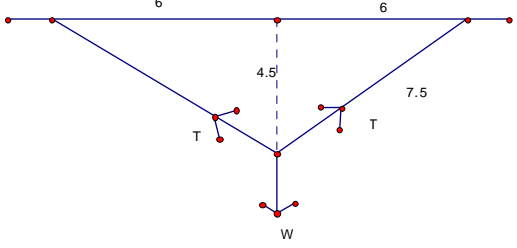
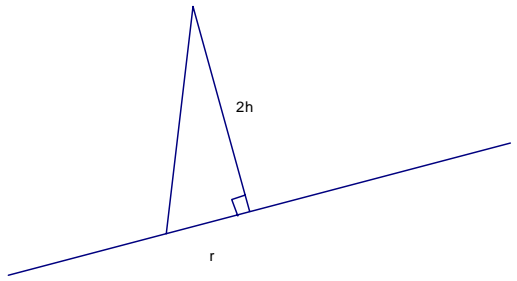


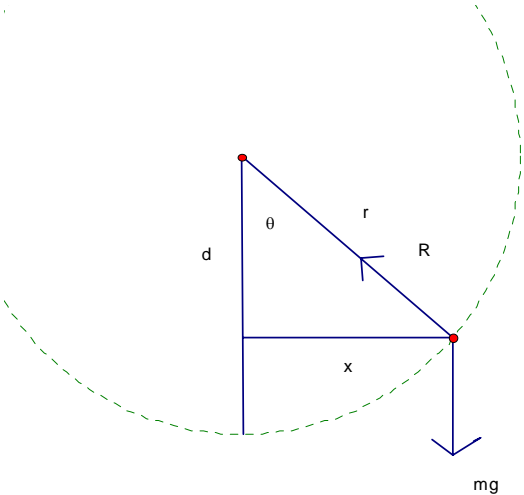
# Mark Scheme (Results) Summer 2009

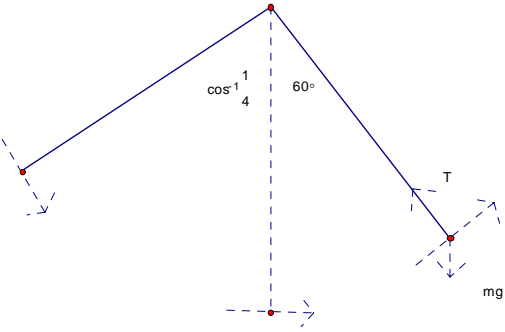
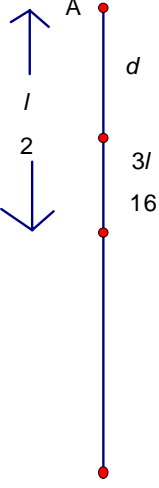
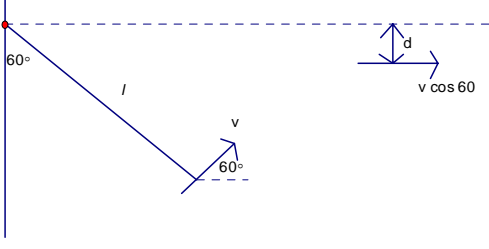
GCE

GCE Mathematics (6679/01)

