Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Further Pure Mathematics FP3 (6669/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper or ag- answer given
- $\llcorner$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to x = ...

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme		Marks
	$P(2, 1, 3)$ and $\mathbf{r}.(\mathbf{i}-2\mathbf{j}-\mathbf{k})=3$		
1.(a)	l is parallel to $\mathbf{i}-2\mathbf{j}-\mathbf{k}$	An appreciation that $\mathbf{i}-2\mathbf{j}-\mathbf{k}$ is the direction of the line (may be implied).	M1
	$"\mathbf{r}"=2\mathbf{i}+\mathbf{j}+3\mathbf{k}+t(\mathbf{i}-2\mathbf{j}-\mathbf{k})$ or $(\mathbf{r}-(2\mathbf{i}+\mathbf{j}+3\mathbf{k}))\times(\mathbf{i}-2\mathbf{j}-\mathbf{k})=\mathbf{0}$ or $\mathbf{r}\times(\mathbf{i}-2\mathbf{j}-\mathbf{k})=(2\mathbf{i}+\mathbf{j}+3\mathbf{k})\times(\mathbf{i}-2\mathbf{j}-\mathbf{k})$ or $\mathbf{r}\times(\mathbf{i}-2\mathbf{j}-\mathbf{k})=5\mathbf{i}+5\mathbf{j}-5\mathbf{k}$	A correct vector **<u>equation</u>** in any form. (Allow any multiple of the direction vector.)	A1
			(2)
(b)	$\begin{pmatrix}2+t\\1-2t\\3-t\end{pmatrix}\begin{pmatrix}1\\-2\\-1\end{pmatrix}=3$	Substitutes a parametric form of their line from part (a) into the equation of the plane. This statement is sufficient.	M1
	$2+t-2(1-2t)-(3-t)=3$	Correct equation **(allow unsimplified)**	A1
	$2+t-2+4t-3+t=3\Rightarrow t=......$	Solves to find a value for t **Dependent on the first M**	**d**M1
	$t=1\Rightarrow l$ meets Π at $\mathbf{r}=3\mathbf{i}-\mathbf{j}+2\mathbf{k}$	Correct position vector (allow as coordinates (3, -1, 2))	A1
			(4)
(c) Way 1	$PQ=\lvert 3\mathbf{i}-\mathbf{j}+2\mathbf{k}-(2\mathbf{i}+\mathbf{j}+3\mathbf{k})\rvert$		
	$=\lvert\mathbf{i}-2\mathbf{j}-\mathbf{k}\rvert=\sqrt{1^2+2^2+1^2}$	Attempts the vector PQ or QP and correct Pythagoras.	M1
	$=\sqrt{6}$	Allow awrt 2.45 or $\frac{6}{\sqrt{6}}$	A1
			(2)
(c) Way 2	$t="1"\Rightarrow\overrightarrow{PQ}="1"\times(\mathbf{i}-2\mathbf{j}-\mathbf{k})$		
	$=\lvert\mathbf{i}-2\mathbf{j}-\mathbf{k}\rvert=\sqrt{1^2+2^2+1^2}$	Attempts the vector PQ using their value for t and their normal vector and correct Pythagoras.	M1
	$=\sqrt{6}$	Allow awrt 2.45 or $\frac{6}{\sqrt{6}}$	A1
			(2)
			Total 8

