AQA Maths Pure Core 4

Mark Scheme Pack

2006-2015

Version 1.0: 0106

ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MPC4 Pure Core 4

Mark Scheme

2006 examination – January series

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It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key To Mark Scheme And Abbreviations Used In Marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)(i)	$f(1)=0$	B1	1	
(ii)	$f(-2)=-24+8+14+2=0$	B1	1	
(iii)	$\frac{(x-1)(x+2)}{3x^3+2x^2-7x+2}=\frac{(x-1)(x+2)}{(x-1)(x+2)(ax+b)}$	B1		Recognising $(x-1),(x+2)$ as factors PI
	$ax^3=3x^3$ $\quad -2b=2$	B1	3	a
	$a=3$ $\quad b=-1$	B1		b
				Or By division M1 attempt started M1 complete division A1 Correct answers
(b)	Use $\frac{1}{3}$	B1		
	$3\left(\frac{1}{3}\right)^3+2\left(\frac{1}{3}\right)^2-7\times\frac{1}{3}+d=2$	M1		Remainder Th$^{\underline{M}}$ with $\pm\frac{1}{3,}\pm 3$
	$d=4$	A1F	3	Ft on $-\frac{1}{3}\left(\text{answer}-\frac{4}{9}\right)$ **Or** by division M1 M1 A1 as above
	Total		**8**	
2(a)	$\frac{dy}{dt}=\frac{-2}{t^2}$ $\quad \frac{dx}{dt}=-4$	M1A1		
	$\frac{dy}{dx}=\frac{dy}{dt}\cdot\frac{1}{\frac{dx}{dt}}=\frac{1}{2t^2}$	m1 A1F	4	Use chain rule Follow on use of chain rule (if f (t)) **Or** eliminate t: M1 $y=f(x)$ attempt to differentiate M1A1 chain rule A1F reintroduce t
(b)	$t=2$ $\quad m_T=\frac{1}{8}$	B1F		follow on gradient (possibly used later)
	$x=-5$ $\quad y=2$	B1		
	$y-2=\frac{1}{8}(x+5)$	M1		Their $(x,y),m$
	$x-8y+21=0$	A1F	4	Ft on (x,y) and m
(c)	$x-3=-4t$ $\quad y-1=\frac{2}{t}$	M1		PI
	$(x-3)(y-1)=-4t\times\frac{2}{t}=(-8)$	M1 A1	3	Attempt to eliminate t AG convincingly obtained
	Total		**11**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$R=\sqrt{13}$ Or 3.6	B1	1	
(b)	$\frac{\sin\alpha}{\cos\alpha}=\tan\alpha=\frac{2}{3}$ $\alpha\approx 33.7$	M1A1	2	Allow M1 for tan $\alpha=\frac{-2}{3}$ or $\pm\frac{3}{2}$
				AG convincingly obtained
(c)	maximum value $=\sqrt{13}$	B1F		
	$\cos(\theta+33.7)=1$ $(\theta=-33.7)$	M1		
	$\theta=326.3$	A1	3	AWRT 326
	Total		**6**	
4(a)	$A=80$	B1	1	
(b)	$5000=80\times k^{56}$	M1		$\left\{\right.$SC1 Verification. Need 62.51 or better
	$k=\sqrt[56]{\frac{5000}{80}}\approx 1.07664$	M1A1	3	**Or** using logs: M1 ln $\left(\frac{5000}{80}\right)=56\ln k$
				A1 $k=e^{\ln\left(\frac{62.5}{56}\right)}$
				Or 3/3 for k = 1.076636
				Or 1.076637 seen
(c)(i)	$V=80\times k^{106}=200707$	M1A1	2	200648 using full register k
(ii)	$\ln 10000=\ln k^{t}$	M1		
	$t=\frac{\ln 10000}{\ln k}=124.7\Rightarrow 2024$	M1A1	3	M1 $t\ln k=\ln 10000$
				A1 CAO
				Or trial and improvement M1expression
				M1 125, 124, A1 2024
	Total		**9**	
5(a)(i)	$(1-x)^{-1}=1+(-1)(-x)+\frac{(-1)(-2)}{2}(-x)^2$	M1		First two terms $+kx^2$
	$=1+x+x^2$	A1	2	
(ii)	$\frac{1}{(3-2x)}=\frac{1}{3}\left(1-\frac{2}{3}x\right)^{-1}$	B1		**Or** directly substitute into formula;
	$\approx *\left(1+\frac{2}{3}x+\left(\frac{2}{3}x\right)^2\right)$	M1		M1 power of 3 M1 other coefficients (allow one error) A1 CAO
	$\approx\frac{1}{3}+\frac{2}{9}x+\frac{4}{27}x^2$	A1	3	AG convincingly obtained
(b)	$(1-x)^{-2}=1+(-2)(-x)+\frac{(-2)(-3)(-x)^2}{2}$	M1		First two terms + kx^2
	$=1+2x+3x^2$	A1	2	

MPC4 (Cont)

Q	Solution	Marks	Total	Comments
5(c)	$2x^2-3=$			
	$A(1-x)^2+B(3-2x)(1-x)+C(3-2x)$	M1		**Or** by equating coefficients
	$x=1 \quad -1=C\times 1 \quad x=\frac{3}{2} \quad \frac{3}{2}=A\times\frac{1}{4}$	M1		M1 same A1 collect terms M1 equate coefficients A1 2 correct A1 3 correct
	$C=-1 \qquad A=6$	A1		Follow on *A* and *C*
	$x=0 \quad (-3=6+3B-3)$			
	or other value ⇒ equation in *A, B, C*	m1		
	$B=-2$	A1	5	
(d)	$\frac{6}{3-2x}-\frac{2}{1-x}-\frac{1}{(1-x)^2}$			
	$\approx\frac{6}{3}\left(1+\frac{2}{3}x+\frac{4}{9}x^2\right)-2\left(1+x+x^2\right)$	M1A1F		Follow on *A B C* and expansions
	$-\left(1+2x+3x^2\right) \quad \approx -1-\frac{8}{3}x-\frac{37}{9}x^2$	A1	3	CAO
	Total		**15**	
6(a)	$\cos 2x=2\cos^2 x-1$	B1B1	2	
(b)	$\cos^2 x=\frac{1}{2}(\cos 2x+1)$	M1		Attempt to express $\cos^2 x$ in terms of $\cos 2x$
	$\frac{1}{2}\int_0^{\frac{\pi}{2}}\cos 2x+1\,\mathrm{d}x=\left[\frac{1}{4}\sin 2x+\frac{x}{2}\right]_0^{\frac{\pi}{2}}$	A1 A1		
	$=\frac{\pi}{4}$	M1A1F	5	Use limits. Ft on integer *a*.
	Total		**7**	
7(a)(i)	$\overrightarrow{AB}=\begin{bmatrix}6\\5\\3\end{bmatrix}-\begin{bmatrix}2\\1\\3\end{bmatrix}=\begin{bmatrix}4\\4\\0\end{bmatrix}$	M1 A1	2	Penalise use of co-ordinates at first occurrence only
(ii)	$\begin{bmatrix}4\\4\\0\end{bmatrix}=4\begin{bmatrix}1\\1\\0\end{bmatrix}\Rightarrow$ parallel	E1	1	Needs comment "same direction" Or "same gradient" (**Or** by scalar product)
(iii)	$\begin{bmatrix}2\\-3\\-1\end{bmatrix}=\begin{bmatrix}6\\1\\-1\end{bmatrix}+\lambda\begin{bmatrix}1\\1\\0\end{bmatrix}$	M1		
	is satisfied by $\lambda=-4$	A1	2	$\lambda=-4$ satisfies 2 equations

MPC4 (cont)

Q	Solution	Marks	Total	Comments
(b)(i)	l_2 has equation $\mathrm{r}=\begin{bmatrix}4\\1\\1\end{bmatrix}+\lambda\left[\begin{bmatrix}4\\1\\1\end{bmatrix}-\begin{bmatrix}2\\-3\\-1\end{bmatrix}\right]=\begin{bmatrix}4\\1\\1\end{bmatrix}+\lambda\begin{bmatrix}2\\4\\2\end{bmatrix}$	M1A1	2	**Or** $r=\begin{bmatrix}2\\-3\\-1\end{bmatrix}+t\begin{bmatrix}2\\4\\2\end{bmatrix}$ M1 calculate and use direction vector A1 all correct
(ii)	$\begin{bmatrix}1\\2\\1\end{bmatrix}\bullet\begin{bmatrix}4\\0\\-4\end{bmatrix}=4-4=0$	M1A1		Clear attempt to use directions of AC and l_2 in scalar product
	$\Rightarrow 90°\,(\text{or perpendicular})$	A1F	3	Accept a correct ft value of $\cos\theta$
	Total		**10**	
8(a)	$\int\frac{\mathrm{d}x}{\sqrt{x-6}}\mathrm{d}x=\int -2\mathrm{d}t$	M1		Attempt to separate and integrate
	$2\sqrt{x-6}=-2t+c$	A1A1		c on either side
	$t=0\quad x=70\quad\Rightarrow\quad c=16$	m1A1F		Follow on c from sensible attempt at integrals $\left(\sqrt{\ }\text{not ln}\right)$
	$t=8-\sqrt{x-6}$	A1	6	CAO (or AEF)
(b)(i)	The liquid level stops falling/flowing/ at minimum depth	B1	1	
	$x=22\qquad t=8-\sqrt{22-6}$	M1		Use x = 22 in their equation provided there is a c **Or** start again using limits M1 $2\sqrt{64}-2\sqrt{16}\ =\pm 2t$, A1 $t=4$
	$t=4$	A1	2	CAO
	Total		**9**	
	Total		**75**	

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MPC4

Q	Solution	Marks	Total	Comments
1 (a)(i)	p(2) = 0	B1	1	
(ii)	See $-\frac{1}{2}$	B1		
	$\text{p}\left(-\frac{1}{2}\right)=6\times\left(-\frac{1}{8}\right)-19\times\frac{1}{4}+9\left(-\frac{1}{2}\right)+10$	M1		Use $\pm\frac{1}{2}$
	$=0$	A1	3	Arithmetic to show = 0 and conclusion. Long division : 0/3
(iii)		B1		$x-2$
	$\text{p}(x)=(2x+1)(x-2)(3x-5)$	B1	2	Complete expression
(b)	$\frac{3x(x-2)}{(2x+1)(x-2)(3x-5)}$	M1		For $\frac{3x(x-2)}{\text{their (a)(iii)}}$
	$=\frac{3x}{(2x+1)(3x-5)}$	A1	2	Or $\frac{3x}{6x^2-7x-5}$ No ISW on A1
	Total		**8**	
2(a)	$(1-x)^{-3}=1+(-3)(-x)+\frac{(-3)(-4)(-x)^2}{2}$	M1		$1\pm3x+x^2$ term
	$=1+3x+6x^2$	A1	2	
(b)	$\left(1-\frac{5}{2}x\right)^{-3}=1+3\left(\frac{5}{2}x\right)+6\left(\frac{5}{2}x\right)^2$	M1		$x\to\frac{5}{2}x$, incl. $\left(\frac{5}{2}x\right)^2$ seen or implied (or start again)
	$=1+\frac{15}{2}x+\frac{75}{2}x^2$	A1	2	CAO OE
………… ………… ………… **(c)**	$\left\lvert\frac{5}{2}x\right\rvert<1 \qquad \lvert x\rvert<\frac{2}{5}$	M1A1	2	Sight of $\frac{\pm5}{2}$ or $\frac{\pm2}{5}$
(d)	$=8(1+\frac{15}{2}x+\frac{75}{2}x^2)=8+60x+300x^2$	M1		k × their $\left(1-\frac{5}{2}x\right)^{-3}$
		A1F	2	ft only on $8\left(1-\frac{5}{2}x\right)^{-3}$
	Alternatively, start again:			
	$8\times$ expression or $k\times\left(1-3\left(\pm\frac{5}{2}x\right)\right)$	(M1)		
	CAO	(A1)		
	Total		**8**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$9x^2-6x+5$ $=3(3x-1)(x-1)+A(x-1)+B(3x-1)$	B1		Or $3+\dfrac{6x+2}{(3x-1)(x-1)}$
	$x=1 \qquad x=\dfrac{1}{3}$	M1		Substitute $x=1$ or $x=\dfrac{1}{3}$
	$B=4 \qquad A=-6$	A1A1	4	Or equivalent method (equating coefficients, simultaneous equations)
(b)	$\int=\int 3-\dfrac{6}{3x-1}+\dfrac{4}{x-1}\,\mathrm{d}x$	M1		Attempt to use partial fractions
	$=3x\ldots$	B1		
	$-2\ln(3x-1)+4\ln(x-1)\left(+c\right)$	M1		$p\ln(3x-1)+q\ln(x-1)$ Condone missing brackets
		A1F	4	Follow through on *A* and *B;* brackets needed.
	Total		**8**	
4(a)(i)	$\sin 2x=2\sin x\cos x$	B1	1	
(ii)	$\cos 2x=2\cos^2 x-1$	B1	1	
(b)	$\sin 2x-\tan x=2\sin x\cos x-\dfrac{\sin x}{\cos x}$	M1		Use of their $\cos 2x$ or $\sin 2x$
	$=\sin x\left(2\cos x-\dfrac{1}{\cos x}\right)$	M1		Use of $\tan x=\dfrac{\sin x}{\cos x}$ and the other double angle identity
	$=\sin x\left(\dfrac{2\cos^2 x-1}{\cos x}\right)=\tan x\cos 2x$	A1	3	AG convincingly obtained
(c)	$\tan x\cos 2x=0 \qquad x=180$	B1		Ignore $x=0$, $x=360°$ & any others outside range
	$\cos 2x=0$ or $\cos^2 x=\dfrac{1}{2}$ $\left(\text{or } \sin^2 x=\dfrac{1}{2}\right)$	M1		
	$x=45$	A1		
	$x=135,225,315$	A1	4	CAO max 3/4 for answers in radians
	Total		**9**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$x=1 \quad y^2-y+3-5=0$	M1		
	$(y-2)(y+1)=0$	M1		Attempt to solve quadratic equation with $x = 1$
	$y=2 \quad y=-1$	A1	3	
(b)(i)	$2y\frac{\mathrm{d}y}{\mathrm{d}x}-x\frac{\mathrm{d}y}{\mathrm{d}x}-y+6x=0$	B1B1		$+6x$; $-5 \to 0$
		B1		Chain rule
		M1A1		Product rule (M1 two terms)
	$6x-y+(2y-x)\frac{\mathrm{d}y}{\mathrm{d}x}=0$	A1	6	Factorise and obtain answer given
	Alternative			
	$\frac{\mathrm{d}y}{\mathrm{d}x}(y-x)^2=(y-x)(0-6x)$	(B1)		$5 \to 0$
		(B1)		$-6x$
	$-\left(5-3x^2\right)\left(\frac{\mathrm{d}y}{\mathrm{d}x}-1\right)$	(M1)		Recognisable attempt at quotient rule
		(A1)		Completely correct OE
	$\frac{\mathrm{d}y}{\mathrm{d}x}\left[(y+x)^2+\left(5-3x^2\right)\right]=(y-x)(-6x)$ $+\left(5-3x^2\right)$	(A1)		Factorise out $\frac{\mathrm{d}y}{\mathrm{d}x}$
	Given answer	(A1)		Correct answer from correct working Be convinced
(ii)	$(1,2) \quad \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{4}{3}$	M1		Substitute $x= 1$ and one y value from (a)
	$(1,-1) \quad \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{7}{3}$	A1F	2	Both; follow on candidates y s OE $\frac{-7}{-3}$; 3SF
(iii)	$y-6x=0$	B1		
	$(6x)^2-x\times 6x+3x^2-5=0$	M1		
	$36x^2-6x^2+3x^2-5=0$			
	$33x^2-5=0$	A1	3	AG convincingly obtained
	Total		**14**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$\overrightarrow{OC}=2\begin{bmatrix}3\\2\\-1\end{bmatrix}=\begin{bmatrix}6\\4\\-2\end{bmatrix}$	B1	1	(Penalise coordinates once only)
(ii)	$\overrightarrow{AB}=\begin{bmatrix}3\\2\\-1\end{bmatrix}-\begin{bmatrix}2\\4\\1\end{bmatrix}=\begin{bmatrix}1\\-2\\-2\end{bmatrix}$	M1 A1	 2	$\overrightarrow{OA}-\overrightarrow{OB}$ or $\overrightarrow{OB}-\overrightarrow{OA}$ or 2/3 correct cpts. A0 for line AB
(b)(i)	$AC^2=(6-2)^2+(4-4)^2+(-1-2)^2=25$ $AC=5$	M1 A1	 2	$\lvert$Components of AC$\rvert$ AG
(ii)	$\overrightarrow{AB}\bullet\overrightarrow{AC}=\begin{bmatrix}1\\-2\\-2\end{bmatrix}\bullet\begin{bmatrix}4\\0\\-3\end{bmatrix}=4+6=10$	M1 A1F		Clear attempt to use $\overrightarrow{AB}$ and $\overrightarrow{AC}$ ft $\overrightarrow{AB}$ from a(ii) and/or $\overrightarrow{AC}$ from b(i)
	$3\times5\times\cos\theta=10$	M1		Use of $\lvert a\rvert\,\lvert b\rvert\cos\theta=\mathbf{a.b}$ with one correct $\lvert\ \rvert$ and **a.b** evaluated
	$\theta=48.189\approx48\,°$	A1	4	CAO (AWRT)
	Alternative: use of cos rule Find 3rd side + use cos rule	(M2) (A1F) (A1)		 ft on previously found vectors CAO (AWRT)
(c)	$\overrightarrow{BP}=\begin{bmatrix}\alpha-3\\\beta-2\\\gamma--1\end{bmatrix}$	B1		
	$\begin{bmatrix}4\\0\\-3\end{bmatrix}\bullet\overrightarrow{BP}=0$	M1		Their $\overrightarrow{BP}$
	$4\alpha-3\gamma-15=0$	A1	3	AG convincingly obtained
	Total		**12**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
7	$\int \frac{\mathrm{d}y}{y^2} = \int 6x\ \mathrm{d}x$	M1		Attempt to separate Either dx or dy in right place
	$-\frac{1}{y} = 3x^2\left(+C\right)$	A1A1		$-\frac{1}{y}$; $3x^2$
	$x = 2 \quad y = 1 \qquad C = -13$	M1 A1		Use (2,1) to find a constant. CAO
	$y = \frac{1}{13 - 3x^2}$	A1	6	CAO OE
	Total		**6**	
8(a)(i)	$(5000 - x)$ seen in a product	B1		Could be implied, eg $5000a - xa$
	$\frac{\mathrm{d}x}{\mathrm{d}t} = kx(5000 - x)$	B1	2	
(ii)	$200 = k \times 1000 \times \left(5000 - 1000\right)$	M1		$\frac{\mathrm{d}x}{\mathrm{d}t} = 200, x = 1000$ in their diff. equation Condone ts and $t = 0$ for M1
	$k = 0.00005$	A1	2	CAO OE
(b)(i)	$t = 4\ln\left(\frac{4 \times 2500}{5000 - 2500}\right) = 5.5$ (hours)	M1 A1	2	$x \to 2500\left(\text{or } 4 \ln 4\right)$ CAO
(ii)	$\mathrm{e}^{\frac{30}{4}}$	B1		
	$\mathrm{e}^{7.5} = \frac{4x}{5000 - x}$	M1		OE
	$5000 \times \mathrm{e}^{7.5} = x\left(4 + \mathrm{e}^{7.5}\right)$	m1		Soluble for x
	$x = 4988.96.. \Rightarrow 4989$ rabbits infected	A1	4	Or 4988 or 4990; integer value only
	Total		**10**	
	TOTAL		**75**	

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B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Jan 07