June 2009  
6679 Mechanics M3  
Mark Scheme

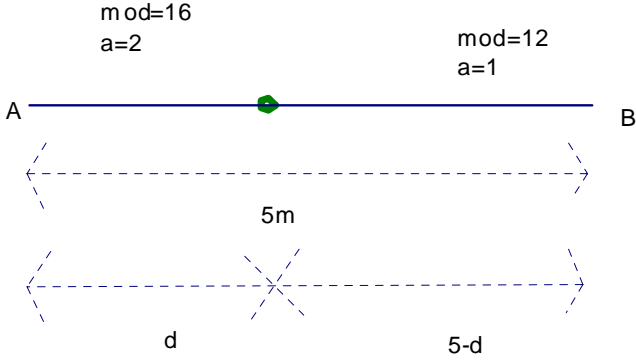
Question Number	Scheme			Marks
Q1 (a)	 <p>Resolving vertically: <math>2T \cos \theta = W</math></p> <p>Hooke's Law: <math>T = \frac{80 \times 3.5}{4}</math> <math>W = 84\text{N}</math></p>			M1A2,1,0
(b)	<p>EPE = <math>2 \times \frac{80 \times 3.5^2}{2 \times 4}</math>, = 245 (or awrt 245)</p> <p>(alternative <math>\frac{80 \times 7^2}{16} = 245</math>)</p>			M1A1ft,A1
Q2 (a)	<p>Object</p> <p>Cone</p> <p>Base</p> <p>Marker</p>	<p>Mass</p> <p><math>m</math></p> <p><math>3m</math></p> <p><math>4m</math></p>	<p>c of m above base</p> <p><math>2h+3h</math></p> <p><math>h</math></p> <p><math>d</math></p>	B1(ratio masses) B1(distances)
(b)	<p><math>m \times 5h + 3m \times h = 4m \times d</math></p> <p><math>d = 2h</math></p>  <p><math>\frac{r}{d} = \frac{1}{12}</math></p> <p><math>6r = h</math></p>			M1A1ft A1 M1A1ft A1

Question Number	Scheme	Marks
<p>Q3 (a)</p> <p>(b)</p>	 $\leftrightarrow R \sin \theta = mx\omega^2$ $R \times \frac{x}{r} = mx \times \frac{3g}{2r}$ $R = \frac{3mg}{2}$ $\downarrow R \cos \theta = mg$ $\frac{3mg}{2} \times \frac{d}{r} = mg$ $d = \frac{2}{3}r$	<p>M1 A1</p> <p>M1</p> <p>A1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>[8]</p>
<p>Q4 (a)</p> <p>(b)</p>	$\text{Volume} = \int_{\frac{1}{4}}^1 \pi y^2 dx = \int_{\frac{1}{4}}^1 \pi \frac{1}{x^4} dx$ $= \left[ \pi \times \frac{-1}{3x^3} \right]_{\frac{1}{4}}^1$ $= \pi \left( \frac{-1}{3} + \frac{64}{3} \right) = 21\pi \quad *$ $21\pi \bar{x} = \rho \int \pi y^2 x dx = \rho \int \pi \frac{1}{x^4} x dx$ $21\pi \bar{x} = \pi \left[ \frac{-1}{2x^2} \right]_{\frac{1}{4}}^1$ $\bar{x} = \frac{1}{21} \left( \frac{-1}{2} + \frac{16}{2} \right) = \frac{5}{14} \quad \text{or awrt } 0.36$ <p><math>\bar{y} = 0</math> by symmetry</p>	<p>M1A1</p> <p>A1ft</p> <p>A1</p> <p>M1A1</p> <p>A1ft</p> <p>A1</p> <p>B1</p> <p>[9]</p>

Question Number	Scheme	Marks
Q5 (a)	 <p>Energy:  <math display="block">\left(\frac{1}{2}mv^2 + \right)mgl\left(\cos\theta - \frac{1}{4}\right) = \frac{1}{2}mv^2</math>                     Resolving:  <math display="block">T - mg \cos\theta = \frac{mv^2}{l}</math>                     Eliminate <math>v^2</math>:  <math display="block">T = mg \cos\theta + \frac{1}{l}\left(2mgl\left(\cos\theta - \frac{1}{4}\right)\right)</math> <math display="block">T = 3mg \cos\theta - \frac{mg}{2} *</math> </p>	M1A1 M1A1 M1 A1
(b)	 <p><math>\theta = 60^\circ \Rightarrow mv^2 = 2mgl\left(\frac{1}{2} - \frac{1}{4}\right)</math>  <math>\Rightarrow v^2 = \frac{gl}{2}</math></p> <p>vertical motion under gravity:  <math>\uparrow 0 = (v \cos 30^\circ)^2 - 2gs</math>  <math>0 = \frac{gl}{2} \times \frac{3}{4} - 2gs \Rightarrow s = \frac{3l}{16}</math></p> <p>Distance below A = <math>\frac{l}{2} - \frac{3l}{16} = \frac{5l}{16}</math></p>	M1 M1 A1 M1A1
Alternative for end of (b) using energy	 <p><math>\frac{1}{2}mv^2 - mgl \cos 60 = \frac{1}{2}m(v \cos 60)^2 - mgd</math>  <math>\frac{gl}{4} - \frac{gl}{2} = \frac{gl}{4} \times \frac{1}{4} - gd</math>  <math>d = \frac{1-4+8}{16}l = \frac{5l}{16}</math></p>	M1A1 M1 A1