Question	Scheme		Marks
2.(a)	$\mathbf{MM}^{\mathrm{T}} = \begin{pmatrix}1&0&2\\0&4&1\\0&5&0\end{pmatrix}\begin{pmatrix}1&0&0\\0&4&5\\2&1&0\end{pmatrix}$	Attempts $\mathbf{MM}^{\mathrm{T}}$ or $\mathbf{M}^{\mathrm{T}}\mathbf{M}$ or scalar product of at least one pair of columns or attempts magnitude of at least one column or finds det**M** or attempts $\mathbf{M}^{-1}$	M1
	$= \begin{pmatrix}5&2&0\\2&17&20\\0&20&25\end{pmatrix} \neq \mathbf{I}$ $\therefore \mathbf{M}$ not orthogonal.	or scalar product $\neq 0$ or magnitude $\neq 1$ or det**M** ≠ ±1 (**must see** ±) or $\mathbf{M}^{-1} \neq \mathbf{M}^{\mathrm{T}}$ **<u>and</u>** conclusion. Note that not all of $\mathbf{MM}^{\mathrm{T}}$ or $\mathbf{M}^{-1}$ is necessary and there may be errors but there must be some correct work (at least one correct relevant element). NB det**M** = -5. See extra notes for $\mathbf{M}^{-1}$	A1
			(2)
(b)	$\begin{vmatrix}1-\lambda&0&2\\0&4-\lambda&1\\0&5&-\lambda\end{vmatrix}=0$	This statement is sufficient. (allow other brackets provided the determinant is implied later)	M1
	$(1-\lambda)\left[(4-\lambda)(-\lambda)-5\right]\left(-0(0-0)+2(0-0)\right)=0$		M1
	Attempts characteristic equation (= 0 may be implied by their value(s) for λ) Allow one slip e.g. – usually the omission of the "-5"		
	$(1-\lambda)((4-\lambda)(-\lambda)-5)=0$		
	$\lambda=1$	$\lambda=1$ with **no errors**	A1**cso**
	$\lambda^2-4\lambda-5=0$ $\Rightarrow \lambda=5, \lambda=-1$	M1: Attempts to find the other 2 eigenvalues from their characteristic equation by solving a 3 term quadratic. A1: $\lambda=5, \lambda=-1$	M1A1
			(5)
(c)	$\begin{pmatrix}1&0&2\\0&4&1\\0&5&0\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}x\\y\\z\end{pmatrix}$ or $\begin{pmatrix}0&0&2\\0&3&1\\0&5&-1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$	A correct statement for the eigenvalue 1. (May be implied by correct equations)	M1
	$\alpha(\mathbf{i}+0\mathbf{j}+0\mathbf{k})$ where α is a constant	Any vector of this form.	A1
			(2)
(d)	$\begin{pmatrix}1&0&2\\0&4&1\\0&5&0\end{pmatrix}\begin{pmatrix}t\\2t\\-t\end{pmatrix}=\begin{pmatrix}-t\\7t\\10t\end{pmatrix}$	M1: Attempt to multiply the parametric form or direction of the line by M. **Condone use of 0 for the *x* component but the line <u>must pass</u> <u>through the origin.</u>** A1: Correct image vector with or without "t"	M1A1
	Cartesian equation $\dfrac{x}{-1}=\dfrac{y}{7}=\dfrac{z}{10}$	M1: Correct method to convert to cartesian form of a straight line **<u>passing through the origin.</u>** A1: Correct equations (any multiple)	M1A1
			(4)
			Total 13