MPC4

Q	Solution	Marks	Total	Comments
1(a)(i)	$\frac{dx}{dt}=2$, $\frac{dy}{dt}=-8t$	B1, B1	2	CAO
(ii)	$\frac{dy}{dx}=\frac{dy}{dt}\frac{dt}{dx}=\frac{-8t}{2}=-4t$	M1 A1F	2	Chain rule in correct form ft on sign coefficient errors (**not** power of *t*)
(b)	$m_T=-4$, $m_N=\frac{1}{4}$	B1F, B1F		ft on $\frac{dy}{dx}$ if f(*t*)
	$x=3$ $y=-3$			
	$\frac{y--3}{x-3}=\frac{1}{4}\Rightarrow\frac{y+3}{x-3}=\frac{1}{4}$	M1 A1	4	Use candidate's (*x*, *y*) and m_N Any correct form; ISW; CAO
(c)	$t=\frac{x-1}{2}$	M1		
	$y=1-4\left(\frac{x-1}{2}\right)^2$	M1A1	3	Substitute for *t* Simplification not required but CAO Or equivalent methods / forms: $y=2x-x^2$, $t^2=\frac{1-y}{4}$, $\left(\frac{x-1}{2}\right)^2=\frac{1-y}{4}$
	Total		**11**	
2(a)	$f\left(\frac{3}{2}\right)=2\left(\frac{3}{2}\right)^3-7\left(\frac{3}{2}\right)^2+13$	M1		Substitute $\pm\frac{3}{2}$ in f(*x*)
	$=4$	A1	2	
(b)	$g\left(\frac{3}{2}\right)=0\Rightarrow d+4=0\Rightarrow d=-4$	M1A1	2	AG (convincingly obtained) SC Written explanation with $g\left(\frac{3}{2}\right)=0$ not seen/clear E2,1,0
(c)	$a=-2$, $b=-3$	B1, B1	2	Inspection expected By division: M1 – complete method A1 CAO Multiply out and compare coefficients: M1 – evidence of use A1 – both *a* and *b* correct
	Total		**6**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\cos 2x = 1 - 2\sin^2 x$	B1	1	
(b)(i)	$3\sin x - \cos 2x = 3\sin x - (1 - 2\sin^2 x)$	M1		Candidate's $\cos 2x$ or $\sin^2 x$
	$= 3\sin x - 1 + 2\sin^2 x$	A1	2	AG
(ii)	$2\sin^2 x + 3\sin x - 2 = 0$	M1		Soluble quadratic form
	$(2\sin x - 1)(\sin x + 2) = 0$	M1		Attempt to solve (allow one error in formula, allow sign errors)
	$\sin x = \frac{1}{2} \quad x = 30 \quad x = 150$	M1 A1	 4	$\sin^{-1}$ and two solutions ($0° < x < 360°$) A0 if radians
	Allow misread for $2\sin^2 x + 3\sin x - 1 = 0$	(M1)		Soluble quadratic form
	$\sin x = \frac{-3 \pm \sqrt{17}}{4}$	(M1)		Use of formula (allow one error)
	$x = 16.3°,\ 163.7°$	(A1)		Max 3/4
(c)	$\int \frac{1}{2}(1 - \cos 2x) = \frac{x}{2} - \frac{\sin 2x}{4} (+c)$	M1A1	2	M1 – solve integral, must have 2 terms for $\sin^2 x$ from (a)
			9	
4(a)(i)	$\frac{3x-5}{x-3} = 3 + \frac{4}{x-3}$	B1, B1	2	By division: B1 for 3, B1 for $\frac{4}{x-3}$ or $B = 4$ By partial fractions: M1 multiply by $x - 3$ and using 2 values of x, A1 both correct
(ii)	$\int 3 + \frac{4}{x-3}\,dx = 3x + 4\ln(x-3)(+c)$	M1A1F	2	M1 $\int 3 + \frac{4}{x-3}\,dx$ and attempt at integrals ft on A and B; condone omission of brackets around $x - 3$
	Alternative: By substitution $u = x - 3$			
	$\int \frac{3x-5}{x-3}\,dx = \int \frac{3u+4}{u}\,du$	(M1)		Integral in terms of u
	$= 3(x-3) + 4\ln(x-3)$	(A1)		Correct, in x
(b)(i)	$6x - 5 = P(2x-5) + Q(2x+5)$	M1		Clear evidence of use of cover-up rule M2
	$x = \frac{5}{2} \qquad x = -\frac{5}{2}$	m1		
	$10 = 10Q \qquad -20 = -10P$			
	$Q = 1 \qquad P = 2$	A1	3	
(ii)	$\int \frac{2}{2x+5} + \frac{1}{2x-5}\,dx$	M1		Attempt at ln integral $\left(a\ln(2x+5) + b\ln(2x-5)\right)$
	$\ln(2x+5) + \frac{1}{2}\ln(2x-5)(+c)$	M1 A1F	 3	 ft on P and Q; must have brackets
	Total		**10**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$(1+x)^{\frac{1}{3}} = 1+\frac{1}{3}x+\frac{1}{3}\left(-\frac{2}{3}\right)\frac{1}{2}x^2$	M1		$1+\frac{1}{3}x+kx^2$
		A1	2	
(b)(i)	$\sqrt[3]{8}\left(1+\frac{3}{8}x\right)^{\frac{1}{3}}$	B1		$8^{\frac{1}{3}}(1+kx)^{\frac{1}{3}}$
	$= 2\left(1+\frac{1}{3}\left(\frac{3}{8}x\right)-\frac{1}{9}\left(\frac{3}{8}x\right)^2\right)$	M1		Replacing *x* with *kx* in answer to (a)
	$= 2+\frac{1}{4}x-\frac{1}{32}x^2$	A1	3	For numerical expression which would evaluate to answer given
	Alternative: B1 – all powers of 8 correct: $8^{\frac{1}{3}}\ 8^{-\frac{2}{3}}\ 8^{-\frac{5}{3}}$ M1 – powers of 3*x* (condone $3x^2$) $2+\frac{1}{8^{\frac{2}{3}}}x-\frac{1}{9}\frac{1}{8^{\frac{5}{3}}}9x^2$ A1 – see some arithmetic processing must see 9s in last term			
(ii)	$x=\frac{1}{3}$: $\sqrt[3]{8+1} = 2+\frac{1}{4}\times\frac{1}{3}-\frac{1}{32}\times\left(\frac{1}{3}\right)^2$	M1		Using $x=\frac{1}{3}$ in given answer
	$\sqrt[3]{9} = \frac{576+24-1}{288} = \frac{599}{288}$	A1	2	Any correct numerical expression = $\frac{599}{288}$
	Total		**7**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$\overrightarrow{BA}=\begin{bmatrix}3\\-2\\4\end{bmatrix}-\begin{bmatrix}5\\4\\0\end{bmatrix}=\begin{bmatrix}-2\\-6\\4\end{bmatrix}$	M1A1	2	Attempt $\pm\overrightarrow{BA}$ $(OA-OB$ or $OB-OA)$
(ii)	$\overrightarrow{BC}=\begin{bmatrix}6\\2\\-4\end{bmatrix}$	B1		Allow $\overrightarrow{CB}$; or $\begin{bmatrix}-6\\-2\\4\end{bmatrix}=\overrightarrow{BC}$ or $\overrightarrow{CB}=\begin{bmatrix}6\\2\\-4\end{bmatrix}$ May not see explicitly
	$\left\lvert\overrightarrow{BA}\right\rvert\left(=\sqrt{(-2)^2+(-6)^2+(4)^2}\right)=\sqrt{56}$	B1F		Calculate modulus of $\overrightarrow{BA}$ or $\overrightarrow{BC}$; for finding modulus of one of vectors they have used
	$\overrightarrow{BA}\bullet\overrightarrow{BC}=\begin{bmatrix}-2\\-6\\4\end{bmatrix}\bullet\begin{bmatrix}6\\2\\-4\end{bmatrix}=-12-12-16$	M1		Attempt at $\overrightarrow{BA}\bullet\overrightarrow{BC}$ with numerical answer; or $\overrightarrow{AB}\bullet\overrightarrow{CB}$
		A1		for –40, or correct if done with multiples of vectors
	$\cos ABC=\dfrac{-40}{\sqrt{56}\sqrt{56}}=-\dfrac{5}{7}$	A1	5	AG (convincingly obtained)
				Cosine rule: M1 attempt to find 3 sides A1 lengths of sides M1 cosine rule A1F correct A1 rearrange to get $\cos ABC=\dfrac{-5}{7}$ (ft on length of sides)

MPC4 (cont)

Q	Solution	Marks	Total	Comments
6 (cont) (b)(i)	$\begin{bmatrix} 8 \\ -3 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 11 \\ 6 \\ -4 \end{bmatrix} \quad (\lambda = 3)$	M1A1	2	$\lambda = 3$ verified in three equations M1 for $\begin{cases} 11 = 8 + \lambda \\ 6 = -3 + 3\lambda \\ -4 = 2 - 2\lambda \end{cases}$ A1 for $\lambda = 3$ shown for all three equations $\lambda \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 11 \\ 6 \\ -4 \end{bmatrix} - \begin{bmatrix} 8 \\ -3 \\ 2 \end{bmatrix} \therefore \lambda = 3$ M1A1 SC: $\lambda = 3$ written and nothing else: SC1
(ii)	$\begin{bmatrix} 2 \\ 6 \\ -4 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$ $\therefore$ same direction or same gradient or parallel	E1	1	
(c)	$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{BA}$	B1		PI; $\overrightarrow{OD}$ = correct vector expression which may involve $\overrightarrow{AD}$
	$= \begin{bmatrix} 11 \\ 6 \\ -4 \end{bmatrix} + \begin{bmatrix} -2 \\ -6 \\ 4 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \\ 0 \end{bmatrix} \quad D \text{ is } (9, 0, 0)$	M1A1	3	M1 for substituting into vector expression for $\overrightarrow{OD}$ NMS 3/3
	Total		**13**	
7(a)	$\tan(x + x) = \dfrac{\tan x + \tan x}{1 - \tan x \tan x} \left(= \dfrac{2\tan x}{1 - \tan^2 x} \right)$	M1 A1	2	$A = B = x$ used
(b)	$2 - 2\tan x - \dfrac{2\tan x(1 - \tan^2 x)}{2\tan x}$	M1		Substitute from (a)
	$2 - 2\tan x - (1 - \tan x)(1 + \tan x)$	M1		Simplification $2 - 2\tan x - (1 - \tan^2 x)$
	$(1 - \tan x)(2 - (1 + \tan x))$	M1		$2 - 2\tan x - 1 + \tan^2 x$
	$(1 - \tan x)^2$	A1	4	AG (convincingly obtained) $= (\tan x - 1)^2 = (1 - \tan x)^2$ Any equivalent method
	Total		**6**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
8(a)(i)	$\int \frac{\mathrm{d}y}{y} = \int \sin t\,\mathrm{d}t$	M1		Attempt to separate and integrate
	$\ln y = -\cos t + C$	A1,A1		A1 for ln y; A1 for –cos$t$; condone missing C
	$y = A\mathrm{e}^{-\cos t}$	A1	4	A present; or $y = \mathrm{e}^{-\cos t + C}$
(ii)	$y = 50,\ t = \pi:\quad 50 = A\mathrm{e}^{-\cos\pi} = A\mathrm{e}$	M1 A1		Substitute y = 50, $t = \pi$ to find constant Can have $50 = \mathrm{e}^{1+C}$ if substituted in above $\mathrm{e}^{C} = \frac{50}{\mathrm{e}}$
	$y = 50\mathrm{e}^{-1}\mathrm{e}^{-\cos t}$	A1	3	AG (convincingly obtained)
	Alternative: Must have a constant in answer to (a)(i) $y = A\mathrm{e}^{-\cos t}$ or $y = \mathrm{e}^{-\cos t + c}$ or $\ln y = -\cos t + c$			**Alternative:** Substitute y = 50, $t = \pi$ into $\ln y = -\cos t + c$ M1 $\ln y = -\cos t + \ln 50 - 1$ A1
	$50 = A\mathrm{e}^{-\cos\pi}$ $50 = \mathrm{e}^{-\cos\pi + c}$ $\ln 50 = -\cos\pi + c$	(M1)		$\ln\frac{y}{50} = -1 - \cos t$ (AG) A1
	$50 = A\mathrm{e}$ $50 = \mathrm{e}^{1+c}$ $\ln y = -\cos t + \ln 50 - 1$	(A1)		
	$y = 50\mathrm{e}^{-1-\cos t}$ $y = \mathrm{e}^{-\cos t}\frac{50}{\mathrm{e}}$ $\ln\left(\frac{y}{50}\right) = -1 - \cos t$	(A1)		
(b)(i)	$t = 6:\ \ y = 50\mathrm{e}^{-1}\mathrm{e}^{-\cos 6} = 7.0417... \approx 7\text{cm}$	M1A1	2	Degrees 6.8 SC1 7 or 7.0 for A1
(ii)	$t = \pi \Rightarrow \left(\sin t = 0 \Rightarrow\right) \frac{\mathrm{d}y}{\mathrm{d}t} = 0$	B1		Condone x for t
	$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = y\cos t + \frac{\mathrm{d}y}{\mathrm{d}t}\sin t$	M1		For attempt at product rule including $\frac{\mathrm{d}y}{\mathrm{d}t}$ term; must have $\frac{\mathrm{d}^2y}{\mathrm{d}t^2} =$
		A1		
	$t = \pi$ $\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = y\cos\pi + \frac{\mathrm{d}y}{\mathrm{d}t}\sin\pi$ $= -50 \Rightarrow$ max	A1	4	Accept = – y, with explanation that y is never negative

MPC4 (cont)

Q	Solution	Marks	Total	Comments
8(b)(ii) (cont)	**Alternative:** $y = 50\mathrm{e}^{-(1+\cos t)} = \frac{50}{\mathrm{e}}\mathrm{e}^{-\cos t}$ $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{50}{\mathrm{e}}\mathrm{e}^{-\cos t} \times \sin t = 0 \text{ at } t = \pi$ $\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} = \frac{50}{\mathrm{e}}\mathrm{e}^{-\cos t} \times \cos t + \frac{50}{\mathrm{e}}\mathrm{e}^{-\cos t} \times \sin^2 t$ Substitute $t = \pi \to -50 \Rightarrow$ max	(B1) (M1) (A1) (A1)		Attempt at product rule Correct
	Total		**13**	
	TOTAL		**75**	

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ASSESSMENT and
QUALIFICATIONS
ALLIANCE

General Certificate of Education

Mathematics 6360

MPC4 Pure Core 4

Mark Scheme

2007 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
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Otherwise we require evidence of a correct method for any marks to be awarded.

June 07

MPC4

Q	Solution	Marks	Total	Comments
1(a)	$2\left(-\frac{1}{2}\right)^2+\left(-\frac{1}{2}\right)-3=-3$	M1A1	2	use of $\pm\frac{1}{2}$
				SC NMS –3 1/2 No ISW, so subsequent answer "3" AO
	Alt algebraic division:			
	$\begin{array}{r} x \\ 2x+1\overline{\smash{)}2x^2+x-3} \\ \underline{2x^2+x} \end{array}$	(M1)		complete division with integer remainder
	-3	(A1)	(2)	remainder = -3 stated, or –3 highlighted
	Alt $\frac{x(2x+1)-3}{2x+1}$	(M1)		attempt to rearrange numerator with $(2x+1)$ as a factor
		(A1)	(2)	remainder $=-3$ stated, or –3 highlighted
(b)	$\frac{(2x+3)(x-1)}{(x+1)(x-1)}$	B1 B1		$\left.\begin{array}{l}\text{numerator}\\ \text{denominator}\end{array}\right\}$ not necessarily in fraction
	$=\frac{2x+3}{x+1}$	B1	3	CAO in this form. Not $\frac{2x+3}{x+1}\frac{\cancel{x-1}}{\cancel{x-1}}$
(b)	**Alternative** $\frac{2x^2-2+x-1}{x^2-1}$			
	$=2+\frac{x-1}{x^2-1}$	(M1)		
	$=2+\frac{x-1}{(x-1)(x+1)}$	(B1)		
	$=2+\frac{1}{x+1}$	(A1)	(3)	
	Total		**5**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
2(a)(i)	$(1+x)^{-1}=1+(-1)x+px^2+qx^3$	M1		$p\neq 0,\ q\neq 0$
	$=1-x+x^2-x^3$	A1	2	SC 1/2 for $=1-x+px^2$
(ii)	$(1+3x)^{-1}=1-3x+(3x)^2-(3x)^3$	M1		x replaced by $3x$ in candidate's (a)(i);condone missing brackets
	$=1-3x+9x^2-27x^3$	A1	2	CAO SC x^3-term : $1-3x+\overset{3}{9}x^2$ 1/2
	Alt (starting again)			
	$(1+3x)^{-1}=1-(3x)+$ $\frac{(-1)(-2)(3x)^2}{2!}+\frac{(-1)(-2)(-3)(3x)^3}{3!}$	(M1)		condone missing brackets accept 2 for 2!, 3.2 for 3!
	$=1-3x+9x^2-27x^3$	(A1)	(2)	CAO
(b)	$\frac{1+4x}{(1+x)(1+3x)}=\frac{A}{1+x}+\frac{B}{1+3x}$	M1		correct partial fractions form, and multiplication by denominator
	$1+4x=A(1+3x)+B(1+x)$			
	$x=-1,\ x=-\frac{1}{3}$	m1		Use (any) two values of x to find A and B
	$A=\frac{3}{2}\ ,\ B=-\frac{1}{2}$	A1	3	A and B both correct
	Alt:			
	$\frac{1+4x}{(1+x)(1+3x)}=\frac{A}{1+x}+\frac{B}{1+3x}$	(M1)		correct partial fractions form, and multiplication by denominator
	$1+4x=A(1+3x)+B(1+x)$			
	$A+B=1,\ 3A+B=4$	(m1)		Set up and solve
	$A=\frac{3}{2}\ ,\ B=-\frac{1}{2}$	(A1)	(3)	A and B both correct
(c)(i)	$\frac{1+4x}{(1+x)(1+3x)}=\frac{3}{2(1+x)}-\frac{1}{2(1+3x)}$	M1		
	$=\frac{3}{2}\left(1-x+x^2-x^3\right)-\frac{1}{2}\left(1-3x+9x^2-27x^3\right)$	m1		multiply candidate's expansions by A and B, and expand and simplify
	$=1-3x^2+12x^3$	A1	3	CAO SC A and B interchanged, treat as miscopy. $\left(1-4x+13x^2-40x^3\right)$
	Alt:			
	$=\frac{1+4x}{(1+x)(1+3x)}=(1+4x)(1+x)^{-1}(1+3x)^{-1}$			
	$=(1+4x)\left(1-x+x^2-x^3\right)\left(1-3x+9x^2-27x^3\right)$	(M1)		write as product, using expansions condone missing brackets on (1 + 4x) only
	$=1-4x+13x^2-40x^3+4x-16x^2+52x^3$	(m1)		attempt to multiply the three expansions up to terms in x^3
	$=1-3x^2+12x^3$	(A1)	(3)	CAO
(ii)	$\lvert x\rvert<1$ and $\lvert 3x\rvert<1$	M1		OE and nothing else incorrect
	$\lvert x\rvert<\frac{1}{3}$ (0.33)	A1	2	OE Condone ≤
	Total		**12**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$R=5$	B1		
	$\tan\alpha=\frac{3}{4}$ (OE) $\alpha=36.9°$ (ISW 216.9)	M1A1	3	SC1 $\tan\alpha=\frac{4}{3}, \alpha=53.1°$ R, α PI in (b)
(b)	$\cos(x-\alpha)=\frac{2}{R}$	M1		
	$x-\alpha=66.4°$	A1		
	$x=103.3°$	A1F		
	$x=330.4°$	A1F	4	accept $330.5°$, -1 each extra ft on acute α
(c)	minimum value $=-5$	B1F		ft on R
	$\cos(x-36.9)=-1$	M1		SC $\cos(x+36.9)$ treat as miscopy
	$x=216.9°$	A1	3	216.9 or better accept graphics calculator solution to this accuracy SC Find max: max = 5 at $(x+36.9)$ stated 1/3 Max 8/10 for work in radians
	Total		**10**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$t=0:\ x=3$	B1	1	
(ii)	$t=14:\ x=15-12\mathrm{e}^{-1}$	M1		or $15-12\,\mathrm{e}^{\frac{-14}{14}}$
	= 10.6	A1	2	CAO
(b)(i)	$-5=-12\mathrm{e}^{-\frac{t}{14}}$	M1		substitute $x=10$; rearrange to form $p=q\mathrm{e}^{-\frac{t}{14}}$
	$\ln\left(\frac{5}{12}\right)=-\frac{t}{14}$ (OE)	m1		take lns correctly
	$t=14\ln\left(\frac{12}{5}\right)$	A1	3	must come from correct working
(ii)	$t=12.256...\approx 12$ days	B1F	1	ft on *a, b* if *a* > *b*; accept *t* = 12 NMS Accept 12 from incorrect working in b(i) Accept 13 if 12.2 or 12.3 seen
(c)(i)	$\frac{\mathrm{d}x}{\mathrm{d}t}=-\frac{1}{14}\times-12\mathrm{e}^{-\frac{t}{14}}$	M1		differentiate; allow sign error condone $\frac{\mathrm{d}y}{\mathrm{d}x}$ used consistently
	$=-\frac{1}{14}(x-15)$	m1		Or $\frac{1}{14}\left(12\mathrm{e}^{-\frac{t}{14}}\right)$ and $12\mathrm{e}^{-\frac{t}{14}}=15-x$ seen
	$=\frac{1}{14}(15-x)$	A1	3	AG – be convinced CSO
	Alt: $t=-14\ln\left(\frac{15-x}{12}\right)$	(M1)		attempt to solve given equation for *t*
	$\frac{\mathrm{d}t}{\mathrm{d}x}=\frac{-14\left(-\frac{1}{12}\right)}{\left(\frac{15-x}{12}\right)}$	(m1)		differentiate wrt *x*, with $\frac{1}{\frac{15-x}{12}}$ seen; OE
	$\frac{\mathrm{d}t}{\mathrm{d}x}=\frac{14}{15-x}\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{1}{14}(15-x)$	(A1)	(3)	AG – be convinced
	Alt: (backwards) $\int\frac{\mathrm{d}x}{15-x}=\int\frac{\mathrm{d}t}{14}=\pm14\ln\left(15-x\right)=t+c$	(M1)		
	Use $\left(0,3\right):-14\ln\left(15-x\right)+14\ln12=t$	(m1)		
	Solve for x: $x=15-12\mathrm{e}^{-\frac{t}{14}}$	(A1)	(3)	All steps shown
(ii)	rate of growth = 0.5 (cm per day)	B1	1	Accept $\frac{7}{14}$
	Total		**11**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$x=1,\ 5a^2-a-4=0$	M1		condone y for a
	$(5a+4)(a-1)=0,\quad a=1$	A1	2	AG – be convinced, both factors seen or $a=-\frac{4}{5}$ or $1\Rightarrow a=1$ A0 for 2 positive roots (substitute $(1,1)\Rightarrow 5=5$ no marks)
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}+4$	B1B1		(Ignore '$\frac{\mathrm{d}y}{\mathrm{d}x}=$' if not used, otherwise loses final A1)
	$=10xy^2+10x^2y\frac{\mathrm{d}y}{\mathrm{d}x}$	M1		attempt product rule, see two terms added
		M1		chain rule, $\frac{\mathrm{d}y}{\mathrm{d}x}$ attached to one term only
		A1		condone 5 × 2 for 10
	$x=1,y=1\quad \frac{\mathrm{d}y}{\mathrm{d}x}+4=10+10\frac{\mathrm{d}y}{\mathrm{d}x}$	M1		two terms, or more, in $\frac{\mathrm{d}y}{\mathrm{d}x}$
	$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{6}{9}=\left(-\frac{2}{3}\right)$	A1	7	CSO
	Alt (for last two marks)			
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{10xy^2-4}{1-10x^2y}$	(M1)		find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x, y and substitute $x=1,\ y=1$ must be from expression with two terms or more in $\frac{\mathrm{d}y}{\mathrm{d}x}$
	$(1,1)\Rightarrow\frac{10-4}{1-10}=-\frac{6}{9}$	(A1)		
(c)	$\frac{y-1}{x-1}=-\frac{2}{3}$ (OE)	B1F	1	ft on gradient ISW after any correct form
	Total		**10**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$\frac{\mathrm{d}x}{\mathrm{d}\theta}=-\sin\theta \qquad \frac{\mathrm{d}y}{\mathrm{d}\theta}=2\cos 2\theta$	B1 B1	2	
(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{2\cos 2\theta}{\sin\theta}, \ \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{2\cos\frac{\pi}{3}}{\sin\frac{\pi}{6}}=-2$	M1		use chain rule $\text{their } \frac{\mathrm{d}y}{\mathrm{d}\theta} \Big/ \text{their } \frac{\mathrm{d}x}{\mathrm{d}\theta}$ and substitute $\theta=\frac{\pi}{6}$
		A1	2	
(b)	$y=2\sin\theta\cos\theta=2\sqrt{1-\cos^2\theta}\cos\theta$	B1 B1		use $\sin 2\theta=2\sin\theta\cos\theta$ use $\sin^2\theta=1-\cos^2\theta$
	$y=2\sqrt{1-x^2}\ x$	M1		$\sin\theta, \cos\theta$ in terms of x
	$y^2=4x^2\left(1-x^2\right)$	A1	4	all correct CSO
	Alt			
	$y^2=\sin^2 2\theta=\left(2\sin\theta\cos\theta\right)^2$	(B1)		use of double angle formula
	$=(4)\sin^2\theta\cos^2\theta=(4)\left(1-\cos^2\theta\right)\cos^2\theta$	(B1)		use of $s^2+c^2=1$ to eliminate $\sin\theta$
	$=(4)\left(1-x^2\right)x^2$	(M1)		Substitute $\cos\theta$ for x
	$=4\left(1-x^2\right)x^2$	(A1)	(4)	CSO
	Total		**8**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\begin{bmatrix}3\\-3\\-1\end{bmatrix}\bullet\begin{bmatrix}1\\2\\-3\end{bmatrix}=3-6+3=0$	M1		attempt at sp, 3 terms, added
	$=0\Rightarrow$ perpendicular	A1	2	$=0\Rightarrow$ perpendicular seen $\left(\text{or } \cos\theta=0\Rightarrow\theta=90^\circ\right)$ Allow $\begin{array}{c}3\\-6\\3\\\hline 0\end{array}$ but not $\begin{bmatrix}3\\-6\\3\end{bmatrix}=0$
(b)	$8+3\lambda=-4+\mu$ $6-3\lambda=2\mu$ $-9-\lambda=11-3\mu$	M1		set up any two equations
	$\lambda=-2,\mu=6$	m1 A1		solve for λ and μ
	verify third equation	m1		substitute λ,μ in third equation
	intersect at $(2,12,-7)$	A1	5	CAO
	Alt (for last two marks)			
	substitute λ into l_1 and μ into l_2	(m1)		
	intersect at $(2,12,-7)$, condone $\begin{pmatrix}2\\12\\-7\end{pmatrix}$	(A1)		$(2,12,-7)$ found from both lines Note: working for (b) done in (a): award marks in (b)
7(c)	$\overrightarrow{AP}=\begin{pmatrix}6\\12\\-18\end{pmatrix}$	M1		$\overrightarrow{AP}=\pm\left\{\text{their } \overrightarrow{OP}-\begin{pmatrix}-4\\0\\11\end{pmatrix}\right\}$
	$AP^2=504$	A1F		ft on P
	$AB^2=2AP^2$	M1		Calculate AB^2
	$AB = 12\sqrt{7}$	A1	4	OE accept 31.7 or better
	Total		**11**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$\int \frac{1}{\sqrt{1+2y}}\,\mathrm{d}y = \int \frac{1}{x^2}\,\mathrm{d}x$	M1		attempt to separate and integrate
	$\int \frac{1}{\sqrt{1+2y}}\,\mathrm{d}y = k\sqrt{1+2y}$	m1		
		A1		OE A1 for $\sqrt{1+2y}$ depends on both Ms
	$\sqrt{1+2y} = -\frac{1}{x}(+c)$	A1		A1 for $-\frac{1}{x}$ depends on first M1 only
	$x=1, y=4 \Rightarrow c=4$	m1		$+c$ must be seen on previous line
		A1F	6	ft on k and $\pm\frac{1}{x}$ only
(b)	$1+2y = \left(4-\frac{1}{x}\right)^2$	m1		need $k\sqrt{1+2y} =$ 'x expression with $+ c$' **and** attempt to square both sides
	$2y = 15 + \frac{1}{x^2} - \frac{8}{x}$	A1	2	terms on RHS in any order AG – be convinced CSO
	Total		**8**	
	TOTAL		**75**	