[11]

Question Number	Scheme	Marks
Q6 (a)	<p>At max <math>v</math>, driving force = resistance</p> $\text{Driving force} = \frac{80}{v}$ $\Rightarrow \frac{80}{20} = k \times 20^2 \Rightarrow k = \frac{1}{100}$ $F = ma \Rightarrow 100a = \frac{80}{v} - kv^2 \quad \left( = \frac{8000 - v^3}{100v} \right)$ $\ast \Rightarrow v \frac{dv}{dx} = \frac{8000 - v^3}{10000v} \quad \ast$ <p>(b)</p> $\int_4^8 \frac{10000v^2}{8000 - v^3} dv = \int_0^D 1 dx$ $D = \left[ -\frac{10000}{3} \ln 8000 - v^3  \right]_4^8$ $= \left( -\frac{10000}{3} \ln \frac{7488}{7936} \right) = 193.7 \dots \approx 194 \text{ m (accept 190)}$ <p>(c)</p> $\frac{dv}{dt} = \frac{8000 - v^3}{10000v} \Rightarrow \int_0^T 1 dt = \int_4^8 \frac{10000v}{8000 - v^3} dv$ $\Rightarrow T \approx \frac{1}{2} \times 2 \times 10000 \times \left\{ \frac{4}{7936} + \frac{2 \times 6}{7784} + \frac{8}{7488} \right\}$ $\Rightarrow T (= 31.1409 \dots) \approx 31$	<p>B1</p> <p>M1A1</p> <p>M1</p> <p>A1</p> <p>M1A1</p> <p>A1</p> <p>M1 A1</p> <p>M1A1</p> <p>M1 A1</p> <p>[14]</p>

Question Number	Scheme	Marks
Q7 (a)	<div style="text-align: center;">  </div> <p>Hooke's law: Equilibrium <math>\Rightarrow \frac{16(d-2)}{2} = \frac{12(4-d)}{1}</math>  <math>\Rightarrow d = 3.2</math>  so extensions are 1.2m and 0.8m.</p> <p>(b) If the particle is displaced distance <math>x</math> towards <b>B</b> then  <math>-m\ddot{x} = \frac{16(1.2+x)}{2} - \frac{12(0.8-x)}{1} (= 20x)</math>  <math>\Rightarrow \ddot{x} = -40x</math> or <math>\ddot{x} = -\frac{20}{m}</math> (<math>\Rightarrow</math> SHM)</p> <p>(c) <math>T = \frac{2\pi}{\sqrt{40}}</math>  <math>a = \frac{\sqrt{10}}{\text{their } \omega}</math>  <math>x = a \sin \omega t</math> their <math>a</math>, their <math>\omega</math>  <math>\frac{1}{4} = \frac{1}{2} \sin \sqrt{40}t</math>  <math>\sqrt{40}t = \frac{\pi}{6} (\Rightarrow t = \frac{\pi}{6\sqrt{40}})</math></p> <p>Proportion <math>\frac{4t}{T} = \frac{4\pi}{6\sqrt{40}} \times \frac{\sqrt{40}}{2\pi} = \frac{1}{3}</math></p>	<p>M1A1A1</p> <p>A1 A1</p> <p>M1A1ft A1ft</p> <p>A1</p> <p>B1ft</p> <p>B1ft</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1A1</p> <p style="text-align: right;">[16]</p>