Question	Scheme		Marks
3.(a)	$x^2-2x+3=(x-1)^2+2$	M1: Attempt to complete the square. Allow $(x-1)^2+k, k\neq 0$ A1: Correct expression	M1A1
	$\int \frac{1}{\sqrt{(x-1)^2+2}}\text{d}x=\alpha\,\text{arsinh}\left(\text{f}(x)\right)$	Allow $\alpha\ln\left(\text{f}(x)+\sqrt{\left(\text{f}(x)\right)^2+\beta}\right)$ (β>0)	M1
	$\left[\text{arsinh}\left(\frac{x-1}{\sqrt{2}}\right)\right]_1^2=\text{arsinh}\frac{1}{\sqrt{2}}$	Any equivalent exact form. Allow $\ln\left(\frac{1+\sqrt{3}}{\sqrt{2}}\right)$ but no other terms e.g. arsinh(0)	A1
			(4)
(b)	$\text{e}^{2x}\sinh x=\text{e}^{2x}\left(\frac{\text{e}^x-\text{e}^{-x}}{2}\right)$	Substitutes the **correct** exponential of sinh$x$	M1
	$\frac{1}{2}(\text{e}^{3x}-\text{e}^x)$	Correct expression with powers of e combined.	A1
	$\int_0^1\frac{1}{2}(\text{e}^{3x}-\text{e}^x)\text{d}x=\left[\frac{1}{2}\left(\frac{1}{3}\text{e}^{3x}-\text{e}^x\right)\right]_0^1$ $=\frac{1}{2}\left(\frac{1}{3}\text{e}^3-\text{e}^1\right)-\frac{1}{2}\left(\frac{1}{3}\text{e}^0-\text{e}^0\right)$	$\int \text{e}^{px}\,dx=q\text{e}^{px}$ at least once and some correct use of the limits 0 and 1 and subtracts the right way round.	M1
	$=\left(\frac{\text{e}^3}{6}-\frac{\text{e}}{2}+\frac{1}{3}\right)$	Any exact equivalent (allow e^1) but all like terms collected but isw following a correct answer.	A1
			(4)
			Total 8
	(b) Integration by parts way 1		
	$I=\left[\frac{1}{2}\text{e}^{2x}\sinh x\right]_0^1-\int_0^1\frac{1}{2}\text{e}^{2x}\cosh x\text{d}x=\left[\frac{1}{2}\text{e}^{2x}\sinh x\right]_0^1-\left[\frac{1}{4}\text{e}^{2x}\cosh x\right]_0^1+\frac{1}{4}I$		
	$\frac{3}{4}I=\left[\frac{1}{2}\text{e}^{2x}\sinh x\right]_0^1-\left[\frac{1}{4}\text{e}^{2x}\cosh x\right]_0^1$		
	M1: Parts twice in the correct direction A1: A correct expression for I or any constant multiple of I		
	$\int_0^1\text{e}^{2x}\sinh x\text{d}x\;=\frac{4}{3}\left(\frac{1}{2}\text{e}^2\sinh 1-\frac{1}{4}\text{e}^2\cosh 1+\frac{1}{4}\right)$ M1A1 oe		
	M1: Correct use of limits having integrated by parts twice A1: Correct expression (oe)		
	(b) Integration by parts way 2		
	$I=\left[\text{e}^{2x}\cosh x\right]_0^1-\int_0^1 2\text{e}^{2x}\cosh x\text{d}x=\left[\text{e}^{2x}\cosh x\right]_0^1-\left[2\text{e}^{2x}\sinh x\right]_0^1+4I$		
	$-3I=\left[\text{e}^{2x}\cosh x\right]_0^1-\left[2\text{e}^{2x}\sinh x\right]_0^1$		
	M1: Parts twice in the correct direction A1: A correct expression for I or any constant multiple of I $\int_0^1\text{e}^{2x}\sinh x\text{d}x\;=-\frac{1}{3}\left(\text{e}^2\cosh 1-1-2\text{e}^2\sinh 1\right)$		
	M1: Correct use of limits having integrated by parts twice A1: Correct expression (oe)		

Question	Scheme		Marks
4.(a)	$\tanh x = \dfrac{\frac{e^x - e^{-x}}{2}}{\frac{e^x + e^{-x}}{2}}$ or $\dfrac{e^x - e^{-x}}{e^x + e^{-x}}$ or $\dfrac{e^{2x}-1}{e^{2x}+1}$		M1
	Use of the correct exponential form of tanh*x*		
	$1-\tanh^2 x = 1-\left(\dfrac{e^x - e^{-x}}{e^x + e^{-x}}\right)^2 = \dfrac{(e^{2x} + e^{-2x} + 2)-(e^{2x}+e^{-2x}-2)}{(e^x + e^{-x})^2}$		**d**M1
	Attempts $1-\tanh^2 x$ with their tanh*x*, obtains a common denominator and expands the numerator correctly – three terms from $(a+b)^2$ at least once		
	$= \dfrac{2e^x.2e^{-x}}{(e^x + e^{-x})^2}$		
	$= \dfrac{4}{(e^x + e^{-x})^2} = \text{sech}^2 x*$	Correct completion with no errors	A1*
	Allow candidates to process both sides and 'meet in the middle' Note that it is possible to start from sech^2x and obtain 1 – tanh^2x by reversing the above work		
			(3)
(b)	**Ignore any imaginary solutions in (b)**		
	$4\left(\dfrac{e^x - e^{-x}}{2}\right) - 3\left(\dfrac{e^x + e^{-x}}{2}\right) = 3$	Substitutes the **correct** exponential forms for sinh*x* and cosh*x*	M1
	$e^x - 7e^{-x} = 6$		
	$e^{2x} - 6e^x - 7 = 0$	Obtains a quadratic in e^x	M1
	$(e^x+1)(e^x-7) = 0 \Rightarrow e^x = ...$	Attempt to solve their 3TQ in e^x as far as e^x = ..	M1
	$x = \ln 7$ or awrt 1.95		A1
			(4)
			Total 7