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ASSESSMENT and
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General Certificate of Education

Mathematics 6360

MPC4 Pure Core 4

Mark Scheme

2008 examination - January series

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Registered address: AQA, Devas Street, Manchester M15 6EX *Dr Michael Cresswell Director General*

Key to mark scheme and abbreviations used in marking

M	mark is for method
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A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	C	candidate
PI	possibly implied	Sf	significant figure(s)
SCA	substantially correct approach	Dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC4

Q	Solution	Marks	Total	Comments
1(a)	$3 = k\left(3 + x + 3 - x\right)$	M1		OE $\frac{A}{3-x} + \frac{B}{3+x} \Rightarrow$ $6A = 3$ $6B = 3$
	$k = \frac{1}{2}$	A1	2	or eg put $x = 0$, $\frac{3}{9} = k\left(\frac{1}{3} + \frac{1}{3}\right) \Rightarrow k = \frac{1}{2}$
(b)	$\int_1^2 \frac{3}{9 - x^2}\,\mathrm{d}x = -\frac{1}{2}\ln\left(3 - x\right) + \frac{1}{2}\ln\left(3 + x\right)$	M1 A1F		$a\ln(3 \pm x)$ ft on k
	$= \frac{1}{2}\left(\left(\ln 5 - \ln 1\right) - \left(\ln 4 - \ln 2\right)\right) = \frac{1}{2}\ln\left(\frac{5}{2}\right)$	A1F	3	accept $\ln\left(\frac{10}{4}\right)$ ft only for sign error in integral: $\frac{1}{2}\ln\left(\frac{5}{8}\right)$
	Total		**5**	

Q	Solution	Marks	Total	Comments
2(a)(i)	$f\left(\frac{1}{2}\right)=2\times\left(\frac{1}{2}\right)^3+3\times\left(\frac{1}{2}\right)^2-18\left(\frac{1}{2}\right)+8$	M1		use of $\pm\frac{1}{2}$ substituted in f (x)
	$=\frac{1}{4}+\frac{3}{4}-9+8=0\Rightarrow$ factor	A1	2	arithmetic seen and conclusion – minimum seen: $2\times\frac{1}{8}+3\times\frac{1}{4}-18\times\frac{1}{2}+8=0$
(ii)	$f(x)=(2x-1)(x^2+2x-8)$	B1B1	2	or $p=2$, $q=-8$
(iii)	$\frac{4x(x+4)}{(2x-1)(x+4)(x-2)}$	M1		numerator correct; attempt to factorise denominator (algebraic fraction not required)
	$=\frac{4x}{(2x-1)(x-2)}$	A1	2	CAO
(b)	$2x^2=A(x+5)(x-3)+B+Cx$	M1		any equivalent method using PFs (see alternative method)
	$A=2$	B1		
	$2A+C=0 \qquad -15A+B=0$	M1		equate coefficients or use 2 values of x to find B and C
	$C=-4 \qquad B=30$	A1	4	both B and C correct
	ALTERNATIVE METHOD 1			
	$\begin{array}{r} 2 \\ x^2+2x-15\,\overline{)\,2x^2} \\ \underline{2x^2+4x-30} \\ -4x+30 \end{array}$	(M1)		complete division
	$A=2$ $B=30$ $C=-4$	(B1) (A1) (A1)		
	ALTERNATIVE METHOD 2			
	$\frac{2x^2}{(x+5)(x-3)}=A+\frac{D}{x+5}+\frac{E}{x-3}$			
	$2x^2=A(x+5)(x-3)+D(x-3)+E(x+5)$			
	$x=3 \qquad 18=8E \qquad E=\frac{9}{4}$ $x=-5 \qquad 50=-8D \qquad D=-\frac{25}{4}$	(M1)		find D and E
	$x=0,\ 0=-15A+\left(-\frac{25}{4}\right)(-3)+\left(\frac{9}{4}\right)(5)$			
	$A=2$	(B1)		
	$\frac{D}{x+5}+\frac{E}{x-3}=\frac{-25}{4(x+5)}+\frac{9}{4(x-3)}$			
	$=\frac{-25(x-3)+9(x+5)}{4(x+5)(x-3)}$			
	$=\frac{120-16x}{4(x+5)(x-3)}$	(M1)		recombine to required form
	$=\frac{30-4x}{(x+5)(x-3)}$	(A1)		CAO
	Total		**10**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$(1+x)^{\frac{1}{2}}=1+\frac{1}{2}x+kx^2$	M1		
	$=1+\frac{1}{2}x-\frac{1}{8}x^2$	A1	2	
(b)	$\left(1+\frac{3}{2}x\right)^{\frac{1}{2}}=1+\frac{1}{2}\left(\frac{3}{2}x\right)-\frac{1}{8}\left(\frac{3}{2}x\right)^2$	M1		x replaced by $\frac{3}{2}x$ – condone missing brackets, but not incorrectly placed brackets eg $\left(\frac{3}{2}\right)x^2$ alternatively, start again and find correct expression
	$=1+\frac{3}{4}x-\frac{9}{32}x^2$	A1	2	correct evaluation
(c)	$\sqrt{\frac{2+3x}{8}}=\sqrt{\frac{2+3x}{4\times 2}}=k\left(1+\frac{3}{2}x\right)^{\frac{1}{2}}$	M1		manipulation to $k\times(\text{answer to (b)})$ and evaluated $\Rightarrow a+bx+cx^2$
	$=\frac{1}{2}+\frac{3}{8}x-\frac{9}{64}x^2$	A1	2	*a*, *b*, *c* fractions or decimals only
				Or use $(a+x)^n$ formula (condone one error for M1)
	Total		**6**	
4(a)(i)	$A=20$	B1	1	
(ii)	$\frac{2000}{A}=k^{60}$	M1		
	$k=(100)^{\frac{1}{60}}=1.079775$	A1	2	AG; or $k=10^{\frac{\log 100}{60}}=10^{0.0333}$ or $\sqrt[60]{100}$ or $\sqrt[30]{10}$ or $\mathrm{e}^{\frac{\ln 100}{60}}=\mathrm{e}^{0.076}$ or $\mathrm{e}^{0.077}$ or 1.0797751(6) seen
(iii)	$P=20\times k^{2008-1885}$	M1		
	$=251780\approx 252000$	A1	2	CAO nearest 1000
(b)	$15\times 1.082709^t=20\times 1.079775^t$	M1		equate prices
	$\frac{15}{20}=\left(\frac{1.079775}{1.082709}\right)^t$	M1		*t* as a single index, or correct log expression at this stage
	$t=\frac{\log 0.75}{\log 0.997290}$	m1		expression for *t*
	$t=106.017\Rightarrow 1991$	A1	4	**SC** Answer only/Trial and error 106 seen (2 out of 4) 1991 (4 out of 4)
	Total		**9**	

Q	Solution	Marks	Total	Comments
5(a)(i)	$t=\frac{1}{2}\quad x=2\times\frac{1}{2}+\frac{1}{\left(\frac{1}{2}\right)^2}\quad y=2\times\frac{1}{2}-\frac{1}{\left(\frac{1}{2}\right)^2}$	M1		
	$x=5\qquad y=-3$	A1	2	
(ii)	$\frac{\mathrm{d}y}{\mathrm{d}t}=2+2t^{-3}\qquad \frac{\mathrm{d}x}{\mathrm{d}t}=2-2t^{-3}$	M1A1		2 and $\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{t^2}\right)$ attempted in both derivatives
	$t=\frac{1}{2}\qquad \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2+\frac{2}{1/8}}{2-\frac{2}{1/8}}=-\frac{9}{7}$	M1 A1		use chain rule; expressions can be in terms of t or evaluated CAO or any equivalent fraction (not decimals)
	$y+3=-\frac{9}{7}(x-5)$	B1F	5	ft on x, y and gradient if $y = mx + c$ used, c must be found correctly and the equation must be re-written
(b)	$x-y=\frac{2}{t^2}\qquad x+y=4t$	M1		either correct expression or both of $x-y=4t$ and $x+y=\frac{2}{t^2}$
	$\frac{2}{(x-y)}=\left(\frac{x+y}{4}\right)^2$	M1		eliminate t
	$32=(x-y)(x+y)^2$	A1	3	or $(x-y)(x+y)^2=\frac{2}{t^2}\times(4t)^2=32$ $k = 32$ alone, no marks
	Total		**10**	
6	$3x\frac{\mathrm{d}y}{\mathrm{d}x}+3y-4y\frac{\mathrm{d}y}{\mathrm{d}x}=0$	M1 A1 A1 B1		attempt implicit differentiation product chain constant
	$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{3}{2}$	A1	5	CSO
	ALTERNATIVE METHOD			
	$x=\frac{2}{3}y+\frac{4}{3y}$	(M1)		solve for x = expression in y and differentiate with respect to y
	$\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{2}{3}-\frac{4}{3y^2}$	(A1A1)		
	$y = 1$, $\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{2}{3}-\frac{4}{3}$	(M1)		substitute $y = 1$
	$\frac{\mathrm{d}x}{\mathrm{d}y}=-\frac{3}{2}$	(A1)		CSO
	Total		**5**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$R=10$	B1		R = 10
	$\tan\alpha=\frac{8}{6}, \quad \alpha=53.1$	B1F	2	For α; ft incorrect R
(ii)	$\sin(2x+53.1)=0.7$	M1		
	$2x+53.1=44.4$	A1F		one correct answer ; ft α and R
	135.6 or 135.7, 404.4, 495.6 or 495.7	A1		3 other correct answers – ignore extras
	$x=41.2$ or 41.3, 175.6 or 175.7, 221.2 or 221.3, 355.6 or 355.7	A1	4	four solutions CAO (with decimal place discrepancies) Answers only: 0/4
(b)(i)	$\sin 2x=2\sin x\cos x$	B1		identities for $\sin 2x$ and $\cos 2x$ in any correct form
	$\cos 2x=\cos^2 x-\sin^2 x$	B1		
	$\frac{\sin 2x}{1-\cos 2x}=\frac{2\sin x\cos x}{1-(1-2\sin^2 x)}=$	M1		use of candidate's double angle formulae
	$\frac{2\sin x\cos x}{2\sin^2 x}=\frac{\cos x}{\sin x}=\frac{1}{\tan x}$	A1	4	AG, CSO
(ii)	$\frac{1}{\tan x}=\tan x \qquad \tan x=\pm1$	M1A1		(see * below)
	x = 45,	B1		x=45
	135, 225, 315	A1	4	if answers given without working, B1 max if $\frac{1}{\tan x}=\tan x$ seen and followed by correct answers without working 4 out of 4
	Total		**14**	

* Comments for 7(b)(ii)

If hence ignored, so working in sines and cosines, must simplify as far as:

$\cos^2 x=\sin^2 x$	or	$\cos^2 x=\frac{1}{2}$	or	$\sin^2 x=\frac{1}{2}$	for M1
$\cos 2x=0$	or	$\cos x=\pm\frac{1}{\sqrt{2}}$	or	$\sin x=\pm\frac{1}{\sqrt{2}}$	for A1

MPC4 (cont)

Q	Solution	Marks	Total	Comments
8	$\int y\,\mathrm{d}y = \int 3\cos 3x\,\mathrm{d}x$	M1		attempt to separate and integrate $py^2 = q\sin 3x$ seen ⇒ implies separation
	$\frac{1}{2}y^2 = \sin 3x \quad (+C)$	A1A1		integrals – accept $\frac{1}{3}\times 3\sin 3x$
	$\left(\frac{\pi}{2},2\right)$ $\frac{1}{2}\times 4 = \sin\frac{3\pi}{2} + C$	M1		use $\left(\frac{\pi}{2},2\right)$ to find constant
	$C = 3$			
	$y^2 = 2\sin 3x + 6$	A1	5	CSO (in any correct form)
	Total		**5**	
9(a)(i)	$\overrightarrow{AB} = \begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix} - \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ -3 \end{bmatrix}$	M1A1	2	M1 for $\pm(\overrightarrow{OA} - \overrightarrow{OB})$
(ii)	$(\mathbf{r} =) \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ -4 \\ -3 \end{bmatrix}$	B1F	1	ft on $\overrightarrow{AB}$; OE
(b)(i)	$\begin{bmatrix} 1 \\ -3 \\ -1 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \\ 5 \end{bmatrix}$	M1		μ found and verified or statement $\mu = -3$ satisfies all components
	$1 + \mu = -2 \qquad \mu = -3$ $-1 - 2\mu = 5 \qquad \mu = -3$	A1	2	$\mu = -3$ alone B1
	ALTERNATIVE METHOD $\mu \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 6 \end{bmatrix}$, which is satisfied by $\mu = -3$			
(ii)	$\overrightarrow{PQ} = \left(\begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ -4 \\ -3 \end{bmatrix} \right) - \begin{bmatrix} -2 \\ -3 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 + 2\lambda \\ 8 - 4\lambda \\ -4 - 3\lambda \end{bmatrix}$	M1 A1		$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$ with $\overrightarrow{OQ}$ in parametric form in terms of λ (can be inferred later) or $\begin{bmatrix} 6+2\lambda \\ 4-4\lambda \\ -7-3\lambda \end{bmatrix}$
	$\begin{bmatrix} 4 + 2\lambda \\ 8 - 4\lambda \\ -4 - 3\lambda \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$	M1		$\overrightarrow{PQ} \bullet \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$ with $\overrightarrow{PQ}$ in terms of λ (can be inferred later)
	$(4 + 2\lambda) + (-2)(-4 - 3\lambda) = 0$	m1		linear expression in λ equated to 0
	$\lambda = -1.5$	A1F		ft on sign/arithmetic error in $\overrightarrow{PQ}$ or equation
	Q is $(-1, 11, 5.5)$	A1	6	CAO
	Total		**11**	
	TOTAL		**75**	

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Mathematics 6360

MPC4 Pure Core 4

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AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC4

Q	Solution	Marks	Total	Comments
1(a)	$\text{f}\left(-\frac{1}{3}\right)=27\times\left(-\frac{1}{3}\right)^3-9\times\left(-\frac{1}{3}\right)+2$	M1		Use of $\pm\frac{1}{3}$ or complete division with integer remainder M1
	$=-1+3+2=4$	A1	2	remainder = 4 indicated A1
(b)(i)	$\text{f}\left(-\frac{2}{3}\right)=-8+6+2=0$	B1	1	AG
(b)(ii)	$\text{f}(x)=(3x+2)(ax^2+bx+c)$	B1		$(3x+2)$ or $\left(x+\frac{2}{3}\right)$ is a factor PI
	$a = 9$ $c = 1$	M1		quadratic factor; find coefficients; 2 correct
	x^2 term $3b+2a=0$ or x term $3c+2b=-9$			equate coefficients and solve for b
	$b = -6$ or (could be shown as) $9x^2-6x+1$	A1		correct quadratic factor or a, b, and c correct or use division or factor theorem to seek another factor (see alternative methods at end of scheme)
	$\text{f}(x)=(3x+2)(3x-1)(3x-1)$	A1	4	SC (see alternative methods at end of scheme)
(b)(iii)	$9x^2+3x-2=(3x-1)(3x+2)$	M1		factorise denominator correctly or complete division
	$\frac{27x^3-9x+2}{9x^2+3x-2}=3x-1$	A1	2	simplified result indicated
	Total		**9**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
2(a)	$\frac{\mathrm{d}x}{\mathrm{d}t}=4 \qquad \frac{\mathrm{d}y}{\mathrm{d}t}=-\frac{1}{2t^2}$	M1 A1		differentiate. 4; at^{-2} seen both derivatives correct
	$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2t^2}\times\frac{1}{4}$	M1		use chain rule candidates' $\frac{\mathrm{d}y}{\mathrm{d}t} \Big/ \frac{\mathrm{d}x}{\mathrm{d}t}$
	$t=\frac{1}{2} \qquad \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2}$	A1	4	CSO
(b)	gradient of normal = 2	B1F		F if gradient $\neq \pm 1$
	$(x,y)=(5,0) \qquad \frac{y}{x-5}=2$	M1 A1F	3	calculate and use (*x*, *y*) on normal F on gradient of normal ACF
(c)	$x-3=4t$ or $y+1=\frac{1}{2t}$	B1		or $t=\frac{x-3}{4}$ or $\frac{1}{t}=2(y+1)$
	$(x-3)(y+1)=2$	M1		eliminate *t*; allow one error
		A1	3	accept $y=\frac{1}{\frac{2(x-3)}{4}}-1$ ACF SC allow marks for part (c) if done in part (a)
	Total		**10**	
3(a)	$\sin(x+2x)=\sin x\cos 2x+\cos x\sin 2x$	M1		
	$=\sin x(1-2\sin^2 x)+\cos x(2\sin x\cos x)$	B1B1		double angles; ACF ISW condone missing *x*
	$=\sin x(1-2\sin^2 x)+2\sin x(1-\sin^2 x)$	A1		all in sin *x*, correct expression
	$=3\sin x-2\sin^3 x-2\sin^3 x$ $=3\sin x-4\sin^3 x$	A1	5	CSO AG
(b)	$\sin^3 x=a\sin x+b\sin 3x$	M1		attempt to solve for $\sin^3 x$ where $a\neq 0$ and $b\neq 0$
	$\int\sin^3 x\,\mathrm{d}x=-a\cos x-\frac{b}{3}\cos 3x$	A1F		either integral correct F on *a*, *b*
	$\int\sin^3 x\ \mathrm{d}x=\frac{1}{4}\left(-3\cos x+\frac{1}{3}\cos 3x\right)\ (+C$	A1	3	CAO alternative method by parts (see end of mark scheme)
	Total		**8**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$(1-x)^{\frac{1}{4}} = 1+\frac{1}{4}(-x)+\frac{1}{2}\times\frac{1}{4}\left(-\frac{3}{4}\right)(-x)^2$	M1		$1\pm\frac{1}{4}x+kx^2$
	$=1-\frac{1}{4}x-\frac{3}{32}x^2$	A1	2	equivalent fractions or decimals
(a)(ii)	$(81-16x)^{\frac{1}{4}} = 81^{\frac{1}{4}}\left(1-\frac{16}{81}x\right)^{\frac{1}{4}}$	B1		
	$=k\left(1-\frac{1}{4}\times\frac{16}{81}x-\frac{3}{32}\left(\frac{16}{81}x\right)^2\right)$	M1		x replaced by $\frac{16}{81}x$ or start binomial again condone one error (missing bracket; x or x^2; sign error)
	= 3( ) $=3-\frac{4}{27}x-\frac{8}{729}x^2$	A1	3	CSO AG use of $(a+bx)^n$ ignoring hence (see end of mark scheme)
(b)	$3-\frac{4}{27}\times\frac{1}{16}-\frac{8}{729}\left(\frac{1}{16}\right)^2$	M1		use $x=\frac{1}{16}$
	= 2.9906979	A1	2	seven decimal places only
	Total		**7**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	$\cos\alpha = \frac{3}{5}$	B1	1	ACF
(a)(ii)	$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$	M1		
	$= \frac{3}{5}\cos\beta + \frac{4}{5}\sin\beta$	A1	2	ACF
(a)(iii)	$\sin\beta = \frac{12}{13}$	B1		
	$\cos(\alpha-\beta) = \frac{63}{65}$	B1	2	$\frac{63}{65}$ NMS B1B1
(b)(i)	$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$	M1		
	$2\tan x = 1-\tan^2 x$ $\tan^2 x + 2\tan x - 1 = 0$	A1	2	CSO AG
(b)(ii)	$\tan x = \frac{-2\pm\sqrt{4+4}}{2}$	M1		must solve quadratic equation by formula or by completing the square condone one slip
	$= -1 \pm \sqrt{2}$	A1		$\pm\sqrt{2}$ required
	$2x = 45° \Rightarrow x = 22\frac{1}{2}^\circ$ is acute			
	$\Rightarrow \tan 22\frac{1}{2}^\circ = \sqrt{2} - 1$	E1	3	explain selection of positive root
	Total		**10**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\frac{2}{\left(x^2-1\right)}=\frac{A}{x-1}+\frac{B}{x+1}$			
	$2=A(x+1)+B(x-1)$	M1		
	$x=1 \qquad x=-1$	m1		use two values of *x* or equate coefficients and solve *A* + *B* = 0 and *A* – *B* = 2
	$A=1 \qquad B=-1$	A1	3	both *A* and *B*
(b)	$\int\frac{2}{x^2-1}\,\mathrm{d}x=p\ln(x-1)+q\ln(x+1)$	M1		ln integrals
	$=\ln(x-1)-\ln(x+1)$	A1F	2	F on *A* and *B* condone missing brackets
(c)	$\int\frac{\mathrm{d}y}{y}=\int\frac{2}{3\left(x^2-1\right)}\,\mathrm{d}x$	M1		separate and attempt to integrate on one side
	$\ln y=\frac{1}{3}\left(\ln(x-1)-\ln(x+1)\right) \quad (+C)$	A1 A1F		left hand side F from part (b) on right hand side
	$(3,1) \qquad \ln 1=\tfrac{1}{3}(\ln 2-\ln 4)+C$	m1		use (3, 1) to attempt to find a constant
	$3\ln y=\ln(x-1)-\ln(x+1)-(\ln 2-\ln 4)$			
	$3\ln y=\left(\ln\left(\frac{x-1}{x+1}\right)+\ln 2\right)$			
	$\ln y^3=\ln\left(\frac{2(x-1)}{x+1}\right)$			
	$y^3=\frac{2(x-1)}{x+1}$	A1	5	CSO AG
	Total		**10**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$AB^2 = (5-3)^2 + (3--2)^2 + (0-1)^2$	M1		use $\pm\left(\overrightarrow{OB}-\overrightarrow{OA}\right)$ in sum of squares of components allow one slip in difference
	$AB = \sqrt{30}$	A1	2	accept 5.5 or better
(b)	$\begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix} = 2+3=5$	M1		$\pm\overrightarrow{AB}\bullet$ direction l evaluated condone one component error
		A1		5 or – 5
	$\cos\theta = \dfrac{5}{\sqrt{30}\sqrt{10}}$	B1F M1		F on either of candidates' vectors use $\lvert a\rvert\lvert b\rvert\cos\theta = a\bullet b$; values needed
	$\theta = 73°$	A1	5	CAO (condone 73.2, 73.22 or 73.22…)
(c)	$\overrightarrow{AC} = \begin{bmatrix} 5+\lambda \\ 3 \\ -3\lambda \end{bmatrix} - \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2+\lambda \\ 5 \\ -1-3\lambda \end{bmatrix}$	M1		for $\overrightarrow{OC}-\overrightarrow{OA}$ or $\overrightarrow{OA}-\overrightarrow{OC}$ with $\overrightarrow{OC}$ in terms of λ condone one component error
		A1		
	$(2+\lambda)^2 + 5^2 + (-1-3\lambda)^2 = 30$	m1		
	$10\lambda^2 + 10\lambda = 0$ $(\lambda = 0 \text{ or})\ \lambda = -1$	A1		
	($\lambda = 0 \Rightarrow (5,3,0)$ is B)			
	$\lambda = -1 \Rightarrow C$ is $(4,3,3)$	A1	5	condone $\begin{bmatrix} 4 \\ 3 \\ 3 \end{bmatrix}$
	Total		**12**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
8(a)(i)	$p\dfrac{\mathrm{d}x}{\mathrm{d}t}=q$	M1		where *p* and *q* are functions
	$\dfrac{\mathrm{d}x}{\mathrm{d}t}=-kx$	A1	2	in any correct combination
(a)(ii)	$-500=-k\,20000$ or $500=k\,20000$	M1		condone sign error or missing 0 *k* can be on either side of the equation
	$k=\dfrac{5}{200}\quad(=0.025)$	A1	2	CSO both (a)(i) and (a)(ii)
(b)(i)	A = 1300	B1	1	
(b)(ii)	$100>A\mathrm{e}^{-0.05\,t}$	M1		condone = for >; condone 99 for 100
	$\ln\left(\dfrac{100}{A}\right)>-0.05t$	m1		take logs correctly condone 0.5
	$t>51.3$	A1		or by trial and improvement (see end of mark scheme)
	population first exceeds 1900 in 2059	A1F	4	F if M1 m1 earned and t>0 following *A*
	Total		**9**	
	TOTAL		**75**	

