OCR

Thursday 22 May 2014 – Morning

A2 GCE MATHEMATICS

4737/01 Decision Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4737/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Six students are choosing their tokens for a board game. The bipartite graph in Fig. 1 shows which token each student is prepared to have.

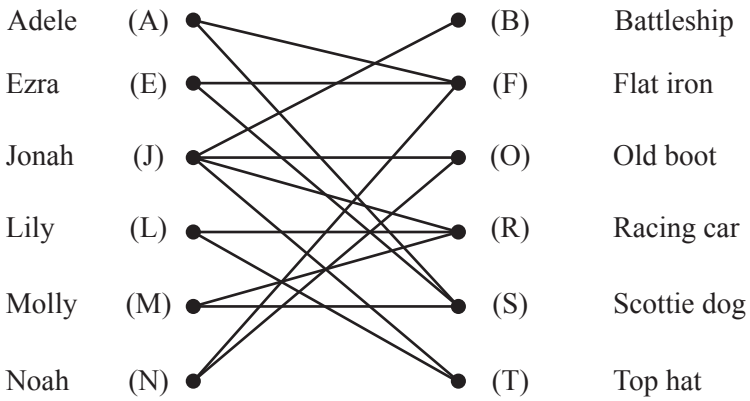


Fig. 1

Initially Ezra takes the flat iron, Jonah the old boot, Lily the racing car and Molly the scottie dog. This leaves Adele and Noah with tokens that they do not want. This incomplete matching is shown in Fig. 2 below.

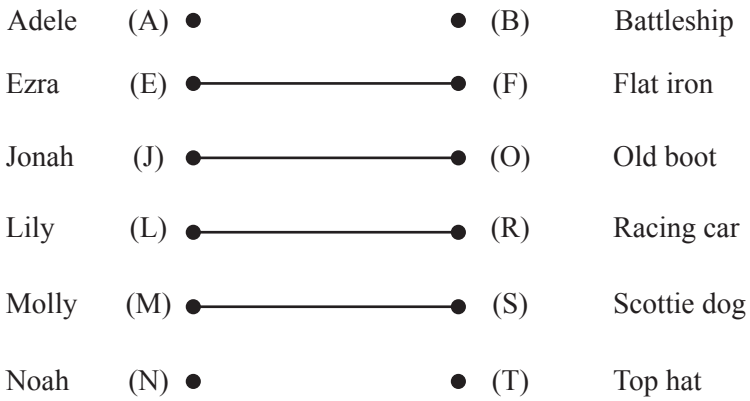


Fig. 2

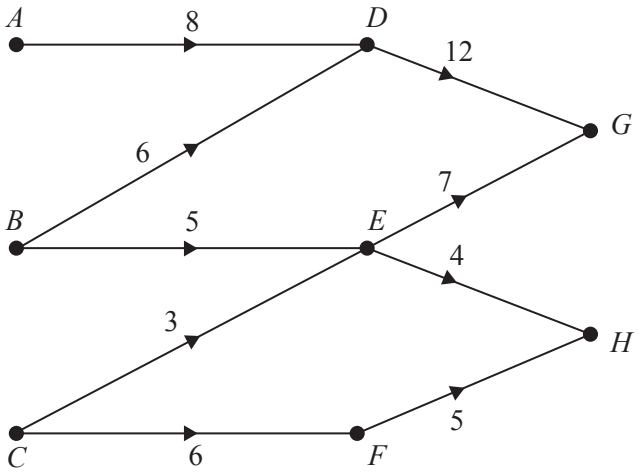
(i) Write down the shortest possible alternating path that starts at A and finishes at either B or T. Hence write down a matching that only excludes Noah and one of the tokens. **[2]**

(ii) Working from the incomplete matching found in part **(i)**, write down the shortest possible alternating path that starts at N and finishes at whichever of B and T has still not been taken. Hence write down a complete matching between the students and the tokens. **[2]**

(iii) By starting at B on Fig. 1, show that there are exactly two complete matchings between the students and the tokens. **[2]**

2 The network models a cooling system in a factory. Coolant starts at A, B and C and flows through the system.

The arcs model components of the cooling system and the weights show the maximum amount of coolant that can flow through each component of the system (measured in litres per second). The arrows show the direction of flow.