	Alternatives for (b)		
	$4\sinh x - 3\cosh x = 3 \Rightarrow 4\sinh x = 3 + 3\cosh x$ $\Rightarrow 7\cosh^2 x - 18\cosh x - 25 = 0$		M1
	M1: Attempt to square correctly and obtains a quadratic in sinh$x$		
	$7\cosh^2 x - 18\cosh x - 25 = 0 \Rightarrow (7\cosh x - 25)(\cosh x + 1) = 0$		
	$\cosh x = \frac{25}{7} \Rightarrow \frac{e^x + e^{-x}}{2} = \frac{25}{7} \Rightarrow 7e^{2x} - 50e^x + 7 = 0$		
	Uses the correct form of cosh$x$ in terms of exponentials to obtain a 3TQ in e^x		M1
	$7e^{2x} - 50e^x + 7 = 0 \Rightarrow (7e^x - 1)(e^x - 7) = 0 \Rightarrow e^x = ...$		M1
	Attempt to solve their 3TQ as far as e^x = ..		
	$x = \ln 7$ or awrt 1.95	No other values	A1
	$4\sinh x - 3\cosh x = 3 \Rightarrow 4\sinh x - 3 = 3\cosh x$ $\Rightarrow 7\sinh^2 x - 24\sinh x = 0$		M1
	M1: Attempt to square correctly and obtains a quadratic in cosh$x$		
	$7\sinh^2 x - 24\sinh x = 0 \Rightarrow \sinh x(7\sinh x - 24) = 0$		
	$\sinh x = \frac{24}{7} \Rightarrow \frac{e^x - e^{-x}}{2} = \frac{24}{7} \Rightarrow 7e^{2x} - 48e^x - 7 = 0$		
	Uses the correct form of sinh$x$ in terms of exponentials to obtain a 3TQ in e^x		M1
	$7e^{2x} - 48e^x - 7 = 0 \Rightarrow (7e^x + 1)(e^x - 7) = 0 \Rightarrow e^x = ...$		M1
	Attempt to solve their 3TQ as far as e^x = ..		
	$x = \ln 7$ or awrt 1.95	No other values	A1
	$4\sinh x - 3\cosh x = 3 \Rightarrow 4\tanh x - 3 = 3\operatorname{sech} x$ $\Rightarrow 25\tanh^2 x - 24\tanh x = 0$		M1
	M1: Attempt to square correctly and obtains a quadratic in tanh$x$		
	$25\tanh^2 x - 24\tanh x = 0 \Rightarrow \tanh x(25\tanh x - 24) = 0$		
	$\tanh x = \frac{24}{25} \Rightarrow \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{24}{25} \Rightarrow e^{2x} = 49$		
	Uses the correct form of tanh$x$ in terms of exponentials to obtain a 2TQ in e^x		M1
	$e^{2x} = 49 \Rightarrow e^x = ...$		M1
	Attempt to solve their 2TQ as far as e^x = ..		
	$x = \ln 7$ or awrt 1.95	No other values	A1