MPC4 (cont)
Alternative methods permitted in the mark scheme

Q	Solution	Marks	Total	Comments
1(b)(ii)	**ALTERNATIVE METHOD 1**			
	$(3x+2)$ is a factor	B1		PI
	use factor theorem	M1		use factor theorem or algebraic division to find another factor
	$\mathrm{f}\left(\frac{1}{3}\right)=0 \Rightarrow (3x-1)$ is a factor			
	$\mathrm{f}(x)=(3x+2)(3x-1)(ax+b)$	A1		
	$\mathrm{f}(x)=(3x+2)(3x-1)(3x-1)$	A1	4	
	ALTERNATIVE METHOD 2			
	$(3x+2)$ is a factor	B1		PI by division
	divide $27x^3-9x+2$ by $(3x+2)$	M1		complete division to ax^2+bx+c
	$9x^2-6x+1$	A1		
	$\mathrm{f}(x)=(3x+2)(3x-1)(3x-1)$	A1	4	
1(b)(ii)	**SPECIAL CASE**			
	$(3x+2)(3x-1)(ax+b)$		2	
2(a)	$y=\frac{2}{x-3}-1$ and differentiate	M1		differentiate expression in *y* and *x*
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-2}{(x-3)^2}$	A1		correct
	$x=5$			
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-2}{(5-3)^2}$	m1		find and therefore use *x* (and *y*)
	$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2}$	A1	4	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
3(b)	**ALTERNATIVE METHOD 1**			
	$\int \sin^3 x\,\mathrm{d}x = \int \sin^2 x \sin x\,\mathrm{d}x$ $=$ $-\sin^2 x\cos x - \int -2\cos x \sin x \cos x\,\mathrm{d}x$	M1		identify parts and attempt to integrate
	$= -\sin^2 x\cos x - \frac{2}{3}\cos^3 x \quad (+C)$	A2	3	
	ALTERNATIVE METHOD 2			
	$\int \sin^3 x\,\mathrm{d}x = \int \sin^2 x\,\mathrm{d}(-\cos x)$ $= \int -\left(1-\cos^2 x\right)\mathrm{d}(\cos x)$	M1		condone sign error
	$= -\cos x + \frac{1}{3}\cos^3 x \quad (+C)$	A2	3	
	ALTERNATIVE METHOD 3			
	$\int \sin x \sin^2 x\,\mathrm{d}x$ $\int \sin x\left(1-\cos^2 x\right)\mathrm{d}x$	M1		this form and attempt to integrate
	$= -\cos x + \frac{1}{3}\cos^3 x \quad (+C)$	A2	3	
4(a)(ii)	$(81-16x)^{\frac{1}{4}} = 81^{\frac{1}{4}} + \frac{1}{4}81^{-\frac{3}{4}}(-16x) + \frac{1}{4}\left(-\frac{3}{4}\right)\frac{1}{2}81^{-\frac{7}{4}}(-16x)^2$	M1 A1		using $(a+bx)^n$ from FB condone one error
	$= \left(3 - \frac{4}{27}x - \frac{8}{729}x^2\right)$	A1	3	CSO completely correct
8(b)(ii)	$t = 51 \rightarrow 101.5$ $t = 52 \rightarrow 96.6$ $\Rightarrow 51 < t < 52$	M1		*t* = 51 or *t* = 52 considered
	population first exceeds 1900 in 2059	A3	4	CAO

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General Certificate of Education

Mathematics 6360

MPC4 Pure Core 4

Mark Scheme

2009 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
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SCA	substantially correct approach	dp	decimal place(s)

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Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC4

Q	Solution	Marks	Total	Comments
1(a)				
(i)	$f(-1)=0$	B1	1	
(ii)	$f\left(-\frac{1}{2}\right)=4\left(-\frac{1}{8}\right)-7\left(-\frac{1}{2}\right)-3$	M1		Use of $\pm\frac{1}{2}$
	$=-\frac{1}{2}+\frac{7}{2}-3=0\Rightarrow$ factor	A1	2	Need to see simplification (at least $\left(-\frac{1}{2}\right)^3=-\frac{1}{8}$), '=0' and conclusion
(iii)	Third factor is $(2x-3)$	B1		PI
	$\frac{(x+1)(2x+1)(2x-3)}{(x+1)(2x+1)}$	M1		$\frac{\text{3 linear factors}}{\text{2 linear factors}}$
	simplifies to $2x-3$	A1		Simplified result stated. Alternative; see end. Use remainder theorem.
	Alternative			
	Complete division to $2x+b$	(M1)		
	Complete division to $2x-3$	(A1)		
	Simplifies to $2x-3$	(A1)	3	Simplified result stated
(b)	$g\left(-\frac{1}{2}\right)=-\frac{1}{2}+\frac{7}{2}+d=2$	M1		
	$d=-1$	A1		
	Alternative			
	Complete division leading to rem = 2	(M1)		Remainder $=d+p=2$
	$d=-1$	(A1)	2	
	Total		**8**	
2(a)	$R=\sqrt{10}$	B1		Accept $R=3.16$ or better.
	$\tan\alpha=3$	M1		OE (Can be implied by 71.57° seen)
	$\alpha=1.25$	A1	3	A0 if extra answers within given range SC 1 $\tan\alpha=\frac{1}{3}$ $\alpha=0.32$
(b)(i)	min value = $-\sqrt{10}$ $\left(\text{or}\ge\sqrt{-10}\right)$	B1F	1	ft on R
(ii)	$\sin(x-\alpha)=-1$	M1		or $\sin^{-1}\frac{3\pi}{2}$
	$x=5.96$	A1F	2	ft on their α (to 2 dp) + $\frac{3\pi}{2}$
	Total		**6**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
3(a) **(i)**	$\frac{2x+7}{x+2}=2+\frac{3}{x+2}$	B1 B1	2	
(ii)	$\int\frac{2x+7}{x+2}=3\ln(x+2)+2x+C$	B1F B1F	2	Either term correct Both correct; constant required; condone missing bracket ft on *A*, *B*
(b)(i)	$28+4x^2=$ $P(5-x)^2+Q(1+3x)(5-x)$ $+R(1+3x)$	M1		
	$x=5 \qquad x=-\frac{1}{3}$	m1		Two values of *x* used to find *R* and *P*.
	$R=8 \qquad P=1$	A1		SC *R* = 8, *P* = 1 NMS can score B1,B1
	$x=0\Rightarrow 28=25P+5Q+R$	m1		Third value of *x* used to find *Q*
	$Q=-1$	A1		.
	Alternative			
	$28+4x^2=$ $P(5-x)^2+Q(1-3x)(5-x)$ $+R(1+3x)$	(M1)		
	$=(25P+5Q+R)+$ $(-10P+14Q+3R)x+(P-3Q)x^2$	(m1)		Collect terms and form equations
	$P-3Q=4$ $14Q+3R-10P=0$ $25P+5Q+R=28$	(A1)		Correct equations
	$P=1 \quad Q=-1 \quad R=8$	(m1) (A1)	5	Solve for *P Q* and *R*
(ii)	$\int\frac{1}{1+3x}-\frac{1}{5-x}+\frac{8}{(5-x)^2}\,\mathrm{d}x$	M1		Use partial fractions
	$=\frac{1}{3}\ln(1+3x)+\ln(5-x)+\frac{8}{5-x}+(C)$	m1 A1F A1F	4	$a\ln(1+3x)+b\ln(5-x)$ OE; both ln integrals correct; needs ( ) Other term correct ft on their *P*, *Q*, *R* SC: If no *P*,*Q*, *R* found in (b)(i), can gain method marks by inserting other values or retaining the letters (max 2/4)
	Total		**13**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
4(a) (i)	$\left(1-x\right)^{\frac{1}{2}}=1+\frac{1}{2}\left(-x\right)+px^2$	M1		
	$=1-\frac{1}{2}x-\frac{1}{8}x^2$	A1	2	
(ii)	$\sqrt{4-x}=2\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$	B1		or $\left(4\right)^{\frac{1}{2}}\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$
	$=\left(2\right)\left(1-\frac{1}{2}\left(\frac{x}{4}\right)-\frac{1}{8}\left(\frac{x}{4}\right)^2\right)$	M1		x replaced by $\frac{x}{4}$; condone missing ( )
				Or start again with $\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$
	$=2-\frac{x}{4}-\frac{x^2}{64}$	A1		CAO or decimal equivalent
	Alternative			
	$\left(4-x\right)^{\frac{1}{2}}=4^{\frac{1}{2}}+\frac{1}{2}\times4^{-\frac{1}{2}}\left(-x\right)$	(M1)		Use of $\left(a+x\right)^n$ from formula book
	$+\dfrac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}4^{-\frac{3}{2}}\left(-x\right)^2$	(A1)		Condone missing brackets and 1 error
	$=2-\frac{x}{4}-\frac{x^2}{64}$	(A1)	3	
(b)	$x=1$ $\sqrt{3}\approx2-\frac{1}{4}-\frac{1}{64}$	M1		$x=1$ used in their expansion
	$=1.734\ \left(3\text{dp}\right)$	A1	2	CSO
	Total		**7**	
5(a)	$\sin2x=2\sin x\cos x$	B1	1	OE, eg sin x cos x + sin x cos x etc
	$\cos x=0$ $x=90,\ 270$	B1		Both required
(b)	$10\sin x+3=0$	M1		
	$x=197.5$ 342.5	A1A1	4	CAO if extra values in given range, max 1/2
(c)	$\cos2x=\cos^2x-\sin^2x$	B1		$\cos2x$ in any correct form
	$2\sin x\cos x+1-2\sin^2x=1+\sin x$	M1		sin 2x expanded and $\cos2x$ in terms of $\sin x$ used
		A1		
	$2\sin x\left(\cos x-\sin x\right)=\sin x$			
	$2\left(\cos x-\sin x\right)=1$	A1	4	CSO; need to see sin x taken out as factor or cancelled
	Total		**9**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
6 **(a)**	$x^2\frac{\mathrm{d}y}{\mathrm{d}x}+2xy$	M1 A1		Product rule used. Allow 1 error
	$+3y^2\frac{\mathrm{d}y}{\mathrm{d}x}$	B1		Chain rule
	$=2$	B1		RHS and equation with no spurious $\frac{\mathrm{d}y}{\mathrm{d}x}$ unless recovered.
	$(2,1),\quad 4\frac{\mathrm{d}y}{\mathrm{d}x}+4+3\frac{\mathrm{d}y}{\mathrm{d}x}=2$	M1		Substitute $(2,1)$
	$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{2}{7}$	A1	6	CSO
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow$	M1		Derivative = 0 used
	$xy=1$	A1		OE
	$x^2\times\frac{1}{x}+\frac{1}{x^3}=2x+1$	m1		Use $xy=k$ to eliminate y on LHS
	$\frac{1}{x^3}=x+1$	A1	4	Answer given; CSO
	Total		**10**	
7(a) **(i)**	$\int\frac{\mathrm{d}x}{\mathrm{e}^{\frac{1}{2}x}}=\int -kt\ \mathrm{d}t$	B1		Separate; condone missing integral signs
	$-2\mathrm{e}^{-\frac{1}{2}x}=-k\frac{t^2}{2}\quad(+C)$	B1B1	3	
(ii)	$-2\mathrm{e}^{-\frac{1}{2}x}=-k\frac{t^2}{2}-2\mathrm{e}^{-3}$	M1		Use $(6,0)$ to find constant
	$\ln\left(\mathrm{e}^{-\frac{1}{2}x}\right)=\ln\left(k\frac{t^2}{4}+\mathrm{e}^{-3}\right)$	M1		Take logarithms correctly; condone one side negative. Must have a constant.
	$-\frac{1}{2}x=\ln\left(k\frac{t^2}{4}+\mathrm{e}^{-3}\right)$			
	$x=-2\ln\left(\frac{kt^2}{4}+\mathrm{e}^{-3}\right)$	A1	3	Answer given; CSO
(b) **(i)**	$t=10\qquad x=-2\ln\left(\frac{0.004\times10^2}{4}+\mathrm{e}^{-3}\right)$	M1		
	$=3.8\ \Rightarrow 3800$	A1	2	CAO
(ii)	$x=0\qquad \frac{0.004\times t^2}{4}+\mathrm{e}^{-3}=1$	M1		
	$t=30.8$	A1	2	CAO Treat 0.04 or 0.0004 as misread (–1)
	Total		**10**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
8(a) **(i)**	$\overrightarrow{AB}=\begin{bmatrix}3\\1\\-2\end{bmatrix}-\begin{bmatrix}2\\1\\-1\end{bmatrix}=\begin{bmatrix}1\\0\\-1\end{bmatrix}$	M1 A1	2	$\pm\left(\overrightarrow{OA}-\overrightarrow{OB}\right)$ A0 if answer as coordinates
(ii)	$\overrightarrow{OB}\bullet\overrightarrow{AB}=3\times1+1\times0+(-2)\times(-1)=5$	M1 A1		Evaluate to single value
	$\cos\theta=\dfrac{\overrightarrow{OB}\bullet\overrightarrow{AB}}{\left\lvert\overrightarrow{OB}\right\rvert\times\left\lvert\overrightarrow{AB}\right\rvert}$	M1		Use formula for $\cos\theta$ with any 2 vectors and at least one of the corresponding modulii 'correct'
	$\left\lvert\overrightarrow{OB}\right\rvert=\sqrt{14}\qquad\left\lvert\overrightarrow{AB}\right\rvert=\sqrt{2}$			
	$\cos\theta=\dfrac{5}{\sqrt{7\times2}\sqrt{2}}=\dfrac{5}{2\sqrt{7}}$	A1		CSO; AG so need to see intermediate step eg $\dfrac{5}{\sqrt{7\times2}\sqrt{2}}$ or $\dfrac{5}{\sqrt{28}}$
	Alternative			
	cos rule attempted with cos B	(M1)		
	cos rule correct with cos B	(A1)		
	derive correct given form	(A2)	4	
(b)	$\mathbf{r}=\begin{bmatrix}6\\2\\-4\end{bmatrix}+\lambda\begin{bmatrix}1\\0\\-1\end{bmatrix}$	M1 A1F	2	$\overrightarrow{OC}+\lambda\overrightarrow{AB}$. Allow one slip ft on $\overrightarrow{AB}$; needs **r** or $\begin{bmatrix}x\\y\\z\end{bmatrix}$
(c)	$\overrightarrow{OD}\bullet\overrightarrow{AB}=\begin{bmatrix}6+\lambda\\2\\-4-\lambda\end{bmatrix}\bullet\begin{bmatrix}1\\0\\-1\end{bmatrix}$	M1		
	$6+\lambda+4+\lambda=0$	m1		
	$\lambda=-5$	A1F		ft on equation of line
	D is $(1,2,1)$	A1		CAO
	Alternative			
	$\begin{bmatrix}a\\b\\c\end{bmatrix}\bullet\begin{bmatrix}1\\0\\-1\end{bmatrix}=a-c=0$	(M1)		Let D be (a,b,c) Scalar product evaluated and equated to 0
	$a=6+\lambda,\quad b=2,\quad c=-4-\lambda$	(m1) (A1)		Use equation of line
	$a+c=2$			
	$a=1\quad b=2\quad c=1$	(A1)	4	
	Total		**12**	
	TOTAL		**75**	

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General Certificate of Education

Mathematics 6360

MPC4 Pure Core 4

Mark Scheme

2009 examination - June series

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Registered address: AQA, Devas Street, Manchester M15 6EX *Dr Michael Cresswell Director General*

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M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

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MPC4

Q	Solution	Marks	Total	Comments
1(a)	$\text{f}\left(\frac{1}{3}\right)=3\times\frac{1}{27}+8\times\frac{1}{9}-3\times\frac{1}{3}-5$	M1		Use $\frac{1}{3}$ in evaluating $\text{f}(x)$
	$=-5$	A1	2	No ISW Evidence of Remainder Theorem
(b)	$\begin{array}{r} x^2+3x \\ 3x-1\overline{)3x^3+8x^2-3x-5} \\ 3x^3-x^2 \\ 9x^2-3x \\ 9x^2-3x \end{array}$	M1		Division with x^2 and an x term seen; x^2+px
	$a=1 \quad b=3$ or $x^2+3x+\frac{c}{3x-1}$	A1		Explicit or in expression
	$c=-5$	B1		Condone $+\frac{-5}{3x-1}$
	Alternative			
	$\frac{(3x-1)(x^2+px)}{3x-1}-\frac{5}{3x-1}$	(M1)		Split fraction and attempt factors
	$x^2+3x \qquad -\frac{5}{3x-1}$	(A1) (B1)		$a=1 \quad b=3$ $c=-5$
	Alternative			
	$\text{f}(x)=3ax^3+(3b-a)x^2-bx+c$	(M1)		Multiply by $(3x-1)$ and attempt to collect terms
	$a=1 \quad b=3$ $c=-5$	(A1) (B1)		
	Alternative			
	$\text{f}(x)=(ax^2+bx)(3x-1)+c$	(M1)		Multiply by $(3x-1)$ and attempt to find a, b, c: substitute 3 values of x and form 3 simultaneous equations, and attempt to solve; or substitute 3 values of x into given equation
	$x=0\Rightarrow c=-5$	(B1)		
	$x=1\Rightarrow 2a+2b+c=3$ $x=2\Rightarrow 20a+10b+c=45$			
	$a=1 \quad b=3$	(A1)	3	
	Total		**5**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
2(a)	$\frac{\mathrm{d}x}{\mathrm{d}t}=-\frac{1}{t^2}$ $\quad$ $\frac{\mathrm{d}y}{\mathrm{d}t}=1-\frac{1}{2t^2}$	B1B1		
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1-\frac{1}{2t^2}}{-\frac{1}{t^2}}$ $\quad$ $\left(=\frac{2t^2-1}{-2}\right)$	M1		Their $\frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}$; condone 1 slip
		A1		CSO; ISW
	Alternative			
	$y=\frac{1}{x}+\frac{x}{2}$	(B1)		
	$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{x^2}+\frac{1}{2}$	(B1)		
	Substitute $x=\frac{1}{t}$	(M1)		
	$\frac{\mathrm{d}y}{\mathrm{d}x}=-t^2+\frac{1}{2}$	(A1)	4	CSO
(b)	$t=1$ $\quad$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2}$	M1		Substitute $t=1$ in $\frac{\mathrm{f}(t)}{\mathrm{g}(t)}\not\equiv k$
	$m_T=-\frac{1}{2}\Rightarrow m_n=2$	B1F		F on $m_T\neq 0$; if in $t\rightarrow$numerical later
	$(x,y)=\left(1,\frac{3}{2}\right)$	B1		PI $\frac{3}{2}=m(\times 1)+c$
	$\left(y-\frac{3}{2}\right)=2(x-1)$ **or** $y=2x+c,\ c=-\frac{1}{2}$	A1	4	ISW, CSO (a) and (b) all correct
(c)	$y=\frac{1}{\frac{1}{t}}+\frac{1}{2}\times\frac{1}{t}$	M1		Attempt to use $t=\frac{1}{x}$ to eliminate t t, or equivalent
	$=\frac{1}{x}+\frac{x}{2}$	A1		
	$2xy=2+x^2\Rightarrow x^2-2xy+2=0$	A1		Correct algebra to AG with $k=2$ allow $k=2$ stated $k=2$, no working or from $\left(1,\frac{3}{2}\right)$: $0/3$
	Alternative $\quad$ **or**			
	$\left(\frac{1}{t}\right)^2-2\left(\frac{1}{t}\right)\left(t+\frac{1}{2t}\right)$ $\quad$ | $\quad$ $xy=\frac{1}{t}\left(t+\frac{1}{2t}\right)$	(M1)		Substitute and multiply out
	$=-2$ $\quad$ | $\quad$ $=1+\frac{x^2}{2}$	(A1)		Eliminate t
	$\Rightarrow x^2-2xy+2=0$	(A1)	3	Conclusion, k = 2
			11	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$(1-x)^{-1}=1+(-1)(-x)+\frac{1}{2}(-1.-2)(-x)^2$	M1		$1\pm x+kx^2$
	$=1+x+x^2$	A1	2	Fully simplified
(b)(i)	$3x-1=A(2-3x)+B(1-x)$	M1		
	$x=1 \qquad x=\frac{2}{3}$	m1		Use 2 values of x or equate coefficients and solve $-3A-B=3$ $2A+B=-1$ condone coefficient errors
	$A=-2 \quad B=3$	A1	3	Both values NMS 3/3 if both correct, 1/3 if one correct
(ii)	$\left(\frac{3x-1}{(1-x)(2-3x)}=\frac{-2}{1-x}+\frac{3}{2-3x}\right)$			
	$\frac{-2}{1-x}=-2-2x-2x^2$	B1F		F on $(1-x)^{-1}$ and A
	$\frac{1}{2-3x}=\frac{1}{2}\left(1-\frac{3}{2}x\right)^{-1}$	B1		
	$=(p)\left(1+kx+(kx)^2\right)$	M1		p, k = candidate's $\frac{1}{2},\frac{3}{2}, k\neq\pm1$
	$=(p)\left(1+\frac{3}{2}x+\frac{9}{4}x^2\right)$	A1		Use (a) or start binomial again; condone missing brackets, and one sign error
	$\frac{3x-1}{(1-x)(2-3x)}=-2(1-x)^{-1}+3(2-3x)^{-1}$	M1		Valid combination of both expansions
	$=-\frac{1}{2}+\frac{1}{4}x+\frac{11}{8}x^2$	A1		CSO
	Alternative			
	$(2-3x)^{-1}=\frac{1}{2}\left(1-\frac{3}{2}x\right)^{-1}$	(B1)		k = candidate's $\frac{3}{2}$ $k\neq\pm1$
	$(1-kx)^{-1}=1+kx+(kx)^2$	(M1)		Use (a) or start binomial again; condone missing brackets and one error
	$=1+\frac{3}{2}x+\frac{9}{4}x^2$	(A1)		
	$\frac{3x-1}{(1-x)(2-3x)}=(3x-1)(1-x)^{-1}(2-3x)^{-1}$	(M1)		$(3x-1)\times$ both expansions
	$\frac{3x-1}{(1-x)(2-3x)}=-\frac{1}{2}+\frac{1}{4}x+\frac{11}{8}x^2$	(m1) (A1)	6	Multiply out; collect terms to form $a+bx+cx^2$ CSO
	Alternative for $(2-3x)^{-1}$			Using $(a+bx)^n$
	$2^{-1}+(-1)(2)^{-2}(-3x)+\frac{(-1)(-2)(2)^{-3}(-3x)^2}{2}$	(M1)		Condone missing brackets, and 1 error
	$=\frac{1}{2}+\frac{3}{4}x+\frac{9}{8}x^2$	(A1) (A1)		First two terms x^2 term

