 M1		Confirm root, at any point Attempt complete division by $(x - 2)$ or equiv
		$520x - 1080x + 800 = 0$ $4x^3 - 21x + 10 = 0$	A1√		Obtain quotient of $ax^2 + 2ax + k$, where a is
		$(x-2)(4x^2+8x-5)=0$	A 1		their coeff of x^3 Obtain $(4x^2 + 8x - 5)$ (or multiple thereof)
		(x-2)(4x + 8x - 5) = 0 (x-2)(2x-1)(2x+5) = 0	A1 M1		Obtain $(4x + 8x - 3)$ (or multiple thereof) Attempt to solve quadratic
		Hence $x = \frac{1}{2}$, $x = -2\frac{1}{2}$	A1	6	l • •
					SR: answer only is B1 B1
			_		
			<u> </u>	2	