Question	Scheme		Marks
5.	$\frac{dy}{dx}=\left(\frac{1}{1-\frac{x^2}{1+x^2}}\right).\left(\frac{(1+x^2)^{\frac{1}{2}}-x^2(1+x^2)^{-\frac{1}{2}}}{(1+x^2)}\right)$ NB $\frac{(1+x^2)^{\frac{1}{2}}-x^2(1+x^2)^{-\frac{1}{2}}}{(1+x^2)}=\frac{1}{(1+x^2)^{\frac{3}{2}}}$	M1: Correct form for the derivative of artanh$x$ using $\frac{x}{\sqrt{1+x^2}}$. M1: Correct quotient or product rule on $\frac{x}{\sqrt{1+x^2}}$ A1: Completely correct expression	M1M A1
	`$=\frac{1}{\sqrt{1+x^2}}$ **ag**	Correct solution with **no errors** seen	A1
			(4)
			Total 4
	Alternative 1		
	$y=\text{artanh}\frac{x}{\sqrt{1+x^2}}\Rightarrow\tanh y=\frac{x}{\sqrt{1+x^2}}$		
	$\text{sech}^2 y\frac{dy}{dx}=\left(\frac{(1+x^2)^{\frac{1}{2}}-x^2(1+x^2)^{-\frac{1}{2}}}{(1+x^2)}\right)$		
	$\frac{dy}{dx}=\left(\frac{1}{1-\frac{x^2}{1+x^2}}\right).\left(\frac{(1+x^2)^{\frac{1}{2}}-x^2(1+x^2)^{-\frac{1}{2}}}{(1+x^2)}\right)$	M1: Divides by the correct form of sech^2y or their simplified sech^2y in terms of x M1: Correct quotient or product rule A1: Completely correct expression	M1M A1
	Then as above		
	Alternative 2		
	$y=\text{artanh}\frac{x}{\sqrt{1+x^2}}\Rightarrow\tanh y=\frac{x}{\sqrt{1+x^2}}\Rightarrow\tanh^2 y=\frac{x^2}{1+x^2}$		
	$2\tanh y\,\text{sech}^2 y\frac{dy}{dx}=\left(\frac{2x(1+x^2)-2x^3}{(1+x^2)^2}\right)$		
	$\frac{dy}{dx}=\left(\frac{1}{1-\frac{x^2}{1+x^2}}\right)\times\frac{(1+x^2)^{\frac{1}{2}}}{2x}\times\frac{2x(1+x^2)-2x^3}{(1+x^2)^2}$		M1M A1
	M1: Divides by the correct form of sech^2y and tanh$y$ in terms of x M1: Correct quotient or product rule A1: Completely correct expression		
	Then as above		

Question	Scheme		Marks
	In this question condone the use of *a* and/or *b* for *α* and *β*		
6(a)	Gradient $m=\dfrac{2\sin\beta-2\sin\alpha}{3\cos\beta-3\cos\alpha}$	Correct attempt at chord gradient – do not allow slips unless a correct method is clear	M1
	$y-2\sin\alpha=m(x-3\cos\alpha)$ or $y-2\sin\beta=m(x-3\cos\beta)$ or $y=mx+c$ and attempts to find c using P or Q	A correct straight line method using their **chord** gradient and the point P or the point Q	M1
	$y-2\sin\alpha=\dfrac{2\sin\beta-2\sin\alpha}{3\cos\beta-3\cos\alpha}(x-3\cos\alpha)$ $y-2\sin\beta=\dfrac{2\sin\beta-2\sin\alpha}{3\cos\beta-3\cos\alpha}(x-3\cos\beta)$ $y-2\sin\alpha=\dfrac{4\cos\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2}}{-6\sin\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2}}(x-3\cos\alpha)$ $y=\dfrac{2\sin\beta-2\sin\alpha}{3\cos\beta-3\cos\alpha}x+2\sin\alpha-\dfrac{3\cos\alpha\left(2\sin\beta-2\sin\alpha\right)}{3\cos\beta-3\cos\alpha}$ $y=-\dfrac{2\cos\frac{\alpha+\beta}{2}}{3\sin\frac{\alpha+\beta}{2}}x+2\sin\alpha+\dfrac{2\cos\alpha\cos\frac{\alpha+\beta}{2}}{\sin\frac{\alpha+\beta}{2}}$		A1
	A correct equation for the chord **in any form**.		
	$3y\sin\frac{1}{2}(\alpha+\beta)+2x\cos\frac{1}{2}(\alpha+\beta)=6(\cos\alpha\cos\frac{1}{2}(\alpha+\beta)+\sin\alpha\sin\frac{1}{2}(\alpha+\beta))$ or $3y\sin\frac{1}{2}(\alpha+\beta)+2x\cos\frac{1}{2}(\alpha+\beta)=6(\cos\beta\cos\frac{1}{2}(\alpha+\beta)+\sin\beta\sin\frac{1}{2}(\alpha+\beta))$		
	$\frac{x}{3}\cos\frac{1}{2}(\alpha+\beta)+\frac{y}{2}\sin\frac{1}{2}(\alpha+\beta)=\cos\frac{1}{2}(\alpha-\beta)$ **ag**		A1cso
	This is cso – there must no errors in applying the factor formulae and sufficient working must be shown to justify the printed answer but allow $\cos\beta\cos\frac{1}{2}(\alpha+\beta)+\sin\beta\sin\frac{1}{2}(\alpha+\beta)=\cos\frac{\alpha-\beta}{2}$		
			(4)
(b)	$\left(\dfrac{3\cos\alpha+3\cos\beta}{2},\dfrac{2\sin\alpha+2\sin\beta}{2}\right)$ or $(3\cos\frac{1}{2}(\beta+\alpha)\cos\frac{1}{2}(\beta-\alpha),2\sin\frac{1}{2}(\beta+\alpha)\cos\frac{1}{2}(\beta-\alpha))$ or $(3\cos\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta),2\sin\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta))$		B1
	Correct coordinates of mid-point in any form Coordinates must be in this order but condone **outer** brackets missing		
			(1)