MPC4 (cont)

Q	Solution	Marks	Total	Comments
(c)	$-2<3x<2$	M1		PI, or any equivalent form
	$\Rightarrow -\frac{2}{3}<x<\frac{2}{3}$	A1	2	Condone $\le$; accept $\pi \ge \frac{2}{3}$ or $x \ge -\frac{2}{3}$ CSO; allow $\lvert \pm x \rvert \le \frac{2}{3}$, or $x<\frac{2}{3}$ and $x>-\frac{2}{3}$
	Total		**13**	
4(a)(i)	$A=12499$	B1	1	Stated in (i) or (ii)
(ii)	$k^{36}=\frac{7000}{\text{their } A}$	M1		$p=\frac{7000}{12499}=0.560044803$
	$k=\sqrt[36]{0.56(00448...)}=0.9840251(26)$ or $\left(0.56(00448...)\right)^{\frac{1}{36}}$ or $k=\sqrt[36]{\frac{7000}{12499}}$ $k=0.984025$	A1	2	Correct expression for k or 7th dp seen. $k=10^{\frac{1}{36}\log p}$ or $k=10^{-0.00699...}$ $k=\mathrm{e}^{\frac{1}{36}\ln p}$ or $k=\mathrm{e}^{-0.016103...}$ AG
(b)	$k^{t}=\frac{5000}{\text{their } A}$	M1		$\frac{5000}{12499}=0.400032...$; condone 4999
	$t\log(k)=\log\left(\frac{5000}{A}\right)$ $(t=56.89)$	m1		Correct use of logs
	$n=57$	A1		n integer; $n=57$ CAO
	Alternative ; trial and improvement on $5000=12499\times0.984025^{t}$ 2 values of $t\ge40$ 1 value of t $50<t<60$ $n=57$	(M1) (m1) (A1)	3	
	Special case, answer only $n=57$ 3/3 $n=56$ 0/3 $n=56.9$ 2/3			
	Total		**6**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
5	$8x+2y\frac{\mathrm{d}y}{\mathrm{d}x}=3y+3x\frac{\mathrm{d}y}{\mathrm{d}x}$			
	$8x$ and $4\to 0$	B1		
	$2y\frac{\mathrm{d}y}{\mathrm{d}x}$	B1		
	$3y+3x\frac{\mathrm{d}y}{\mathrm{d}x}$	M1 A1		Two terms with one $\frac{\mathrm{d}y}{\mathrm{d}x}$
	at $(1,3)$ (gradient) $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{3}$	A1	5	CSO
	Total		**5**	
6(a)(i)	$\cos 2x=2\cos^2 x-1$	B1		Seen in question, in consistent variable
	$3(2\cos^2 x-1)+7\cos x+5$	M1		Substitute candidate's $\cos 2x$ in terms of $\cos x$
	$6\cos^2 x+7\cos x+2(=0)$	A1	3	
(ii)	$(2\cos x+1)(3\cos x+2)$	M1		Attempt factors; formula ('a' and 'c' correct; allow one slip)
	$\cos x=-\frac{1}{2}$ $\quad\cos x=-\frac{2}{3}$	A1	2	Accept $-0.5, -0.67$ $x=\cos^{-1}\left(-\frac{1}{2}\right)$; $\cos^{-1}\left(-\frac{2}{3}\right)$
(b)(i)	$R=\sqrt{58}$	B1		Accept 7.6 or better
	$\alpha=\sin^{-1}\left(\frac{3}{\text{their } R}\right)$	M1		OE $\alpha=\sin^{-1}\left(\frac{3}{7}\right)$
	$=23.2°$	A1	3	AWRT 23.2° (23.1985…)
(ii)	$\alpha+\theta=\sin^{-1}\left(\frac{4}{\text{their } R}\right)$	M1		Candidate's R , α
	$\theta=8.5°$	A1F		F on α, AWRT, condone 8.6
	$\theta=125.1°$	A1	3	Two solutions only, but ignore out of range
(c)(i)	$h^2=1+(2\sqrt{2})^2$	M1		Pythagoras with h or sec x
	$h=3\Rightarrow\cos\beta=\frac{1}{3}$	A1	2	AG
(ii)	$\sin 2\beta=2\sin\beta\cos\beta$	M1		
	$\sin 2\beta=\frac{4}{9}\sqrt{2}$	A1	2	CSO; accept $p=\frac{4}{9}$ (not 0.444…)
	Total		**15**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\left(AB^2=\right)(4-3)^2+(0--2)^2+(1-5)^2$	M1		Condone one sign error in one bracket
	$AB=\sqrt{21}$	A1	2	Accept 4.58 or better
(b)	$4=6+2\lambda \quad \Rightarrow \quad \lambda=-1$	M1		$\lambda=-1$
	$0=-1+(-1)\times(-1)$ $1=5+(-1)\times4$	A1	2	$\lambda=-1$ confirmed in other two equations Accept for M1A1 $\begin{bmatrix}6\\-1\\5\end{bmatrix}-\begin{bmatrix}2\\-1\\4\end{bmatrix}=\begin{bmatrix}4\\0\\1\end{bmatrix}$
	Special case $\begin{bmatrix}6\\-1\\5\end{bmatrix}+\lambda\begin{bmatrix}2\\-1\\4\end{bmatrix}=\begin{bmatrix}4\\0\\1\end{bmatrix}, \quad \lambda\begin{bmatrix}2\\-1\\4\end{bmatrix}=\begin{bmatrix}-2\\1\\-4\end{bmatrix}$ $\lambda=-1$	(B2)		M1 condone 1 slip
(c)	$\begin{bmatrix}3\\-2\\5\end{bmatrix}+\mu\begin{bmatrix}-1\\3\\8\end{bmatrix}=\begin{bmatrix}6\\-1\\5\end{bmatrix}+\lambda\begin{bmatrix}2\\-1\\4\end{bmatrix}$	M1		Equate vector equations PI by two equations in λ or μ
	$\left.\begin{array}{r}3-\mu=6+2\lambda\\-2+3\mu=-1-\lambda\end{array}\right\}$ eliminate λ *or* μ	m1		Form (any) two simultaneous equations and solve for λ or μ
	$\lambda=-2$ or $\mu=1$	A1		
	C has coordinates $(2, 1, -3)$	A1		CAO condone $\begin{bmatrix}2\\1\\-3\end{bmatrix}$
	$BC^2=(2-4)^2+(0-1)^2+(1--3)^2$ $BC=\sqrt{21}$	M1		Use C to find BC or AC or to find two angles
	$AB=BC \quad (=\sqrt{21})$	A1	6	AB = BC or $\angle A=\angle C$ $(=20.2°)$ stated
	Total		**10**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$\int x \text{ d}x = \int 150\cos 2t \text{ d}t$	B1		Correct separation; condone missing ∫ signs; must see dx, dt
	$\frac{1}{2}x^2 = 75\sin 2t \quad (+C)$	B1B1		Correct integrals Accept $\frac{1}{2}\times 150$
	$\left(20, \frac{\pi}{4}\right) \quad \frac{1}{2}\times 20^2 = 75\sin\left(2\times\frac{\pi}{4}\right)+C$	M1		C present. Use $\left(20, \frac{\pi}{4}\right)$ to find C
	$C = 125$	A1F		F on $x^2 = k\sin 2t$
	$x^2 = 150\sin 2t + 250$	A1	6	Correct integrals and evaluation of C
(b)(i)	$t = 13 \quad x^2 = 150\sin 26 + 250 \; (= 364.38)$	M1		Evaluate $x^2 = \text{f}\,(13)$; $x^2 = k\sin 2t + c$ with numerical k and t
	$x = 19.1 \text{ (cm)}$	A1	2	AWRT
(ii)	$\left.\begin{array}{l} x = 11 \quad \sin 2t = -\frac{129}{150} \quad (= -0.86) \\ \text{or} \quad 2t = -1.035.. , 4.176... \end{array}\right\}$	M1		
	$t = 2.1 (\text{seconds})$	A1	2	AWRT
	Total		**10**	
	TOTAL		**75**	

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General Certificate of Education

Mathematics 6360

MPC4 Pure Core 4

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2010 examination - January series

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Q	Solution	Marks	Total	Comments
1(a)(i)	$\text{f}(-1) = -15+19-4=0$	B1	1	
(ii)	$\text{f}\left(\frac{2}{5}\right)$	M1		evaluate **or** complete division leading to a numerical remainder
	$\left(15\times\frac{8}{125}+19\times\frac{4}{25}-4\right) = 0 \Rightarrow$ factor	A1	2	Or decimal equivalent (0.96 + 3.04 – 4) **or** zero remainder ⇒ factor
(b)	$(x+1)$ is a factor	B1		Stated or implied.
	Third factor is $(3x+2)$	M1 A1		Any appropriate method to find third factor
	$\frac{15x^2-6x}{\text{f}(x)} = \frac{3x(5x-2)}{(x+1)(5x-2)(3x+2)}$	M1		$\left\{\begin{array}{l}(5x-2)(3x^2\pm 5x\pm 2)+\text{attempt}\\ \text{to factorise}\\ \text{Factorise numerator correctly}\\ \text{and attempt to simplify}\end{array}\right.$
	$=\frac{3x}{(x+1)(3x+2)}$	A1	5	CSO no ISW
	Total		**8**	
2(a)	$R=\sqrt{10}$	B1		Accept $R = 3.16$ or better
	$\tan\alpha = 3$	M1		OE
	$\alpha = 1.249$ ignore extra out of range	A1	3	AWRT 1.25 SC $\alpha = 0.322$ B1 radians only
(b)(i)	minimum value $=-\sqrt{10}$	B1F	1	F on *R*
(ii)	$\cos(x-\alpha) = -1$	M1		
	$x = 4.391$	A1F	2	AWRT 4.39 51.56° or .. .57° or better
(c)	$\cos(x-\alpha) = \frac{2}{\sqrt{10}}$	M1		
	$x-\alpha = \pm$ 0.886 5.397 ignore extra out of range	A1		Two values, accept 2dp and condone 5.4 condone use of degrees
	x = 0.36296.. 2.13512..	A1F		F on $x-\alpha$, either value. AWRT
	$x = 0.363$ 2.135	A1	4	CSO 3dp or better
	Total		**10**	
(c)	**Alternative** $10\sin^2 x-12\sin x+3=0$	M1		Or equivalent quadratic using cos *x* $(\text{ie } \sin^2 x+\cos^2 x=1 \text{ used})$
	$\sin x$ = two numerical answers $-1 \le$ ans ≤ 1	A1F		Or equivalent using cos *x*
	x = one correct answer	A1F		
	$x = 0.363$ 2.135	A1		CSO 3 dp or better

MPC4 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)	$(1+x)^{\frac{1}{3}} = 1 \pm \frac{1}{3}x + kx^2$	M1		$1 \pm \frac{1}{3}x + kx^2$
	$= 1 - \frac{1}{3}x + \frac{2}{9}x^2$	A1	2	
(ii)	$\left(1+\frac{3}{4}x\right)^{-\frac{1}{3}} = 1 - \frac{1}{3}\times\frac{3}{4}x + \frac{2}{9}\left(\frac{3}{4}x\right)^2$	M1		x replaced by $\frac{3}{4}x$ or start binomial again; condone missing brackets
	$= 1 - \frac{1}{4}x + \frac{1}{8}x^2$	A1	2	
(b)	$\sqrt[3]{\frac{256}{4+3x}} = k\left(1+\frac{3}{4}x\right)^{-\frac{1}{3}}$	M1		$k \neq 1$
	$= 4\left(1 - \frac{1}{4}x + \frac{1}{8}x^2\right)$	A1F		F on (a)(ii) $k = 4$, accept $\sqrt[3]{64}$ or $64^{\frac{1}{3}}$
	$= 4 - x + \frac{1}{2}x^2$ or $a = 4 \quad b = -1 \quad c = \frac{1}{2}$	A1	3	CSO fully simplified Be convinced
	Total		**7**	
4(a)	$10x^2 + 8 = 2(x+1)(5x-1) +$	M1		A and B terms correct
	$A(5x-1) + B(x+1)$	A1		
	$x = -1 \quad x = \frac{1}{5}$	m1		Use two values of x to find A and B, or set up and solve
	$A = -3 \quad B = 7$	A1	4	$8 + 5A + B = 0$ $-2 - A + B = 8$ SC1 NWS A & B correct $4/4$ SC2 NWS A or B correct $1/4$
(b)	$\int \frac{10x^2+8}{(x+1)(5x-1)}\,dx = \int 2 - \frac{3}{x+1} + \frac{7}{5x-1}\,dx$	M1		Use the partial fractions
	$= 2x + C$	B1		
		M1		$a\ln(x+1) + b\ln(5x-1)$ condone missing brackets
	$-3\ln(x+1) + \frac{7}{5}\ln(5x-1)$	A1F	4	F on A and B
	Total		**8**	
5	$x^2 + xy = e^y$			
	$2x + y + x\frac{dy}{dx} = e^y\frac{dy}{dx}$	B1		$2x$
		M1		Use product rule
		A1		
		B1		RHS
	$(-1, 0) \quad \frac{dy}{dx} = -1$	A1	5	CSO
	Total		**5**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta$	B1 B1	2	OE condone use of x etc, but variable must be consistent
(ii)	$\sin\theta = \frac{4}{5} \Rightarrow \sin 2\theta = 2\times\frac{4}{5}\times\frac{3}{5} = \frac{24}{25}$ **or** $2\times\sin\left(\cos^{-1}\frac{3}{5}\right)\times\frac{3}{5}$	B1		AG Use of 106.26° …. B0
	$\cos 2\theta = \frac{9}{25} - \frac{16}{25} = -\frac{7}{25}$	B1	2	- 0.28
(b)(i)	$\frac{dx}{d\theta} = 6\cos 2\theta$, $\frac{dy}{d\theta} = -8\sin 2\theta$	M1 A1		Attempt both derivatives. ie $p\cos2\theta$ Both correct. $q\sin2\theta$
	$\frac{dy}{dx} = -\frac{4}{3}\frac{\sin 2\theta}{\cos 2\theta}$ ISW	A1	3	CSO OE
(ii)	$P\left(\frac{72}{25}, -\frac{28}{25}\right)$	B1F		(2.88,- 1.12)
	Gradient = $= -\frac{4}{3}\times-\frac{24}{7}$	M1		Their $\frac{q\sin2\theta}{p\cos2\theta}$ **or** $\frac{p\cos2\theta}{q\sin2\theta}$ must be working with rational numbers
	Tangent $y + \frac{28}{25} = \frac{32}{7}\left(x - \frac{72}{25}\right)$ ISW	A1	3	Any correct form. $7y = 32x - 100$ Fractions in simplest form Equation required
	Total		**10**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
7	$\int y\ \mathrm{d}y = \int \cos\left(\frac{x}{3}\right)\mathrm{d}x$	B1		Separate; condone missing integral signs.
	$\frac{1}{2}y^2 = 3\sin\left(\frac{x}{3}\right) + (C)$	B1 B1		Accept $\dfrac{\sin\left(\frac{x}{3}\right)}{\frac{1}{3}}$
	$\left(\frac{\pi}{2},1\right) \quad \frac{1}{2} = 3\sin\frac{\pi}{6} + C$	M1		Use $\left(\frac{\pi}{2},1\right)$ to find C must be in form $\text{p}y^2 = q\sin\left(\frac{x}{3}\right) + C$
	$C = -1$	A1F		
	$y^2 = 6\sin\left(\frac{x}{3}\right) - 2$	A1	6	CSO
	Total		**6**	
8(a)	$0 = 2 + \lambda \Rightarrow \lambda = -2$	M1		
	Check $\quad -1 + -2 \times -3 = -1 + 6 = 5$			
	$-5 - 2 \times 2 = -5 \times -4 = -9$	A1	2	OE
(b)	$\overrightarrow{BC} = \begin{bmatrix} 9 \\ 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 0 \\ 5 \\ -9 \end{bmatrix} = \begin{bmatrix} 9 \\ -3 \\ 12 \end{bmatrix}$	M1 A1	2	$\pm\left(\overrightarrow{OC} - \overrightarrow{OB}\right)$
(c)(i)	$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = \overrightarrow{OA} + 2\overrightarrow{BC}$	M1		
	$\overrightarrow{OD} = \begin{bmatrix} 2 \\ -1 \\ -5 \end{bmatrix} + \begin{bmatrix} 18 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 20 \\ -7 \\ 19 \end{bmatrix}$			
	D is $(20, -7, 19)$	A1	2	AG
(ii)	$\overrightarrow{PD} = \overrightarrow{OD} - \overrightarrow{OP} =$			
	$\begin{bmatrix} 20 \\ -7 \\ 19 \end{bmatrix} - \begin{bmatrix} 2 + p \\ -1 - 3p \\ -5 + 2p \end{bmatrix} = \begin{bmatrix} 18 - p \\ -6 + 3p \\ 24 - 2p \end{bmatrix}$	M1 A1		Find $\overrightarrow{PD}$ in terms of p condone $\overrightarrow{PD} = \overrightarrow{OP} - \overrightarrow{OD}$ here
	$\overrightarrow{PD} \bullet \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} = 0$	B1		
	$(18 - p) \times 1 + (-6 + 3p) \times -3 + (24 - 2p) \times 2 = 0$	m1		
	$p = 6$	A1	5	CSO OE working with $\overrightarrow{DP}$
	Total		**11**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
9(a)(i)	$t=0 \qquad h=A(1-1)=0$	B1	1	
(ii)	$57=A\left(1-\mathrm{e}^{-\frac{12}{4}}\right)$	M1		
	$A=\dfrac{57}{\left(1-\mathrm{e}^{-3}\right)}\approx 60$	A1	2	Or 59.9… seen. A = correct expression $\approx$ 60 2 sf
(b)(i)	$h=48 \qquad \dfrac{48}{60}=1-\mathrm{e}^{-\frac{1}{4}t}$	M1		
	$\ln\left(\mathrm{e}^{-\frac{1}{4}t}\right)=\ln\left(\dfrac{1}{5}\right)$	m1		
	$-\dfrac{1}{4}t=-\ln 5 \Rightarrow t=4\ln 5$	A1	3	
(ii)	$\dfrac{\mathrm{d}h}{\mathrm{d}t}=-\dfrac{1}{4}\times -60\times \mathrm{e}^{-\frac{1}{4}t}$	M1		Differentiate, condone sign errors
	$60e^{-\frac{1}{4}t}=60-h \Rightarrow \dfrac{\mathrm{d}h}{\mathrm{d}t}=\dfrac{1}{4}(60-h)$	m1		Eliminate $\mathrm{e}^{-\frac{1}{4}t}$
	$\dfrac{\mathrm{d}h}{\mathrm{d}t}=15-\dfrac{h}{4}$	A1	3	CSO, AG
(iii)	$h=8$	B1	1	
	Total		**10**	
	TOTAL		**75**	

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General Certificate of Education
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Mathematics MPC4

Pure Core 4

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC4

Q	Solution	Marks	Total	Comments
1(a)	$\mathrm{f}\left(\frac{1}{4}\right)=8\times\frac{1}{64}+6\times\frac{1}{16}-14\times\frac{1}{4}-1$	M1		Use $x=\frac{1}{4}$ in evaluation
	$=-4$	A1	2	NMS 2/2; no ISW
(b)(i)	$\mathrm{g}\left(\frac{1}{4}\right)$ = number(s) + d = 0	M1		Use factor theorem to find d See some processing
	$d=3$	A1	2	NMS 2/2
(ii)	$\mathrm{g}(x)=(4x-1)(2x^2+bx-3)$	B1F		$a=2 \quad c=-3$; F on d $(c=-d)$
	$x^2 \quad 6=4b-2$ or $x \quad -14=-b-12$	M1		Any appropriate method; PI
	$b=2$	A1	3	NMS 2/2
	Total		**7**	
	Alternatives:			
(a)	$2x^2+2x-3$ $4x-1\overline{)8x^3+6x^2-14x-1}$ $8x^3-\underline{2x^2}$ $8x^2-14x$ $8x^2\underline{-2x}$ $-12x-1$ $-12x\underline{+3}$ -4	(M1)		Complete division with integer remainder
		(A1)	(2)	Remainder = –4 stated
(b)(i)	Division as for (a) $\Rightarrow d-3$ last line	(M1)		Candidate's -3
	$d=3$	(A1)	(2)	
2(a)	$\frac{\mathrm{d}x}{\mathrm{d}t}=-3 \quad \frac{\mathrm{d}y}{\mathrm{d}t}=6t^2$	B1		Both derivatives correct; PI
	$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{6t^2}{3}$	M1		Correct use of chain rule
	$=-2t^2$	A1	3	CSO
(b)	$t=1 \quad m_{\mathrm{T}}=-2 \quad m_{\mathrm{N}}=\frac{1}{2}$	M1		Substitute $t=1 \quad m_N=-\frac{1}{m_T}$
		A1F		F on gradient; $m_{\mathrm{T}}\neq\pm1$
	Attempt at equation of normal using $(x,y)=(-2,3)$	M1		Condone one error
	Normal has equation $y-3=\frac{1}{2}(x+2)$	A1	4	CSO; ACF
(c)	$t=\frac{1-x}{3}$ or $t=\sqrt[3]{\frac{y-1}{2}}$	M1		Correct expression for t in terms of x or y
	$y=1+2\left(\frac{1-x}{3}\right)^3$	A1	2	ACF
	Total		**9**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)	$7x-3=A(3x-2)+B(x+1)$	M1		
	$x=-1 \qquad x=\frac{2}{3}$	m1		Substitute two values of *x* and solve for *A* and *B*
	$A=2 \qquad B=1$	A1	3	Or solve $\left.\begin{array}{r}7=3A+B\\-3=-2A+B\end{array}\right\}$ condone one error
(ii)	$\int\frac{7x-3}{(x+1)(3x-2)}\,\mathrm{d}x=$			
	$p\ln(x+1)+q\ln(3x-2)$	M1		Condone missing brackets
	$=2\ln(x+1)+\frac{1}{3}\ln(3x-2)\ (+c)$	A1F	2	F on *A* and *B*; constant not required
(b)	$\frac{6x^2+x+2}{2x^2-x+1}=\frac{6x^2-3x+3+4x-1}{2x^2-x+1}$	M1		
	$=3+\frac{4x-1}{2x^2-x+1}$	B1 A1	3	$P=3$ $Q=4$ and $R=-1$
	Total		**8**	
	Alternatives:			
(a)(i)	By cover up rule			
	$x=-1 \quad A=\frac{-7-3}{-5}$			
	$x=\frac{2}{3} \quad B=\frac{\frac{14}{3}-3}{\frac{5}{3}}$	(M1)		$x=-1$ and $x=\frac{2}{3}$ and attempt to find *A* and *B*
	$A=2 \quad B=1$	(A1,A1)	(3)	SC NMS *A* and *B* both correct 3/3 One of *A* or *B* correct 1/3
(b)	$\begin{array}{rl} & \qquad\qquad 3 \\ 2x^2-x+1 & \overline{)6x^2+\ \ x+2} \\ & \ \ \underline{6x^2-3x+3} \\ & \qquad\quad 4x-1\end{array}$	(M1) (B1) (A1)	(3)	Complete division, with *ax* + *b* remainder *P* = 3 stated *Q* = 4 and *R* = –1 stated or written as expression
	or			
	$6x^2+x+2=P(2x^2-x+1)+Qx+R$			
	$=2Px^2+(Q-P)x+P+R$	(M1)		Multiply across and equate coefficients or use numerical values of *x*
	$P=3$	(B1)		*P* = 3 stated
	$Q-P=1$			
	$P+R=2$			
	$Q=4$ and $R=-1$	(A1)	(3)	*Q* = 4 and *R* = –1 stated or written as expression