(i) Add a supersource, S, and a supersink, T, to the copy of the network in your answer book. Connect S and T to the network using appropriately weighted arcs. **[1]**

(ii) **(a)** Find the capacity of the cut that separates A, B, C and E from D, F, G and H. **[1]**

(b) Find the capacity of the cut that separates A, B, C, D, E and F from G and H. **[1]**

(c) What can you deduce from this value about the maximum flow through the system? **[1]**

(iii) Find the maximum possible flow through the system and prove that this is the maximum. **[3]**

(iv) Describe what effect increasing the capacity of CE would have on the maximum flow. **[2]**

3 Each of five jobs is to be allocated to one of five workers, and each worker will have one job. The table shows the cost, in £, of using each worker on each job. It is required to find the allocation for which the total cost is minimised.

Worker \ Job	Plastering	Rewiring	Shelving	Tiling	Upholstery
Gill	25	50	34	40	25
Harry	36	42	48	44	45
Ivy	27	50	45	42	26
James	40	46	28	45	50
Kelly	34	48	34	50	40

(i) Construct a reduced cost matrix by first reducing rows and then reducing columns. Cross through the 0's in your reduced cost matrix using the least possible number of horizontal or vertical lines. [Try to ensure that the values in your table can still be read.] **[4]**

(ii) Augment your reduced cost matrix and hence find a minimum cost allocation. Write a list showing which job should be given to which worker for your minimum cost allocation, and calculate the total cost in this case. **[4]**

Gill decides that she does not like the job she has been allocated and increases her cost for this job by £100. New minimum cost allocations can be found, each allocation costing just £1 more than the minimum cost allocation found in part **(ii)**.

(iii) Use the grid in your answer book to show the positions of the 0's and 1's in the augmented reduced cost matrix from part **(ii)**. Hence find three allocations, each costing just £1 more than the minimum cost allocation found in part **(ii)** and with Gill having a different job to the one allocated in part **(ii)**. **[5]**

4 Ross and Collwen are playing a game in which each secretly chooses a magic spell. They then reveal their choices, and work out their scores using the tables below. Ross and Collwen are both trying to get as large a score as possible.

		Collwen's choice		
	Score for Ross	*Fire*	*Ice*	*Gale*
Ross's choice	*Fire*	1	7	2
	Ice	6	2	4
	Gale	5	1	3

		Collwen's choice		
	Score for Collwen	*Fire*	*Ice*	*Gale*
Ross's choice	*Fire*	7	1	6
	Ice	2	6	4
	Gale	3	7	5

(i) Explain how this can be rewritten as the following zero-sum game.

		Collwen's choice		
		Fire	*Ice*	*Gale*
Ross's choice	*Fire*	–3	3	–2
	Ice	2	–2	0
	Gale	1	–3	–1

[2]

(ii) If Ross chooses *Ice* what is Collwen's best choice? **[1]**

(iii) Find the play-safe strategy for Ross and the play-safe strategy for Collwen, showing your working. Explain how you know that the game is unstable. **[5]**

(iv) Show that none of Collwen's strategies dominates any other. **[3]**

(v) Explain why Ross would never choose *Gale*, hence reduce the game to a 2×3 zero-sum game, showing the pay-offs for Ross. **[2]**

Suppose that Ross uses random numbers to choose between *Fire* and *Ice*, choosing *Fire* with probability p and *Ice* with probability $1 - p$.

(vi) Use a graphical method to find the optimal value of p for Ross. **[3]**

5 Following a promotion at work, Khalid needs to clear out his office to move to a different building. The activities involved, their durations (in hours) and immediate predecessors are listed in the table below. You may assume that some of Khalid's friends will help him and that once an activity is started it will be continued until it is completed.