Question	Scheme		Marks
(c)	Centre of chord is $(3\cos\frac{1}{2}(\beta+\alpha)\cos\frac{1}{2}(\beta-\alpha), 2\sin\frac{1}{2}(\beta+\alpha)\cos\frac{1}{2}(\beta-\alpha))$ Attempt factor formulae on both coordinates of mid-point at any stage in (c) May be implied by their $\pm\frac{y}{x}$ below		M1
	$\pm\frac{y}{x}=\pm\frac{2\sin\frac{1}{2}(\beta+\alpha)\cos\frac{1}{2}(\beta-\alpha)}{3\cos\frac{1}{2}(\beta+\alpha)\cos\frac{1}{2}(\beta-\alpha)}\left(=\pm\frac{2\sin\frac{1}{2}(\beta+\alpha)}{3\cos\frac{1}{2}(\beta+\alpha)}\right)$ Or $\pm\frac{x}{y}=\pm\frac{3\cos\frac{1}{2}(\beta+\alpha)\cos\frac{1}{2}(\beta-\alpha)}{2\sin\frac{1}{2}(\beta+\alpha)\cos\frac{1}{2}(\beta-\alpha)}\left(=\pm\frac{3\cos\frac{1}{2}(\beta+\alpha)}{2\sin\frac{1}{2}(\beta+\alpha)}\right)$		**d**M1A1
	M1: Obtains an expression for k or $-k$ or $\frac{1}{k}$ or $-\frac{1}{k}$ **Dependent on the previous M1 (factor formulae must have been used)** A1: Correct expression in any form		
	$m=-\frac{2\cos\frac{1}{2}(\beta+\alpha)}{3\sin\frac{1}{2}(\beta+\alpha)}$	Must be seen or used in (c)	B1
	$\frac{\sin\frac{1}{2}(\beta+\alpha)}{\cos\frac{1}{2}(\beta+\alpha)}=-\frac{2}{3m}$ So $\frac{y}{x}=\frac{2}{3}\left(-\frac{2}{3m}\right)\Rightarrow k=\frac{4}{9m}$		A1**cso**
			(5)
			Total 10

Question	Scheme		Marks
7.(a)	$y=\sqrt{r^2-x^2} \Rightarrow \frac{dy}{dx}=-x(r^2-x^2)^{-\frac{1}{2}}$ Or $2x+2y\frac{dy}{dx}=0$	$\frac{dy}{dx}=px(r^2-x^2)^{-\frac{1}{2}}$ Or $px+qy\frac{dy}{dx}=0$	M1
	Attempts to differentiate explicitly or implicitly to give one of the given forms		
	$1+\left(\frac{dy}{dx}\right)^2=1+\frac{x^2}{r^2-x^2}$ or $1+\frac{x^2}{y^2}$	Substitutes their derivative into $1+\left(\frac{dy}{dx}\right)^2$	M1
	$=\frac{r^2-x^2+x^2}{r^2-x^2}=\frac{r^2}{r^2-x^2}$ *	cso	A1*
	This is cso and so there must be no errors e.g. $\frac{dy}{dx}=\frac{x}{y}$ could give the correct answer but loses the A1 but allow to show equivalence of lhs and rhs		
			(3)
(b)	$S=(2\pi)\int y\sqrt{\frac{r^2}{r^2-x^2}}\,dx$	M1: Use of $\int y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$ using their answer to part (a) **(must be *y* and not y^2)** 2π **not** required here A1: Correct expression including 2π (may be implied by later work but must appear before any integration)	M1A1
	$=(2\pi)\int\sqrt{r^2-x^2}\sqrt{\frac{r^2}{r^2-x^2}}\,dx$	Substitutes for *y* in terms of *x*. **Dependent on first M.**	**d**M1
	$=[2\pi rx]_{-r}^{r}$ or $[2\pi rx]_{0}^{r}$	Substitutes the limits *r* and –*r* or 0 and *r* into an expression of the form $\underline{k\pi rx}$ and subtracts. The use of the 0 limit can be taken on trust if omitted. **Dependent on both previous method marks.**	**dd**M1
	If they reach $2\pi r^2$ correctly then double, then some justification is needed e.g. some mention of symmetry		
	$=4\pi r^2$ *	cso	A1
			(5)
	Note that $S=2\times 2\pi\int_0^r y\sqrt{\frac{r^2}{r^2-x^2}}\,dx$ followed by correct work could score full marks as could the correct use of $S=(2\pi)\int y\sqrt{\frac{r^2}{y^2}}dx$		
(c)	arc length $=\frac{\pi}{2}$	Ignore any working	B1
			(1)
			Total 9