MPC4 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$(1+x)^{\frac{3}{2}}=1+\frac{3}{2}x+kx^2$	M1		
	$=1+\frac{3}{2}x+\frac{3}{8}x^2$	A1	2	
(ii)	$(16+9x)^{\frac{3}{2}}=16^{\frac{3}{2}}\left(1+\frac{9}{16}x\right)^{\frac{3}{2}}$	B1		
	$=k\left(1+\frac{3}{2}\times\frac{9}{16}x+\frac{3}{8}\left(\frac{9}{16}x\right)^2\right)$	M1		x replaced by $\frac{9}{16}x$ or start binomial again Condone missing brackets
	$=64+54x+\frac{243}{32}x^2$	A1	3	Accept $7.59375x^2$
(b)	$x=-\frac{1}{3}$	M1		Use $x=-\frac{1}{3}$
	$13^{\frac{3}{2}}\approx 46+\frac{27}{32}$	A1	2	46 seen with a = 27 b =32, or $\left(\frac{k\times 27}{k\times 32}\right)$
	Total		**7**	
	Alternative:			
(a)(ii)	$(16+9x)^{\frac{3}{2}}=$ $16^{\frac{3}{2}}+\frac{3}{2}\times 16^{\frac{1}{2}}\times 9x+\frac{3}{2}\times\frac{1}{2}\times\frac{1}{2}\times 16^{-\frac{1}{2}}\times(9x)^2$	(M1)		Use $(a+bx)^n$ from FB. Allow one error. Condone missing brackets.
	$=64+54x+\frac{243}{32}x^2$	(A2)	(3)	Accept $7.59375x^2$
5(a)(i)	$\cos 2x=1-2\sin^2 x$	B1		ACF in terms of sin (PI later)
	$3\left(1-2\sin^2 x\right)+2\sin x+1=0$	M1		Substitute candidate's cos2x in terms of sin x (at least 2 terms)
	$-6\sin^2 x+2\sin x+4=0$			
	$3\sin^2 x-\sin x-2=0$	A1	3	AG
(ii)	$(3\sin x+2)(\sin x-1)=0$	M1		Factorise correctly or use formula correctly
	$\sin x=-\frac{2}{3}$ $\quad\sin x=1$	A1	2	Both; condone -0.67 or -0.66 or better
(b)(i)	$R=\sqrt{13}$	B1		Accept 3.6 or better
	$\tan\alpha=\frac{2}{3}$ $\quad\alpha=33.7$	M1A1	3	OE; accept α = 33.69(0)
(ii)	$2x-\alpha=\cos^{-1}\left(\frac{-1}{R}\right)$	M1		Candidate's R. Or $\cos(2x-\alpha)=\frac{-1}{R}$
	$2x-\alpha=106.1°,\ 253.9°$			
	$x=69.9^\circ,\ 143.8^\circ$	A1		One correct answer
		A1	3	Both correct, no extras in range
	Total		**11**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$x^3 + \cos\pi = 7 \Rightarrow x^3 - 1 = 7$	M1		Or $x = \sqrt[3]{7 - \cos\pi}$
	$x = 2$	A1	2	CSO
(b)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^3 y\right) = 3x^2 y + x^3 \frac{\mathrm{d}y}{\mathrm{d}x}$	M1		2 terms added, one with $\frac{\mathrm{d}y}{\mathrm{d}x}$
		A1		
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\cos\pi y\right) = -\pi\sin\left(\pi y\right)\frac{\mathrm{d}y}{\mathrm{d}x}$	B1		
	At $(2,1)$ $\quad 3\times 4 + 8\frac{\mathrm{d}y}{\mathrm{d}x} - \pi\sin\pi\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	M1		Substitute candidate's *x* from (a) and *y* = 1 with 0 on RHS and both derivatives attempted and no extra derivatives
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{3}{2}$	A1	5	CSO; OE
	Total		**7**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\overrightarrow{OB}=\begin{bmatrix}1\\-1\\2\end{bmatrix}$	B1		PI
	$\overrightarrow{AB}=\begin{bmatrix}1\\-1\\2\end{bmatrix}-\begin{bmatrix}4\\-3\\2\end{bmatrix}=\begin{bmatrix}-3\\2\\0\end{bmatrix}$	M1 A1	3	Use $\pm\left(\overrightarrow{OB}-\overrightarrow{OA}\right)$
(b)(i)	$4+2\lambda=-1+\mu$ $-3\quad=3-2\mu$ $2+\lambda\quad=4-\mu$	M1		$\begin{bmatrix}4+2\lambda\\-3\\2+\lambda\end{bmatrix}=\begin{bmatrix}1+\mu\\3-2\mu\\4-\mu\end{bmatrix}$ or set up 3 equations
	$-6=-2\mu\qquad\mu=3$	m1		Solve for λ and μ
	$\lambda=4-3-2\qquad\lambda=-1$	A1		Both correct
	$4+2\lambda=4-2=2$ $-1+\mu=-1+3=2$	A1	4	Independent check with conclusion: minimum "intersect"
(ii)	P is $(2,-3,1)$	B1	1	
(c)	$\overrightarrow{OC}=\overrightarrow{OA}+\overrightarrow{AC}$ $=\overrightarrow{OA}+\overrightarrow{PB}$			Or $\overrightarrow{OC}=\overrightarrow{OB}+\overrightarrow{BC}$ $=\overrightarrow{OB}+\overrightarrow{PA}$
	$\overrightarrow{OC}=\begin{bmatrix}4\\-3\\2\end{bmatrix}+\begin{bmatrix}1-2\\-1--3\\2-1\end{bmatrix}$	M1		$\overrightarrow{OA}+\overrightarrow{PB}$ in components
	C is $(3,-1,3)$	A1		
	or $\overrightarrow{OC}=\overrightarrow{OB}+\overrightarrow{BC}$ $=\overrightarrow{OB}+\overrightarrow{AP}$			
	$\overrightarrow{OC}=\begin{bmatrix}1\\-1\\2\end{bmatrix}+\begin{bmatrix}2-4\\-3--3\\1-2\end{bmatrix}$	M1		$\overrightarrow{OB}+\overrightarrow{AP}$ in components
	C is $(-1,-1,1)$	A1	4	
	Total		**12**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
7(c)	**Alternative:** $\overrightarrow{AP}=\overrightarrow{BC}$ $\lvert \overrightarrow{AP} \rvert = \lvert \overrightarrow{BC} \rvert =$ $\sqrt{(2-4)^2+(-3--3)^2+(1-2)^2}$ $=\sqrt{5}$	(M1)		
	$\overrightarrow{BC}=k\begin{pmatrix}2\\0\\1\end{pmatrix} \qquad \lvert \overrightarrow{BC} \rvert = \sqrt{k}\sqrt{5}$ so $k=\pm 1$	(A1*)		For $k=1$ and $k=-1$
	$\overrightarrow{OC}=\overrightarrow{OB}+k\begin{pmatrix}2\\0\\1\end{pmatrix}$ $=\begin{pmatrix}1\\-1\\2\end{pmatrix}+\begin{pmatrix}2\\0\\1\end{pmatrix}$ or $\begin{pmatrix}1\\-1\\2\end{pmatrix}-\begin{pmatrix}2\\0\\1\end{pmatrix}$	(M1)		Either
	$=\begin{pmatrix}3\\-1\\3\end{pmatrix}$ or $\begin{pmatrix}-1\\-1\\1\end{pmatrix}$	(A1)	(4)	Both
	*****If k = 1 or $k=-1$ (ie only one k), one correct point gets 2/4			

MPC4 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$\int \frac{\mathrm{d}x}{\sqrt{x+1}} = \int -\frac{1}{5}\,\mathrm{d}t$	B1		Correct separation; or $\frac{\mathrm{d}t}{\mathrm{d}x} = -5(x+1)^{-\frac{1}{2}}$ Condone missing integral signs
	$2\sqrt{x+1} = -\frac{1}{5}t \quad (+C)$	B1B1		Correct integrals; condone $\frac{\sqrt{x+1}}{\frac{1}{2}}$
	$x = 80 \quad t = 0 \quad C = 2\sqrt{81}$	M1		Use $(0, 80)$ to find a constant C
	$= 18$	A1F		F on integrals if in form $\sqrt{x+1} = qt + c$
	$x = \left(9 - \frac{1}{10}t\right)^2 - 1$	A1	6	OE; CSO; x = correct expression in t
(b)	$t = 60 \quad x = \mathrm{f}(60)$	M1		Evaluate $\mathrm{f}(60)$, ie x = … (C not required)
	$= 8$	A1	2	CSO
(c)(i)	$\frac{\mathrm{d}A}{\mathrm{d}t} = kA(9 - A)$	M1		$\frac{\mathrm{d}A}{\mathrm{d}t}$ = product involving A; k required Condone terms in t
		A1	2	
(ii)	$4.5 = \frac{9}{1 + 4\mathrm{e}^{-0.09t}}$	M1		Condone one slip in denominator
	$\mathrm{e}^{-0.09t} = \frac{1}{4}$	A1		
	$-0.09t = \ln\left(\frac{1}{4}\right)$	m1		Take ln correctly
	$t = \frac{\ln\left(\frac{1}{4}\right)}{-0.09}$			
	$= 15.4 \text{ (hours)}$	A1	4	CAO; condone more than 3sf if correct 15.40327068 Allow 15h 24m
	Total		**14**	
	TOTAL		**75**	

Version 1.0

General Certificate of Education (A-level)
January 2011

Mathematics MPC4

(Specification 6360)

Pure Core 4

Mark Scheme

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Registered address: AQA, Devas Street, Manchester M15 6EX.

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

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MPC4

Q	Solution	Marks	Total	Comments
1(a)	$R=\sqrt{29}$	B1		Accept 5.4 or 5.38, 5.39, 5.385….
	$R\sin\alpha=5$ or $R\cos\alpha=2$ or $\tan\alpha=\frac{5}{2}$	M1		
	$\alpha=68.2^{\circ}$	A1	3	Condone $\alpha=68.20^{\circ}$
(b)(i)	(maximum value =) $\sqrt{29}$	B1ft	1	ft on *R*
(ii)	$\sin(x+\alpha)=1$	M1		Or $x+\alpha=90$, $x+\alpha=\frac{\pi}{2}$
	$x=21.8^{\circ}$ only	A1	2	No ISW
	Total		**6**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
2 (a)(i)	$\text{f}\left(-\frac{1}{3}\right)=9\left(-\frac{1}{3}\right)^3+18\left(-\frac{1}{3}\right)^2-\left(-\frac{1}{3}\right)-2$ $=9\left(-\frac{1}{27}\right)+18\left(\frac{1}{9}\right)-\left(-\frac{1}{3}\right)-2$ $=-\frac{1}{3}+2+\frac{1}{3}-2=0$	M1		$\text{f}\left(-\frac{1}{3}\right)$ attempted **NOT** long division
	$\Rightarrow (3x+1)$ is a factor	A1	2	Shown = 0 plus statement
(ii)	**(f**$(x)=$) $(3x+1)\left(3x^2+kx-2\right)$	M1		3 and -2
	$k=5$	A1		
	(f$(x)=$) $(3x+1)(3x-1)(x+2)$	A1	3	
(iii)	$9x^3+21x^2+6x=x\left(9x^2+21x+6\right)$	M1		x and attempt to factorise quadratic equation.
	$=3x(3x+1)(x+2)$	A1		Correct factors
	$\dfrac{9x^3+21x^2+6x}{\text{f}(x)}=\dfrac{3x}{3x-1}$	A1	3	cso no ISW
(b)	$9\left(\frac{2}{3}\right)^3+p\left(\frac{2}{3}\right)^2-\frac{2}{3}-2=-4$	M1		Condone missing brackets, but must have $=-4$
	$p=-9$	A1	2	
			10	
2(a)(ii)	**Alternative** Using long division			
	$3x^2+5x-2$ $3x+1\overline{)9x^3+18x^2-x-2}$ $9x^3+3x^2$ $\overline{15x^2-x}$ $15x^2+5x$	(M1)		$3x^2+ax+b$
	$\overline{-6x-2}$ $-6x-2$	(A1)		$3x^2+5x-2$
	(f $(x)=$) $(3x+1)(3x-1)(x+2)$	(A1)	(3)	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
2(a)(iii)	**Alternative** $\dfrac{\mathrm{f}(x)+\mathrm{q}(x)}{\mathrm{f}(x)}$, where q is a quadratic expression $= 1+\dfrac{(3x+1)(x+2)}{(3x+1)(3x-1)(x+2)}$ $=1+\dfrac{1}{3x-1}$	(M1) (A1) (A1)	(3)	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$3+9x=A(3+5x)+B(1+x)$	M1		PI by correct A and B
	$x=-1$ $\quad x=-\frac{3}{5}$	m1		Substitute two values of *x* and solve for *A* and *B*.
	$A=3$ $\quad B=-6$	A1	3	
	Alternative Equating coefficients			
	$3+9x=A(3+5x)+B(1+x)$	(M1)		
	$3=3A+B$ $9=5A+B$	(m1)		Set up simultaneous equations and solve. Condone 1 error.
	$A=3$ $\quad B=-6$	(A1)	(3)	
	Alternative Cover up rule			
	$x=-1 \quad A=\frac{3-9}{3-5}$ $x=-\frac{3}{5} \quad B=\frac{3-\frac{27}{5}}{1-\frac{3}{5}}$	(M1)		$x=-1$ and $x=-\frac{3}{5}$ and attempt to find *A* and *B*.
	$A=3$ $\quad B=-6$	(A1 A1)	(3)	SC NMS *A* and *B* both correct; 3/3 One of *A* and *B* correct 1/3
(b)	$(1+x)^{-1}=1-x+kx^2$	M1		
	$=1-x+x^2$	A1		
	$(3+5x)^{-1}=3^{-1}\left(1+\frac{5}{3}x\right)^{-1}$	B1		
	$\left(1+\frac{5}{3}x\right)^{-1}=1-\frac{5}{3}x+\left(\frac{5}{3}x\right)^2$	M1		Condone missing brackets; allow one sign error
	$=1-\frac{5}{3}x+\frac{25}{9}x^2$	A1		
	$\frac{3+9x}{(1+x)(3+5x)}$ $=3\left(1-x+x^2\right)-6\times3^{-1}\left(1-\frac{5}{3}x+\frac{25}{9}x^2\right)$	M1		Use PFs and simplify to $a+bx+cx^2$ **or** expand product of $(3+9x)$ and binomial expansions and simplify to $a+bx+cx^2$
	$=1+\frac{1}{3}x-\frac{23}{9}x^2$	A1	7	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
(c)	$\frac{5x}{3} < 1$ oe or $\frac{5x}{3} > -1$ oe	M1		Condone $\leq$ instead of $<$
	$\lvert x \rvert < \frac{3}{5}$ or $-\frac{3}{5} < x < \frac{3}{5}$	A1	**2**	CAO
			12	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$\frac{\mathrm{d}x}{\mathrm{d}t} = 3e^{t}$ $\quad\frac{\mathrm{d}y}{\mathrm{d}t} = 2e^{2t} + 2e^{-2t}$	M1 A1		Both derivatives attempted and one correct Both correct
	$t = 0$ $\quad$ gradient $= \frac{4}{3}$	A1	3	cso Condone $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4}{3}$
(ii)	$y = \frac{4}{3}(x-3)$ oe	B1ft	1	ft on non-zero gradient
(b)	$\mathrm{e}^{2t} = \frac{x^2}{9}$ or $9\mathrm{e}^{2t} = x^2$ or $\mathrm{e}^{t} = \frac{x}{3}$ or $\mathrm{e}^{2t} = \left(\frac{x}{3}\right)^2$ or $t = \ln\left(\frac{x}{3}\right)$ or $2t = \ln\left(\frac{x^2}{9}\right)$	M1		
	$y = \frac{x^2}{9} - \frac{9}{x^2}$	A1	2	Equation required
			6	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$m = 10 \times 2^{-\frac{14}{8}}$	M1		
	≈ 3 (gm)	A1	2	Condone 2.97 or better NOT 2.9 as final answer
(b)	$2^{-\frac{d}{8}} = \frac{1}{16}$	M1		
	$\frac{d}{8} = 4 \Rightarrow d = 32$	A1	2	cso
(c)	$0.01m_0 = m_0 \times 2^{-\frac{t}{8}}$	M1		m_0 can be numerical
	$\ln(0.01) = -\frac{t}{8}\ln(2)$	M1		Take logs correctly from their equation leading to a linear equation in *t*.
	$t = 53.15$			
	$n = 54$	A1	3	cso
			7	

Q	Solution	Marks	Total	Comments
6(a)(i)	$\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$	B1		Condone numerator as tan x + tan x
	$2\tan x + \tan x\left(1-\tan^2 x\right) = 0$	M1		Multiplying throughout by their denominator
	$\tan x = 0$ or $\left(2+1-\tan^2 x\right) = 0 \Rightarrow \tan^2 x = 3$	A1	3	**AG** Must show tan x = 0 **and** $\tan^2 x = 3$
	Alternative			
	$\tan 2x = \dfrac{\sin 2x}{\cos 2x} = \dfrac{2\sin x\cos x}{\cos^2 x - \sin^2 x}$			
	$\dfrac{2\sin x\cos x}{\cos^2 x - \sin^2 x} + \dfrac{\sin x}{\cos x} = 0$	(B1)		
	$2\sin x\cos^2 x + \sin x\left(\cos^2 x - \sin^2 x\right) = 0$			
	$\sin x(2\cos^2 x + \cos^2 x - \sin^2 x) = 0$	(M1)		
	$\left.\begin{array}{l}\Rightarrow \sin x = 0\\ \Rightarrow \tan x = 0\end{array}\right\}\begin{array}{c}\text{and}\\ \text{and}\end{array}\left.\begin{array}{c}3\cos^2 x = \sin^2 x\\ \tan^2 x = 3\end{array}\right\}$	(A1)	(3)	
(ii)	$x = 60$ **AND** $x = 120$	B1	1	Condone extra answers outside interval eg 0 and 180
(b)(i)	$2\sin x\cos x = \cos x.\mathrm{f}\,(x)$	M1		Where f(x) = $\cos^2 x - \sin^2 x$ or $2\cos^2 x - 1$ or $1 - 2\sin^2 x$
	$2\sin x\cos x = \cos x\left(1-2\sin^2 x\right)$	A1		
	$\left(\cos x \neq 0\right)$ $2\sin x = 1 - 2\sin^2 x$ $2\sin^2 x + 2\sin x - 1 = 0$	A1	3	**AG**

(ii)	$\sin x = \dfrac{-2 \pm \sqrt{4 - 4 \times 2 \times (-1)}}{2 \times 2}$	M1		Correct use of quadratic formula or completing the square or correct factors
	$\sin x = \dfrac{-2 \pm 2\sqrt{3}}{4}$	A1		$\sqrt{12}$ must be simplified and must have ±
	$\left.\begin{array}{l}\sin x = \dfrac{-1-\sqrt{3}}{2} \text{ has no solution} \\ \sin x = \dfrac{\sqrt{3}-1}{2}\end{array}\right\}$	E1	3	Reject one solution and state correct solution.
			10	

MPC4

Q	Solution	Marks	Total	Comments
7 **(a)(i)**	$\int \frac{\mathrm{d}x}{\sqrt{x}} = \int \sin\left(\frac{t}{2}\right) \mathrm{d}t$	B1		Correct separation; condone missing integral signs.
	$2\sqrt{x} = -2\cos\left(\frac{t}{2}\right)$ $(+\,k)$	M1		$p\sqrt{x} = q\cos\left(\frac{t}{2}\right)$ Condone missing + k
	$x = \left(-\cos\left(\frac{t}{2}\right) + C\right)^2$	A1	3	Must have previous line correct
(ii)	$(1,0) \quad 2 = -2 + k$ or $1 = (-1 + C)^2$	M1		Use $(1,0)$ to find a constant
	$k = 4$ or $C = 2$	A1ft		ft on $C = p - q$ from (a)(i)
	$x = \left(2 - \cos\left(\frac{t}{2}\right)\right)^2$	A1	3	cso applies to (a)(ii)
(b)(i)	Greatest height when $\cos(bt) = -1$	M1		
	Greatest height = 9 (m)	A1ft	2	ft is (their a + 1)^2
(ii)	$\cos\left(\frac{t}{2}\right) = 2 - \sqrt{5}$	M1		$\cos bt = a - \sqrt{5}$
	$t = 2\cos^{-1}\left(2 - \sqrt{5}\right) = 3.6$ (seconds 1dp)	A1	2	condone 3.6 or better (3.618…..)
			10	

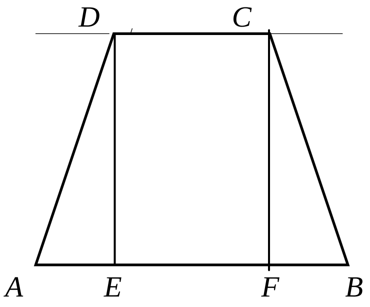
MPC4 (cont)