Activity		Duration (hours)	Immediate predecessors
A	Sort through cupboard and throw out rubbish	4	-
B	Get packing boxes	1	-
C	Sort out items from desk and throw out rubbish	3	-
D	Pack remaining items from cupboard in boxes	2	*A*, *B*
E	Put personal items from desk into briefcase	0.5	*C*
F	Pack remaining items from desk in boxes	1.5	*B*, *C*
G	Take certificates down and put into briefcase	1	-
H	Label boxes to be stored	0.5	*D*, *F*

(i) Represent this project using an activity network. **[4]**

(ii) Carry out a forward pass and a backward pass through the activity network, showing the early event time and late event time at each vertex of your network. State the minimum project completion time and list the critical activities. **[5]**

(iii) How much longer could be spent on sorting the items from the desk and throwing out the rubbish (activity *C*) without it affecting the overall completion time? **[1]**

Khalid says that he needs to do activities *A*, *C*, *E* and *G* himself. These activities take a total of 8.5 hours.

(iv) By considering what happens if Khalid does *A* first, and what happens if he does *C* first, show that the project will take more than 8.5 hours. **[2]**

(v) Draw up a schedule to show how just two people, Khalid and his friend Mia, can complete the project in 9 hours. Khalid must do *A*, *C*, *E* and *G* and activities cannot be shared between Khalid and Mia. **[2]**

6 The table below shows an incomplete dynamic programming tabulation to solve a maximin problem. Do not write your answer on this copy of the table.

Stage	State	Action	Working	Suboptimal maximin
3	0	0	6	6
	1	0	1	1
	2	0	3	3
2	0	0	min(3, 6) = 3	3
	1	0	min(1, 6) = 1	
		1	min(1, 1) = 1	
		2	min(2, 3) = 2	2
	2	2	min(1, 3) = 1	1
1	0	0	min (3,) =	
		1	min (4,) =	
	1	1	min (3,) =	
	2	1	min (3,) =	
		2	min (1,) =	
0	0	0	min (5,) =	
		1	min (3,) =	
		2	min (4,) =	

(i) Complete the working and suboptimal maximin columns on the copy of the table in your answer book. **[4]**

(ii) Use your answer to part **(i)** to write down the maximin value and the corresponding route. Give your route using (stage; state) variables. **[3]**

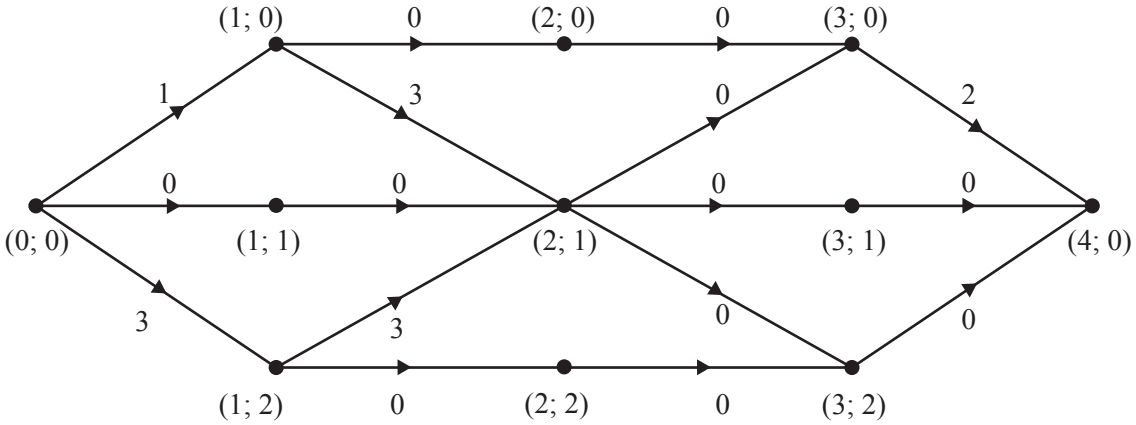
(iii) Draw the network that is represented in the table. **[3]**

The network represents a system of pipes and the arc weights show the capacities of the pipes, in litres per second.

(iv) What does the answer to part **(ii)** represent in this network? **[1]**

Question 6 continues on page 8.

The weights of the arcs in the maximin route are each reduced by the maximin value and then a maximin is found for the resulting network. This is done until the maximin value is 0. At this point the network is as shown below.



(v) (a) Describe how this solves the maximum flow problem on the original network. **[1]**

(b) Draw this maximum flow and draw a cut with value equal to the value of the flow. **[2]**

END OF QUESTION PAPER