<table>
<tr><th>Question</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td rowspan="6">8.(a)</td><td colspan="2">$\mathbf{OA}=\begin{pmatrix}1\\-1\\0\end{pmatrix},\mathbf{OB}=\begin{pmatrix}1\\1\\1\end{pmatrix},\mathbf{OC}=\mathbf{AB}=\begin{pmatrix}0\\2\\1\end{pmatrix},\mathbf{BC}=\begin{pmatrix}-1\\1\\0\end{pmatrix},\mathbf{AC}=\begin{pmatrix}-1\\3\\1\end{pmatrix}$</td><td></td></tr>
<tr><td>$\mathbf{AB}\times\mathbf{AC}=-\mathbf{i}-\mathbf{j}+2\mathbf{k}$
Or e.g.
$\mathbf{BA}\times\mathbf{BC}=\mathbf{i}+\mathbf{j}-2\mathbf{k}$</td><td>M1: Attempt vector product for two sides of the triangle. If the method is unclear, at least 2 components must be correct.
A1: Correct vector</td><td>M1A1</td></tr>
<tr><td>Area ABC = $\frac{1}{2}\sqrt{1^2+1^2+2^2}$</td><td>Attempts $\frac{1}{2}\left|\text{their}\mathbf{AB}\times\mathbf{AC}\right|$
Dependent on the first M</td><td>dM1</td></tr>
<tr><td>$\frac{1}{2}\sqrt{6}$</td><td>Accept equivalents or awrt 1.22</td><td>A1</td></tr>
<tr><td colspan="2">Note that triangles OAB and OBC have the same area but score 0/4
It must be triangle ABC</td><td></td></tr>
<tr><td colspan="2"></td><td>(4)</td></tr>
<tr><td rowspan="2">(b)</td><td>$\mathbf{b}\times\mathbf{c}=(\mathbf{i}+\mathbf{j}+\mathbf{k})\times(2\mathbf{j}+\mathbf{k})=-\mathbf{i}-\mathbf{j}+2\mathbf{k}$</td><td>Attempt $\mathbf{b}\times\mathbf{c}$. If the method is unclear, at least 2 components must be correct.</td><td>M1</td></tr>
<tr><td>$=\left(\frac{1}{6}\right)(\mathbf{i}-\mathbf{j}).(-\mathbf{i}-\mathbf{j}+2\mathbf{k})=\left(\frac{1}{6}\right)(-1+1)=0$</td><td>M1: Attempt scalar product of a with their $\mathbf{b}\times\mathbf{c}$ to obtain a number not a vector.
A1: Obtains = 0 with no errors (allow omission of $\frac{1}{6}$ for all 3 marks)
Just $=\mathbf{a}.(-\mathbf{i}-\mathbf{j}+2\mathbf{k})=0$ would lose the A1</td><td>M1A1</td></tr>
<tr><td></td><td></td><td></td><td>(3)</td></tr>
<tr><td></td><td colspan="2">Alternative</td><td></td></tr>
<tr><td></td><td>$(\mathbf{a}.\mathbf{b}\times\mathbf{c}=)\begin{vmatrix}1&-1&0\\1&1&1\\0&2&1\end{vmatrix}$</td><td>Writes this statement (allow other brackets provided the determinant is implied later)</td><td>M1</td></tr>
<tr><td></td><td>$=(1-2)+1(1)-0=0$</td><td>M1: Clear attempt at determinant
A1: Obtains = 0 with no errors (allow omission of $\frac{1}{6}$ for all 3 marks)</td><td>M1A1</td></tr>
<tr><td rowspan="2">(c)</td><td colspan="2">Volume of tetrahedron (OABC) = 0
$\mathbf{a}=\mathbf{b}-\mathbf{c}$ oe or $\mathbf{c}=\mathbf{b}-\mathbf{a}$ oe
b x c is perpendicular to a or a is parallel to CB
All vectors/points lie in the same plane
OABC is a parallelogram
a, b and c are linearly dependent
Do not isw – if there are contradictory or wrong statements award B0</td><td>B1</td></tr>
<tr><td colspan="2"></td><td>(1)</td></tr>
<tr><td></td><td colspan="2"></td><td>Total 8</td></tr>
</table>