Q	Solution	Marks	Total	Comments
8(a)(i)	$\overrightarrow{AB}=\begin{bmatrix}6\\0\\3\end{bmatrix}-\begin{bmatrix}3\\-2\\4\end{bmatrix}=\begin{bmatrix}3\\2\\-1\end{bmatrix}$	M1 A1	2	$\pm\left(\overrightarrow{OB}-\overrightarrow{OA}\right)$ implied by 2 correct components
(ii)	$\begin{bmatrix}3\\2\\-1\end{bmatrix}\bullet\begin{bmatrix}2\\-1\\3\end{bmatrix}=6-2-3=1$	M1 A1ft		Scalar product with correct vectors; allow one component error. ft on $\overrightarrow{AB}$
	$\cos\theta=\frac{sp}{\sqrt{14}\sqrt{14}}$	m1		Correct form for $\cos\theta$ with one correct modulus
	$\cos\theta=\frac{1}{14}\qquad\theta=85.9^\circ$	A1	4	cso 85.9 or better
(b)(i)	$\overrightarrow{OD}=\begin{bmatrix}3\\-2\\4\end{bmatrix}+2\begin{bmatrix}2\\-1\\3\end{bmatrix}=\begin{bmatrix}7\\-4\\10\end{bmatrix}$	M1		Implied by 2 correct components
	line $l_2\quad\mathbf{r}=\begin{bmatrix}7\\-4\\10\end{bmatrix}+\mu\begin{bmatrix}3\\2\\-1\end{bmatrix}$	A1ft	2	$\mathbf{r}=$ or $\begin{bmatrix}x\\y\\z\end{bmatrix}$ required ft on $\overrightarrow{AB}$
(ii)	$\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}=\begin{bmatrix}1+3p\\-4+2p\\7-p\end{bmatrix}$	M1		$\mu=p$ at C Find $\overrightarrow{BC}$ in terms of p
	$\overrightarrow{AD}=\begin{bmatrix}4\\-2\\6\end{bmatrix}\qquad\left\|\overrightarrow{BC}\right\|=\sqrt{56}$	B1ft		PI B1 is for $\left\|\overrightarrow{BC}\right\|=\sqrt{56}$
	$(1+3p)^2+(-4+2p)^2+(7-p)^2=56$	m1		
	$14p^2-24p+66=56$ $7p^2-12p+5=0$ $(7p-5)(p-1)=0$	m1		ft on $\overrightarrow{BC}$ Simplification to quadratic equation with all terms on one side
	$p=\frac{5}{7}$ and $p=1$	A1		Exact fraction required
	C is at $\left(9\frac{1}{7},-2\frac{4}{7},9\frac{2}{7}\right)$	A1	6	cso Accept as column vector
			14	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
8(b)(ii)	**Alternative** : Using equal angles			$\mu = p$ at C
	$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \begin{bmatrix} 1+3p \\ -4+2p \\ 7-p \end{bmatrix}$	(M1)		Find $\overrightarrow{BC}$ in terms of p
	$\overrightarrow{AD} = \begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix}$ $\quad \lvert\overrightarrow{BC}\rvert = \sqrt{56}$	(B1ft)		
	$(\cos\theta) = \dfrac{\overrightarrow{BA} \bullet \overrightarrow{BC}}{\sqrt{14}\sqrt{56}} = \dfrac{\begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix} \bullet \begin{bmatrix} 1+3p \\ -4+2p \\ 7-p \end{bmatrix}}{\sqrt{14}\sqrt{56}} = \dfrac{1}{14}$	(m1)		Condone $\overrightarrow{AB}$ used. Allow $\lvert\overrightarrow{BC}\rvert$ in terms of p, in which case previous B1 is implied
	$-3-9p+8-4p+7-p=2$	(m1)		Reduce to linear or quadratic equation in p.
	$p = \dfrac{5}{7}$	(A1)		
	C is at $\left(9\dfrac{1}{7}, -2\dfrac{4}{7}, 9\dfrac{2}{7}\right)$	(A1)	(6)	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
8(b)(ii)	**Alternative** : using symmetry (i)			
	$\left\lvert\overrightarrow{AD}\right\rvert=\left\lvert\overrightarrow{BC}\right\rvert=\sqrt{56}$	(B1ft)		$\overrightarrow{AD}=\begin{bmatrix}4\\-2\\6\end{bmatrix}$
	$\left\lvert\overrightarrow{DC}\right\rvert=\left\lvert\overrightarrow{AB}\right\rvert-\left\lvert\overrightarrow{AD}\right\rvert\cos\theta-\left\lvert\overrightarrow{BC}\right\rvert\cos\theta$	(M1)		Substitute values and evaluate $\left\lvert\overrightarrow{AB}\right\rvert-\left\lvert\overrightarrow{AD}\right\rvert\cos\theta-\left\lvert\overrightarrow{BC}\right\rvert\cos\theta$
	$\left\lvert\overrightarrow{DC}\right\rvert=\frac{10}{\sqrt{14}}$	(A1ft)		F on $\overrightarrow{AB}$ and $\cos\theta$
	$\left\lvert\overrightarrow{DC}\right\rvert=p\left\lvert\overrightarrow{AB}\right\rvert\Rightarrow\frac{10}{\sqrt{14}}=p\sqrt{14}$	(m1)		Set up equation in *p*
	$p=\frac{5}{7}$	(A1)		
	C is at $\left(9\frac{1}{7},-2\frac{4}{7},9\frac{2}{7}\right)$	(A1)	(6)	
	Alternative using symmetry (ii) $\left\lvert\overrightarrow{AD}\right\rvert=\sqrt{56}$	(B1ft)		
	$\left\lvert\overrightarrow{AE}\right\rvert=\left\lvert\overrightarrow{AD}\right\rvert\cos\theta=\sqrt{56}\times\frac{1}{14}=\frac{2}{\sqrt{14}}$	(M1) (A1ft)		Substitute values and evaluate for $\left\lvert\overrightarrow{AD}\right\rvert\cos\theta$. F on $\cos\theta$
	$\left\lvert\overrightarrow{AE}\right\rvert=q\left\lvert\overrightarrow{AB}\right\rvert\Rightarrow\frac{2}{\sqrt{14}}=q\sqrt{14}$	(m1)		Set up equation to find *p*
	and $\left\lvert\overrightarrow{AE}\right\rvert=\left\lvert\overrightarrow{FB}\right\rvert\Rightarrow p=1-2q$			
	$q=\frac{2}{14}\qquad p=\frac{5}{7}$	(A1)		
	C is at $\left(9\frac{1}{7},-2\frac{4}{7},9\frac{2}{7}\right)$	(A1)	(6)	
	TOTAL		**75**	

Version 1.0

General Certificate of Education (A-level)
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Mathematics MPC4

(Specification 6360)

Pure Core 4

Final

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AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
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Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	$\left(\text{f}\left(-2\right)=\right)0$	B1	1	ISW (0 seen is B1)
(b)	$\text{f}\left(\frac{3}{2}\right)=4\left(\frac{3}{2}\right)^3-13\left(\frac{3}{2}\right)+6$	M1		Clear attempt at $\text{f}\left(\frac{3}{2}\right)$ with 3 terms Factor theorem required; **NOT** long division
	$4\times\frac{27}{8}-13\times\frac{3}{2}+6$ or $13.5-19.5+6$			Must see this, or equivalent
	$=0\Rightarrow\left(2x-3\right)$ is a factor	A1	2	Shown = 0 and statement.
(c)	Any appropriate method to find third factor	M1		Full long division Compare coefficients Factor Theorem $\text{f}\left(\frac{1}{2}\right)$
	$\left(x+2\right)\left(2x-3\right)\left(2x-1\right)$	A1		Or $\left(2x^2+x-6\right)\left(2x-1\right)$ NMS M1A1 **SC1** (2x + 1) or (1 – 2x) or $\left(x-\frac{1}{2}\right)$ or $\left(\frac{1}{2}-x\right)$ for third factor
	$2x^2+x-6=\left(x+2\right)\left(2x-3\right)$	M1		Factorise **numerator** correctly or cancel $2x^2+x-6$
	$\frac{2x^2+x-6}{\text{f}\left(x\right)}=\frac{1}{2x-1}$	A1	4	No ISW
			7	

Q	Solution	Marks	Total	Comments
2(a)(i)	$(A=)80$	B1	1	Ignore units
(ii)	$2000 = A \times k^{25}$	M1		A or their value from (a)(i)
	$k = \sqrt[25]{25}$ or $25^{\frac{1}{25}}$ or $k = 10^{0.04\log 25}$ or $\mathrm{e}^{0.04\ln 25}$ $\Rightarrow k = 1.137411$ **AG**	A1	2	Correct expression for k, or 1.13741146….seen, **and** correct answer to 6 d.p.
(b)	$\ln\left(\dfrac{100000}{their\ A}\right) = t\ln k$	M1		Take logs correctly. Condone miscopied k $\ln 1250 = t\ln k$ or t = $\log_k 1250$
	$t = 55.38$	A1		Condone 55.3 or 55.4 PI
	$\Rightarrow 2016$	A1	3	
			6	
2(b)	**Alternative** By trial and improvement $1250 = k^t$	M1		Attempt to calculate k^{55} and k^{56}.
	$t = 56$ or $55 < t < 56$	A1		
	$\Rightarrow$ 2016	A1	3	

Q	Solution	Marks	Total	Comments
3 **(a)(i)**	$(1-x)^{\frac{1}{3}} = 1-\frac{1}{3}x$	M1		Condone $1^{\frac{1}{3}} + -\frac{1}{3}x$ for M1
	$= 1-\frac{1}{3}x-\frac{1}{9}x^2$	A1	2	Must simplify coefficients including signs
(ii)	$(125-27x)^{\frac{1}{3}} = 125^{\frac{1}{3}}\left(1-\frac{27}{125}x\right)^{\frac{1}{3}}$	B1		May have 5 instead of $125^{\frac{1}{3}}$
	$\left(1-\frac{27}{125}x\right)^{\frac{1}{3}} = \left(1-\frac{1}{3}\times\frac{27}{125}x-\frac{1}{9}\left(\frac{27}{125}x\right)^2\right)$	M1		Attempt to replace x by $\pm\frac{27}{125}x$ condone missing brackets, or start binomial again.
	$= 5-\frac{9}{25}x-\frac{81}{3125}x^2$	A1	3	Condone $5+\frac{-9}{25}x+\frac{-81}{3125}x^2$
(b)	$x=\frac{2}{9}$ **used** in answer to (a)(ii)	M1		Condone $x=\frac{6}{27}$ or x = 0.222 or better
	$\sqrt[3]{119} \approx 5-\frac{9}{25}\times\frac{2}{9}-\frac{81}{3125}\left(\frac{2}{9}\right)^2$			
	$= 4.91872$	A1	2	This answer only and must follow from correct expansion
			7	
3(a) **(ii)**	**Alternative** using $(a+bx)^n$			
	$(125-27x)^{\frac{1}{3}} = 125^{\frac{1}{3}} + \frac{1}{3}\times125^{-\frac{2}{3}}\times(-27x)$	M1		Allow one error; condone missing brackets
	$+\frac{1}{3}\left(-\frac{2}{3}\right)\frac{1}{2}\times125^{-\frac{5}{3}}(-27x)^2$			
	$= 5-\frac{9}{25}x-\frac{81}{3125}x^2$	A2	3	

Q	Solution	Marks	Total	Comments
4 **(a)(i)**	$\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}=\right)-6\sin 2\theta$, $\left(\frac{\mathrm{d}y}{\mathrm{d}\theta}=\right)-2\sin\theta$	M1		$\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}=\right)p\sin 2\theta$ or $r\sin\theta\cos\theta$ $\left(\frac{\mathrm{d}y}{\mathrm{d}\theta}=\right)q\sin\theta$
		A1		Both correct.
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-2\sin\theta}{-6\sin 2\theta}$	M1		Use chain rule $\frac{\frac{\mathrm{d}y}{\mathrm{d}\theta}}{\frac{\mathrm{d}x}{\mathrm{d}\theta}}$; condone one slip
	$=\frac{2\sin\theta}{6\times 2\sin\theta\cos\theta}=\frac{1}{6\cos\theta}$	A1	4	$k=6$ must come from correct working seen **AG**
(ii)	$\theta=\frac{\pi}{3}$ $m_{\mathrm{T}}=\frac{1}{3}$	B1ft		ft on k $\left(\frac{1}{k\times\frac{1}{2}}\right)$ k need not be numerical
	$m_{\mathrm{N}}=-3$	B1ft		ft on m_{T}
	$(x,y)=\left(-\frac{3}{2},1\right)$	B1		
	Normal $y-1=-3\left(x+\frac{3}{2}\right)$	B1	4	CAO; any correct form, ISW. $2y+6x+7=0$
(b)	$\sin^2 x=\frac{1}{2}(1-\cos 2x)$	M1 A1		$p+q\cos 2x$; Allow different letters for x or mixture eg θ even for A1and the following A1ft
	$\int p\ \mathrm{d}x=px$ $\int q\cos 2x=\frac{1}{2}q\sin 2x$	A1ft		Both integrals correct; ft on p and q
	$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\sin^2 x\ \mathrm{d}x=\left[\frac{x}{2}-\frac{1}{4}\sin 2x\right]$			
	$=\left(\frac{\pi}{8}-\frac{1}{4}\right)-\left(-\frac{\pi}{8}-\left(-\frac{1}{4}\right)\right)$	m1		Correct use of limits; $\mathrm{F}\left(\frac{\pi}{4}\right)-\mathrm{F}\left(-\frac{\pi}{4}\right)$ or $2\mathrm{F}\left(\frac{\pi}{4}\right)$ $\mathrm{F}(x)=px+r\sin 2x$ and $\sin\frac{\pi}{2}$, $\sin\left(-\frac{\pi}{2}\right)$ must be evaluated correctly for m1
	$=\frac{\pi}{4}-\frac{1}{2}$	A1	5	CSO OE ISW
			13	

4 (b)	**Alternative**			
	$\int \sin^2 x\ \mathrm{d}x = -\sin x \cos x - \int -\cos x \cos x\ \mathrm{d}x$	M1		Use parts; condone sign slips
	$= -\sin x \cos x + \int 1 - \sin^2 x\ \mathrm{d}x$	m1		Use $\cos^2 x = 1 - \sin^2 x$
	$2\int \sin^2 x\ \mathrm{d}x = -\sin x \cos x + x$	A1		
	$2\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x\ \mathrm{d}x = \mathrm{G}\left(\frac{\pi}{4}\right) - \mathrm{G}\left(-\frac{\pi}{4}\right)$	m1		Correct use of limits
	$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x\ \mathrm{d}x = \frac{\pi}{4} - \frac{1}{2}$	A1	5	

Q	Solution	Marks	Total	Comments
5 (a)	$\overrightarrow{AB}=\begin{bmatrix}4\\-1\\3\end{bmatrix}-\begin{bmatrix}5\\1\\-2\end{bmatrix}=\begin{bmatrix}-1\\-2\\5\end{bmatrix}$	B1		$\pm\left(\overrightarrow{OA}-\overrightarrow{OB}\right)$ Co-ordinate form only is B0 Condone one component incorrect
	Line through *A* and *B*	M1		$\overrightarrow{OA}+\lambda\mathbf{d}$ or $\overrightarrow{OB}+\lambda\mathbf{d}$ where $\mathbf{d}=\overrightarrow{AB}$ or $\overrightarrow{BA}$ all in components and identified.
	$\mathbf{r}=\begin{bmatrix}5\\1\\-2\end{bmatrix}+\lambda\begin{bmatrix}-1\\-2\\5\end{bmatrix}$ or $\mathbf{r}=\begin{bmatrix}4\\-1\\3\end{bmatrix}+\lambda\begin{bmatrix}-1\\-2\\5\end{bmatrix}$	A1	3	OE **r** or $\begin{bmatrix}x\\y\\z\end{bmatrix}$ required Condone missing brackets on $\overrightarrow{OA}$ or $\overrightarrow{OB}$
(b)(i)	$5-\lambda=-8+5\mu$ $1-2\lambda=5$ $-2+5\lambda=-6-2\mu$	M1		Clear attempt to set up and solve at least two simultaneous equations in μ and a different parameter. Allow in column vector form.
	$\lambda=-2 \quad \mu=3$	A1		One of λ **or** μ correct OE
	$-2+5\times-2=-12 \qquad -6-2\times3=-12$ Both equal -12 so intersect	E1		Verify intersect, λ **and** μ correct or verify $(7,5,-12)$ is on both lines; **statement required**
	P is $(7,5,-12)$	B1	4	CAO condone *P* = $\begin{bmatrix}7\\5\\-12\end{bmatrix}$ OE and missing brackets
(ii)	$\overrightarrow{BC}=\begin{bmatrix}-8+5\mu\\5\\-6-2\mu\end{bmatrix}-\begin{bmatrix}4\\-1\\3\end{bmatrix}$	B1		$\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}$ or $\overrightarrow{CB}=\overrightarrow{OB}-\overrightarrow{OC}$
	$\begin{bmatrix}3\\6\\-15\end{bmatrix}\bullet\overrightarrow{BC}=0$	M1		Clear attempt at $\pm\overrightarrow{BP}$ or $\pm\overrightarrow{AB}$ or $\pm\overrightarrow{AP}$ in components sp with $\overrightarrow{BC}=0$
	$-36+15\mu+36+135+30\mu=0$	m1		Linear equation in μ using *their* $\overrightarrow{BC}$ and solved for μ. Condone one arithmetical or sign slip
	$\mu=-3$	A1		
	C is $(-23,5,0)$	A1	5	CSO Condone column vector.
			12	

Q	Solution	Marks	Total	Comments
6 **(a)**	$(C=)\dfrac{2}{e}$ or $2e^{-1}$ or $2\left(\dfrac{1}{e}\right)$ or $2\left(e^{-1}\right)$	B1	1	One of these answers only. Not 0.736 but allow ISW.
(b)	$\dfrac{d}{dx}(2y)=2\dfrac{dy}{dx}$	B1		
	$\dfrac{d}{dx}\left(e^{2x}y^2\right)=2e^{2x}y^2+e^{2x}2y\dfrac{dy}{dx}$	M1		Product; 2 terms added, one with $\dfrac{dy}{dx}$;
		A1 A1		A1 for each term
	$\dfrac{d}{dx}\left(x^2+C\right)=2x$	B1		
	$\dfrac{dy}{dx}=$	M1		Solve *their* equation correctly for $\dfrac{dy}{dx}$
	$\dfrac{x-e^{2x}y^2}{e^{2x}y+1}$	A1	7	Condone factor of 2 in both numerator and denominator. ISW
(c)	Evaluate $\dfrac{dy}{dx}$ at $\left(1,\dfrac{1}{e}\right)$	M1		Substitute $x=1$ and $y=\dfrac{1}{e}$ into numerator of $\dfrac{dy}{dx}$; allow one slip
	numerator $=1-e^2e^{-2}=0\Rightarrow$ stationary point	A1	2	Conclusion required; must score full marks in part (b) Allow $1-1=0$ or $2-2=0$
			10	

Q	Solution	Marks	Total	Comments
Q7 (a)	$\frac{\mathrm{d}A}{\mathrm{d}t}$	B1		
	$= -k$	B1	2	
(b)(i)	$A = -kt (+ C)$	M1		Integrate
	$C = 4\pi \times 60^2$	A1		C correct from $A = \pm kt + C$
	$4\pi \times 30^2 = -9k + 4\pi \times 60^2$	m1		Use $r = 30 \quad t = 9$ and attempt to find k, as far as $k = \ \ldots$ $k = 1200\pi$
	$A = -1200\pi t + 14400\pi$ $= 1200\pi(12 - t)$	A1	4	**AG** CSO
(ii)	$t = 12 \quad (\text{days})$	B1	1	
			7	

Q	Solution	Marks	Total	Comments
Q8 (a)	$1=A(1-x)^2+B(1-x)(3-2x)+C(3-2x)$	M1		Attempt to clear fractions
	$\left.\begin{array}{lll} x=1 & x=\frac{3}{2} & x=0 \\ C=1 & 1=A\left(-\frac{1}{2}\right)^2 & 1=A+3B+3C \end{array}\right\}$	m1		Use any two (or three) values of *x* to set up two (or three) equations
	$A=4 \quad B=-2 \quad C=1$	A1 A1	4	Two values correct All values correct
(b)	$\int \frac{1}{2\sqrt{y}}\,\mathrm{d}y=\int \frac{4}{3-2x}-\frac{2}{1-x}+\frac{1}{(1-x)^2}\,\mathrm{d}x$	B1ft		Separate using partial fractions; correct notation; condone missing integral signs but d*y* and d*x* must be in correct place. ft on their *A, B, C* and on each integral.
	$\int \frac{1}{2\sqrt{y}}\,\mathrm{d}y=\sqrt{y}=$	B1		OE $\int \frac{k}{\sqrt{y}}\,\mathrm{d}y=2k\sqrt{y}$ is B1
	$-2\ln(3-2x)$	B1ft		
	$+2\ln(1-x)$	B1ft		Condone missing brackets on one ln integral.
	$+\frac{1}{1-x} \quad (+C)$	B1ft		Condone omission of +*C*
	$x=0 \quad y=0 \quad \Rightarrow 0=-2\ln 3+0+1+C$	M1		Use $(0,0)$ to find *C*. Must get to *C* = …..
	$C=2\ln 3-1$	A1		Correct *C* found from correct equation. *C* must be exact, in any form but not decimal.
	$\sqrt{y}=2\ln\left(\frac{3-3x}{3-2x}\right)+\frac{1}{1-x}-1$	m1		Correct use of rules of logs to progress towards requested form of answer . *C* must be of the form *r* ln*s* + *t*
	$y^{\frac{1}{2}}=2\ln\left(\frac{3-3x}{3-2x}\right)+\frac{x}{1-x}$	A1	9	OE CSO condone B0 for separation
			13	
	TOTAL		**75**	

Q8 (a)	**Alternative** $1 = A(1-x)^2 + B(1-x)(3-2x) + C(3-2x)$	M1		
	$1 = A + 3B + 3C$ $0 = -2A - 5B - 2C$ $0 = A + 2B$	m1		Set up three simultaneous equations
		A1		Two values correct
	$A = 4 \quad B = -2 \quad C = 1$	A1	4	All values correct

General Certificate of Education (A-level)
January 2012

Mathematics MPC4

(Specification 6360)

Pure Core 4

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC4: January 2012 - Mark scheme

Q	Solution	Marks	Total	Comments
1(a)	$2x+3=A(2x+1)+B(2x-1)$	M1		
	$x=\frac{1}{2} \qquad x=-\frac{1}{2}$	m1		Use two values of *x* to find *A* and *B*
	$A=2 \qquad B=-1$	A1	3	Both
(b)	$\begin{array}{r} 3x \\ 4x^2-1\overline{)12x^3-7x-6} \\ 12x^3-\underline{3x} \\ -4x-6 \end{array}$	M1		Complete division leading to values for *C* and *D*
	$C=3$	A1		$C=3 \qquad D=-2$ stated or written in expression.
	$D=-2$	A1	3	SC B1 $C=3$, *D* not found or wrong; $D=-2$, *C* not found or wrong.
(c)	$\int 3x-2\left(\frac{2}{2x-1}-\frac{1}{2x+1}\right)\,\mathrm{d}x$	M1		Use parts (a) and (b) to obtain integrable form
	$3\frac{x^2}{2}$	A1ft		ft on *C*
	$-2\left(\ln(2x-1)-\frac{1}{2}\ln(2x+1)\right)$	A1ft		Both correct; ft on *A*, *B* and *D* Condone missing brackets
	$\frac{3}{2}(4-1)-2\left(\left(\ln 3-\frac{1}{2}\ln 5\right)-\left(\ln 1-\frac{1}{2}\ln 3\right)\right)$	m1		Correct substitution of limits
	$\frac{9}{2}-3\ln 3+\ln 5=\frac{9}{2}+\ln\left(\frac{5}{27}\right)$	A1	5	$p=\frac{9}{2} \qquad q=\frac{5}{27}$
		Total	**11**	

(a) Condone poor algebra for M1 if continues correctly.

(b) Complete division for M1; obtain a value for *C* (*Cx*) and a remainder $ax+b$

(c) Form $\int Cx+\left(\frac{P}{2x-1}+\frac{Q}{2x+1}\right)\mathrm{d}x$ using candidate's *P*, *Q* , *C* for M1. Condone missing d*x*.

$\int Cx\,\mathrm{d}x=C\frac{x^2}{2}$ for A1ft ISW extra terms eg $\frac{12}{4x^2-1}$ for first three terms only; max 3/5

Candidate's C; must have a value.

$\int\frac{4x+6}{4x^2-1}\,\mathrm{d}x=\int\frac{4x}{4x^2-1}+\frac{6}{4x^2-1}\,\mathrm{d}x$ is an integrable form, as $\int\frac{1}{x^2-a^2}\,\mathrm{d}x=\frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right)$ is in the formula book, but they **must** try to integrate to show they know this, **or** use partial fractions again with $\frac{6}{4x^2-1}=\frac{3}{2x-1}-\frac{3}{2x+1}$ for M1

Substitute limits into $C\frac{x^2}{2}+m\ln(2x-1)+n\ln(2x+1)$, or equivalent, for m1; substitution must be completely correct.