Question	Scheme		Marks
9(a)	$\int (x^2+1)^{-n}\,dx = x(x^2+1)^{-n} + \int xn(x^2+1)^{-n-1}2x\,dx$		M1A1
	M1: Integration by parts in the correct direction A1: Correct expression (If the parts formula is not quoted and the expression is wrong, score M0A0)		
	$= x(x^2+1)^{-n} + 2n\int x^2(x^2+1)^{-n-1}dx$		
	$= x(x^2+1)^{-n} + 2n\int (x^2+1)^{-n} - (x^2+1)^{-n-1}dx$	Use of $x^2 = x^2+1-1$ or equivalent. **Dependent on the previous method mark.**	dM1
	$I_n = x(x^2+1)^{-n} + 2nI_n - 2nI_{n+1}$	Correctly replaces $\int (x^2+1)^{-n}dx$ and $\int (x^2+1)^{-n-1}dx$ by I_n and I_{n+1}. **Dependent on both previous method marks.**	ddM1
	$I_{n+1} = \dfrac{x(x^2+1)^{-n}}{2n} + \dfrac{2n-1}{2n}I_n$	Correct completion to the printed answer with **no errors**.	A1cso
			(5)
(b)	$I_2 = \dfrac{x(x^2+1)^{-1}}{2} + \dfrac{1}{2}I_1$	Correct application of the given reduction formula **using *n* = 1 only**	M1
	$I_1 = \int \dfrac{dx}{x^2+1} = \arctan x\,(+C)$	$I_1 = k\arctan x$ (must be x and not just for arctan)	M1
	$I_2 = \dfrac{x}{2(x^2+1)} + \dfrac{1}{2}\arctan x\,(+C)$	Cao (constant not needed)	A1
			(3)
			Total 8

Extra Notes

2. (a) $\mathbf{M}^{-1} = \frac{1}{5}\begin{pmatrix} 5 & -10 & 8 \\ 0 & 0 & 1 \\ 0 & 5 & -4 \end{pmatrix}$

$\mathbf{M}^{\mathrm{T}}\mathbf{M} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 41 & 4 \\ 2 & 4 & 5 \end{pmatrix}$

3. (b) Parts once then exponentials

$$I = \left[\tfrac{1}{2}e^{2x}\sinh x\right]_0^1 - \int_0^1 \tfrac{1}{2}e^{2x}\cosh x\,dx = \left[\tfrac{1}{2}e^{2x}\frac{e^x - e^{-x}}{2}\right]_0^1 - \int_0^1 \tfrac{1}{2}e^{2x}\frac{e^x + e^{-x}}{2}\,dx$$

M1 integrates by parts and writes cosh$x$ as exponentials
A1 Correct expression

$$= \left[\tfrac{1}{2}e^{2x}\frac{e^x - e^{-x}}{2}\right]_0^1 - \left[\frac{1}{12}e^{2x} + \frac{1}{4}e^x\right]_0^1 = \left[\tfrac{1}{2}\left(\tfrac{1}{3}e^{3x} - e^x\right)\right]_0^1 == \tfrac{1}{2}\left(\tfrac{1}{3}e^3 - e^1\right) - \tfrac{1}{2}\left(\tfrac{1}{3}e^0 - e^0\right)$$

M1 $\int e^{px}\,dx = qe^{px}$ at least once and correct use of the limits 0 and 1

$$= \left(\frac{e^3}{6} - \frac{e}{2} + \frac{1}{3}\right) \text{ A1}$$

Any exact equivalent (allow e^1) but all like terms collected but isw following a correct answer.

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