Condone $\frac{9}{2}-\ln\left(\frac{27}{5}\right)$ for A1

Q	Solution	Marks	Total	Comments
1 (a)	Alternative; equating coefficients			
	$2x+3=A(2x+1)+B(2x-1)$	M1		
	x term $\quad 2=2A+2B$			Set up simultaneous equations and solve.
	constant $\quad 3=A-B$	m1		
	$A=2 \quad B=-1$	A1	3	Both
	Alternative; cover up rule			
	$x=\frac{1}{2} \quad A=\frac{2\times\frac{1}{2}+3}{2\times\frac{1}{2}+1} \quad \left(=\frac{4}{2}\right)$ $x=-\frac{1}{2} \quad B=\frac{2\times\left(-\frac{1}{2}\right)+3}{2\times\left(-\frac{1}{2}\right)-1} \quad \left(=\frac{2}{-2}\right)$	M1		$x=\frac{1}{2}$ and $x=-\frac{1}{2}$ used to find A and B SC NMS
	$A=2 \quad B=-1$	A1A1	3	A and B both correct 3/3 One of A or B correct 1/3
1 (b)	Alternative			
	$\frac{12x^3-7x-6}{4x^2-1}=\frac{12x^3-3x-4x-6}{4x^2-1}$ $=3x-\frac{2(2x+3)}{4x^2-1}$	M1		
	$C=3$ $D=-2$	A1 A1	3	$C=3 \quad D=-2$ stated or written in expression SC B1 $C=3$, D not found or wrong; $D=-2$, C not found or wrong.
	Alternative $12x^3-7x-6=4Cx^3-Cx+2Dx+3D$	M1		Complete method for C and D
	$C=3$ $D=-2$	A1 A1	3	C = 3, D = -2 stated or written in expression. SC B1 $C=3$, D not found or wrong; $D=-2$, C not found or wrong.
	Alternative $x=0 \qquad x=1$ $6=-3D \qquad -\frac{1}{3}=C+\frac{5}{3}D$	M1		Use two values of x to set up simultaneous equations
	$C=3$ $D=-2$	A1 A1	3	$C=3 \quad D=-2$ stated or written in expression. SC B1 $C=3$, D not found or wrong; $D=-2$, C not found or wrong.

Q	Solution	Marks	Total	Comments
2(a)(i)	$\tan\alpha = \frac{4}{3}$	B1	1	Fraction required Allow 1.333 …. (recurring)
(ii)	$1,\ 2,\ \sqrt{3}$ seen (from Pythagoras) or $4 = 1 + \cot^2\beta$	M1		Use $\operatorname{cosec}^2\beta = 1 + \cot^2\beta$
	$\tan\beta = -\frac{1}{\sqrt{3}}$	A1	2	SC B1 $\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$
(b)	$\tan(\alpha+\beta) = \dfrac{\frac{4}{3} - \frac{1}{\sqrt{3}}}{1 - \frac{4}{3}\left(-\frac{1}{\sqrt{3}}\right)}$	M1		Use $\tan(\alpha+\beta)$ formula
	Remove fractions within fractions	m1		Correct manipulation to form $\frac{a+b\sqrt{3}}{c+d\sqrt{3}}$ $a\ b\ c\ d$ integers
	$= \frac{4\sqrt{3}-3}{3\sqrt{3}+4}$	A1	3	$m = 4 \quad n = 3$ or any multiple
		Total	**6**	
(b)	Alternative $\tan(\alpha+\beta)$			
	$= \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \dfrac{\frac{4}{5}\times\left(-\frac{\sqrt{3}}{2}\right) + \frac{3}{5}\times\frac{1}{2}}{\frac{3}{5}\times\left(-\frac{\sqrt{3}}{2}\right) - \frac{4}{5}\times\frac{1}{2}}$	M1		Use formulae for $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$
	Remove fractions within fractions	m1		Correct manipulation to form $\frac{a+b\sqrt{3}}{c+d\sqrt{3}}$ $a\ b\ c\ d$ integers
	$= \frac{-4\sqrt{3}+3}{-3\sqrt{3}-4} \quad \left(= \frac{4\sqrt{3}-3}{3\sqrt{3}+4}\right)$	A1		$m = -4 \quad n = -3$ or any multiple

(a)(ii) Special case B1 for $\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$

(b) M1 for substituting candidates values for $\tan\alpha$ and $\tan\beta$ into correct formula.
Completely correct or completely correct ft on $\tan\alpha$, $\tan\beta$.
Special case answer is $\frac{12+3\sqrt{3}}{9-4\sqrt{3}}$ or $\times\frac{a}{a}$ where a is integer or $\sqrt{3}$ for M1m1A0

Q	Solution	Marks	Total	Comments
3 **(a)**	$(1+6x)^{\frac{2}{3}}=1+\frac{2}{3}\times 6x+kx^2$	M1		
	$=1+4x-4x^2$	A1	2	Simplified coefficients required
(b)	$(8+6x)^{\frac{2}{3}}=8^{\frac{2}{3}}\left(1+\frac{6}{8}x\right)^{\frac{2}{3}}$	B1		OE
	$\left(1+\frac{6}{8}x\right)^{\frac{2}{3}}=1+4\left(\frac{x}{8}\right)-4\left(\frac{x}{8}\right)^2$	M1		x replaced by $\frac{x}{8}$ in answer to (a) Condone missing brackets, allow one error.
	$(8+6x)^{\frac{2}{3}}=4+2x-\frac{1}{4}x^2$	A1	3	Simplified coefficients required.
(c)	$(100=10^2 \quad 8+6x=10 \quad x=\frac{1}{3})$			
	$4+2\times\frac{1}{3}-\frac{1}{4}\times\left(\frac{1}{3}\right)^2$	M1		Use $x=\frac{1}{3}$ in binomial expansion from part (b)
	$=\frac{167}{36}$	A1	2	$\sqrt[3]{100}\approx\frac{167}{36}$
		Total	**7**	
3 **(b)**	**Alternative**			
	$(8+6x)^{\frac{2}{3}}=8^{\frac{2}{3}}\left(1+\frac{6}{8}x\right)^{\frac{2}{3}}$			OE
	$\left(1+\frac{6}{8}x\right)^{\frac{2}{3}}=1+\frac{2}{3}\left(\frac{6}{8}x\right)+\frac{2}{3}\left(\frac{2}{3}-1\right)\frac{1}{2}\left(\frac{6}{8}x\right)^2$			Condone missing brackets, allow one error.
	$(8+6x)^{\frac{2}{3}}=4+2x-\frac{1}{4}x^2$			
	Alternative			
	$8^{\frac{2}{3}}+\frac{2}{3}\times 8^{-\frac{1}{3}}\times 6x+\frac{2}{3}\left(\frac{2}{3}-1\right)\frac{1}{2}\times 8^{-\frac{4}{3}}\times(6x)^2$			Use binomial formula; condone one error and missing brackets.
	$4+2x-\frac{1}{4}x^2$			
(a)(b)	Condone $1^{\frac{2}{3}}$ for 1 for M1			

Q	Solution	Marks	Total	Comments
4 (a)	$P=500e^{\frac{1}{8}\times 60}$	M1		Must use $t=$ 60
	$=904\ 000$	A1	2	Nearest thousand required 904000 only
(b)(i)	$\left(e^{\frac{1}{8}t}\right)^2=\frac{500000}{500}$	M1		
	$t=8\ln\sqrt{1000}$	M1		OE Take logs correctly leading to expression for *t*.
	$t=27.6$ (minutes)	A1	3	Accept 27.631
(ii)	$500e^{\frac{1}{8}T}-500000e^{-\frac{1}{8}T}=45000$	M1		Set up equation; condone one error; allow in t. Condone inequality.
	$\times\frac{e^{\frac{1}{8}T}}{500}\Rightarrow\left(e^{\frac{1}{8}T}\right)^2-1000=90e^{\frac{1}{8}T}$ $\left(e^{\frac{1}{8}T}\right)^2-90e^{\frac{1}{8}T}-1000=0$	A1		Multiply by $\frac{e^{\frac{1}{8}T}}{500}$ and rearrange to AG, be convinced.
	$e^{\frac{1}{8}T}=100$ $\quad$ ($e^{\frac{1}{8}T}=-10$ rejected)	M1		Solve quadratic equation (retaining positive root).
	$t=36.8$ (minutes)	A1	4	CAO
		Total	**9**	
4 (b)(i)	Alternative $e^{\frac{1}{8}t}=1000e^{-\frac{1}{8}t}\Rightarrow e^{\frac{1}{4}t}=\frac{500000}{500}$	M1		
	$t=4\ln 1000$	M1		Take logs correctly leading to expression for *t*.
	$t=27.6$ (minutes)	A1	3	
	Alternative $e^{\frac{1}{8}t}=1000e^{-\frac{1}{8}t}\Rightarrow\ln\left(e^{\frac{1}{8}t}\right)=\ln 1000+\ln\left(e^{-\frac{1}{8}t}\right)$	M1		Take logs correctly.
	$t=4\ln 1000$	M1		
	$t=27.6$ (minutes)	A1	3	

(b)(ii) M1 for solve quadratic equation Let $x=e^{\frac{1}{8}t}$ solve quadratic equation $x^2-90x-1000=0$ by

inspection, $x=100$ seen;

factors $(x-100)(x+10)$ with 100 and 10 seen;

complete square $x=45\pm\sqrt{3025}$ all correct

formula $x=\frac{90\pm\sqrt{90^2+4000}}{2}$ all correct

Final answer ; must have $t=36.8$ for A1

(b)(i) 27.6 as final answer NMS 3/3

27.6 following wrong working AO (FIW) but could still score M mark(s)

Q	Solution	Marks	Total	Comments
5(a)	$xy^2+3y=\left(8t^2-t\right)\left(\frac{3}{t}\right)^2+3\left(\frac{3}{t}\right)$	M1		Substitute and expand
	$=72-\frac{9}{t}+\frac{9}{t}=72$	A1	2	$k=72$
(b)(i)	$\frac{dx}{dt}=16t-1 \quad \frac{dy}{dt}=-\frac{3}{t^2}$	B1B1		
	$t=\frac{1}{4} \quad \frac{dy}{dx}=\frac{-\frac{3}{\left(\frac{1}{4}\right)^2}}{16\times\frac{1}{4}-1}$	M1		Use chain rule $\left(\frac{dy}{dx}=\frac{-3}{16t^3-t^2}\right)$ and calculate gradient using $t=\frac{1}{4}$
	$=-16$	A1		
	$t=\frac{1}{4} \quad x=\frac{8}{16}-\frac{1}{4} \quad y=\frac{3}{\frac{1}{4}}$	M1		Calculate x and y using $t=\frac{1}{4}$
	$x=\frac{1}{4} \quad y=12$	A1		Both correct
	tangent $y=-16x+16$	A1	7	ACF CSO $y-12=-16\left(x-\frac{1}{4}\right)$ ISW
(ii)	$y=-16\times\frac{3}{2}+16=-8$	M1		Substitute $x=\frac{3}{2}$ into candidate’s tangent; calculate y
	$\frac{3}{2}(-8)^2+3\times(-8)=96-24=72$	A1	2	$y=-8$ used to verify 72
		Total	**11**	
5(a)	Alternative			
	$x=8\left(\frac{3}{y}\right)^2-\frac{3}{y}$	M1		Eliminate t
	$xy^2+3y=72$	A1	2	$k=72$
(b)(i)	Alternative			
	$2xy\frac{dy}{dx}+y^2$	M1A1		Product rule attempted; two terms added, one with $\frac{dy}{dx}$
	$+3\frac{dy}{dx}=0$	B1		
	$t=\frac{1}{4} \quad x=\frac{8}{16}-\frac{1}{4} \quad y=\frac{3}{\frac{1}{4}}$	M1		Calculate x and y using $t=\frac{1}{4}$
	$x=\frac{1}{4} \quad y=12$	A1		Both correct.
	$\left(\frac{dy}{dx}=\frac{-y^2}{2xy+3}\right) \quad \frac{dy}{dx}=-16$	m1		Calculate gradient from candidate’s expression.
	tangent $y=-16x+16$	A1	7	ACF CSO $y-12=-16\left(x-\frac{1}{4}\right)$ ISW

Q	Solution	Marks		Total
5**(b)(i)**	Alternative $x=\frac{72-3y}{y^2}$	M1		Correct expression for x from candidate's implicit equation. Quotient rule attempted; y^4 and two terms subtracted.
	$\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{y^2(-3)-(72-3y)\times 2y}{y^4}$	A1 A1		Numerator; first term; second term
	$\left(\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{3y-144}{y^3}\right)$			
	$t=\frac{1}{4}\quad y=\frac{3}{\frac{1}{4}}=12$	B1		
	$\frac{\mathrm{d}x}{\mathrm{d}y}=-\frac{1}{16}\quad\frac{\mathrm{d}y}{\mathrm{d}x}=-16$	m1		Use $t=\frac{1}{4}$ to calculate y
	$t=\frac{1}{4}\quad x=\frac{8}{16}-\frac{1}{4}=\frac{1}{4}$	B1		Evaluate and invert.
	$y=-16x+16$	A1		
	Alternative for $\frac{\mathrm{d}x}{\mathrm{d}y}$			Use $t=\frac{1}{4}$ to calculate x ACF CSO
	$x=\frac{72}{y^2}-\frac{3}{y}$	M1		
	$\frac{\mathrm{d}x}{\mathrm{d}y}=-\frac{144}{y^3}+\frac{3}{y^2}$	A1 A1		
	$\left(\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{3y-144}{y^3}\right)$			Correct expression for x from candidate's implicit equation and attempt derivatives

Q	Solution	Marks	Total	Comments
6(a)	$16\left(\frac{3}{4}\right)^3+11\left(\frac{3}{4}\right)-15$	M1		Evaluate $\mathrm{f}\left(\frac{3}{4}\right)$; not long division.
	$=\frac{27}{4}+\frac{33}{4}-15=0\Rightarrow$ factor	A1	2	Processing and conclusion.
(b)	$27\cos\theta\left(2\cos^2\theta-1\right)+$	B1		Use acf of $\cos 2\theta$ formula
	$19\sin\theta\left(2\sin\theta\cos\theta\right)-15=0$	B1		Use acf of $\sin 2\theta$ formula
	$54\cos^3\theta-27\cos\theta+38\left(1-\cos^2\theta\right)\cos\theta$ $-15=0$	M1		All in cosines.
	$16\cos^3\theta+11\cos\theta-15=0$ $x=\cos\theta\Rightarrow 16x^3+11x-15=0$	A1	4	Simplification and substitute $x=\cos\theta$ to obtain AG CSO.
(c)	$16x^3+11x-15=\left(4x-3\right)\left(4x^2+3x+5\right)$	M1A1		Factorise $\mathrm{f}\left(x\right)$
	$b^2-4ac=3^2-4\times4\times5 \quad \left(=-71\right)$	m1		Find discriminant of quadratic factor; or seen in formula
	$b^2-4ac<0$, no solution $\left(\text{to } 4x^2+3x+5=0\right)$ $\Rightarrow$ $\left(\text{only}\right)$ solution is $\cos\theta=\frac{3}{4}$	A1	4	Conclusion; CSO Condone $x=\frac{3}{4}$ is $\left(\text{only}\right)$ solution
		Total	**10**	

(a) For A1; minimum processing seen; $16\times\frac{27}{64}+11\times\frac{3}{4}-15=0$; $15-15=0$ and no other working is A0

minimum conclusion $=0$ hence factor

(b) For M1 mark; $\cos 2\theta$ (eventually) in form $a\cos^2\theta+b$; $19\sin\theta\sin 2\theta$ in form $c\cos\theta\sin^2\theta$ and use $\sin^2\theta=1-\cos^2\theta$ to obtain $c\cos\theta\left(1-\cos^2\theta\right)$

(c) M1 $\left(4x-3\right)\left(4x^2+kx\pm5\right)$ A1 fully correct

m1 candidate's values of *a, b, c* used in expression for b^2-4ac

or complete square to obtain $\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}$

A1 b^2-4ac correct or $\left(x+\frac{3}{8}\right)^2=\frac{9}{64}-\frac{5}{4}$ $\left(=-\frac{71}{64}\right)$ and stated to be negative so no solution

or solutions are not real (imaginary)

Accept imaginary solutions from calculator if stated to be imaginary.

Condone $\sqrt{-71}$ is negative, or similar, so no solution.

Conclusion $x=\frac{3}{4}$ is solution, or $\cos\theta=\frac{3}{4}$ is solution

Q	Solution	Marks	Total	Comments
7	$\int \frac{\mathrm{d}y}{y^2} = \int x\sin 3x \ \mathrm{d}x$	B1		Correct separation and notation; condone missing integral signs
	$\int \frac{\mathrm{d}y}{y^2} = -\frac{1}{y}$	B1		
	$\int x\sin 3x \ \mathrm{d}x = x\left(-\frac{1}{3}\cos 3x\right)$	M1		Use parts $u = x$ $\frac{\mathrm{d}v}{\mathrm{d}x} = \sin 3x$ $\frac{\mathrm{d}u}{\mathrm{d}x} = 1$ $v = k\cos 3x$
	$-\int -\frac{1}{3}\cos 3x \ \mathrm{d}x$	A1		with correct substitution into formula
	$= -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x$	A1		CAO
	$-\frac{1}{y} = -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x + C$			
	$-1 = -\frac{1}{3}\times\frac{\pi}{6}\cos\left(\frac{\pi}{2}\right) + \frac{1}{9}\sin\left(\frac{\pi}{2}\right) + C$	M1		Use $x = \frac{\pi}{6}$ $y = 1$ to find C
	$C = -\frac{10}{9}$	A1		CAO
	$-\frac{1}{y} = -\frac{1}{9}\left(3x\cos 3x - \sin 3x + 10\right)$	m1		And invert to $-y = -\frac{9}{(.....)}$
	$y = \frac{9}{3x\cos 3x - \sin 3x + 10}$	A1	9	CSO, condone first B1 not given
		Total	**9**	

Second M1 finding C; substitute $x = \frac{\pi}{6}$ $y = 1$ into $\mathrm{f}\left(y\right) = px\cos 3x + q\sin 3x + C$ and evaluate using radians. Must calculate a value of C.

m1 for reaching form $\pm\frac{k}{y} = \frac{1}{9}\left(Px\cos 3x + Q\sin 3x + R\right)$ where P and Q are ± 3 or $\pm\frac{1}{3}$ or ± 1

and inverting to $\pm\frac{y}{k} = \frac{9}{\left(Px\cos 3x + Q\sin 3x + R\right)}$

Q	Solution	Marks	Total	Comments
8 **(a)(i)**	$\overrightarrow{AB}=\begin{bmatrix}2\\0\\-1\end{bmatrix}-\begin{bmatrix}4\\-2\\3\end{bmatrix}=\begin{bmatrix}-2\\2\\-4\end{bmatrix}$	M1 A1	2	$\pm\left(\overrightarrow{OB}-\overrightarrow{OA}\right)$ implied by two correct components Allow as $(-2,\ 2,\ -4)$
(ii)	$\begin{bmatrix}1\\5\\-2\end{bmatrix}\bullet\overrightarrow{AB}=-2+10+8=16$	M1 A1ft		ft on $\overrightarrow{AB}$
	$\cos\theta=\dfrac{16}{\sqrt{24}\sqrt{30}}$	M1		Correct formula for $\cos\theta$ with consistent vectors and correct moduli, in form $\sqrt{a^2+b^2+c^2}$
	$\theta=53°$	A1	4	CSO Accept 53.4°, 53.40° SC B1 90° following *sp* = 0
(b)	$\overrightarrow{AB}\bullet\overrightarrow{BC}=\begin{bmatrix}-2\\2\\-4\end{bmatrix}\bullet\left(\begin{bmatrix}4+p\\-2+5p\\3-2p\end{bmatrix}-\begin{bmatrix}2\\0\\-1\end{bmatrix}\right)$	M1		Set up scalar product. $\mu=p$ at *C*. Any letter for *p*. Clear attempt to find $\overrightarrow{BC}$ in terms of *p*.
	$\overrightarrow{BC}=\begin{bmatrix}2+p\\-2+5p\\4-2p\end{bmatrix}$	B1		$\overrightarrow{BC}$ or $\overrightarrow{CB}$ correct
	$-4-2p-4+10p-16+8p=0$	m1		Expand scalar product and solve for *p*; ($=0$ possibly implied)
	$16p=24 \qquad p=\dfrac{3}{2}$	A1		
	$\overrightarrow{OD}=\overrightarrow{OA}+\overrightarrow{BC}=\begin{bmatrix}4\\-2\\3\end{bmatrix}+\begin{bmatrix}\frac{7}{2}\\\frac{11}{2}\\1\end{bmatrix}\quad\left(=\begin{bmatrix}\frac{15}{2}\\\frac{7}{2}\\4\end{bmatrix}\right)$	m1		Correct vector expression to find $\overrightarrow{OD}$ written in components
	D is at $\left(\dfrac{15}{2},\dfrac{7}{2},4\right)$	A1	6	CAO; condone column vector
		Total	**12**	
	Alternative for last 2 marks			
	$\overrightarrow{OD}=\overrightarrow{OC}+\overrightarrow{BA}=\begin{bmatrix}4\\-2\\3\end{bmatrix}+\dfrac{3}{2}\begin{bmatrix}1\\5\\-2\end{bmatrix}+\begin{bmatrix}2\\-2\\4\end{bmatrix}$	m1		
	D is at $\left(\dfrac{15}{2},\dfrac{7}{2},4\right)$	A1		

Part (b) **NB** $p=\frac{3}{2}$ can come from wrong working where candidate uses $\overrightarrow{OC}$ in place of $\overrightarrow{BC}$.
This is M0 and scores **no further marks**, (unless they happen to find and go on to use it correctly).

Version 1.0

General Certificate of Education (A-level)
June 2012

Mathematics MPC4

(Specification 6360)

Pure Core 4

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC4

Q	Solution	Marks	Total	Comments
1(a)(i)	$5x-6=A\left(x-3\right)+Bx$	M1		Multiply by denominator and use two values of *x*.
	$x=0 \quad x=3$			
	$A=2 \quad B=3$	A1	2	
	Alternative: equate coefficients			
	$-6=-3A \quad 5=A+B$	(M1)		Set up and solve simultaneous equations for values of *A* and *B*.
	$A=2 \quad B=3$	(A1)		
(ii)	$\left(\int \frac{2}{x}+\frac{3}{x-3}\ \mathrm{d}x=\right)2\ln x$	B1ft		their *A* ln *x*
	$+3\ln\left(x-3\right) \ \left(+C\right)$	B1ft	2	their *B* ln (*x* – 3) and no other terms; condone *B* ln *x* – 3
(b)(i)	$\begin{array}{r} 2x^2-x+3 \\ 2x+1\overline{\left)4x^3+5x-2\right.} \\ 4x^3+\underline{2x^2} \\ -2x^2+5x \\ -2x^2\underline{-x} \\ 6x-2 \\ 6x+\underline{3} \\ -5 \end{array}$	M1		Division as far as $2x^2+px+q$ with $p\neq0,\ q\neq0$, PI
	$p=-1$	A1		PI by $2x^2-x+q$ seen
	$q=3$	A1		PI by $2x^2-x+3$ seen
	$r=-5$	A1	4	and must state *p*= –1, *q* = 3, *r* = –5 explicitly or write out full correct RHS expression
	Alternative 1:			
	$4x^3+5x-2=$ $4x^3+\left(2+2p\right)x^2+\left(p+2q\right)x+q+r$			
	$2+2p=0$	(M1)		Clear attempt to equate coefficients, PI by *p* = –1
	$p+2q=5$			
	$q+r=-2$			
	$p=-1$	(A1)		
	$q=3 \quad r=-5$	(A1A1)		
	Alternative 2:			
	$4x^3+5x-2=\left(2x+1\right)\left(2x^2+px+q\right)+r$			
	$x=-\frac{1}{2} \quad 4\times\left(-\frac{1}{2}\right)^3+5\left(-\frac{1}{2}\right)+2=r$	(M1)		$x=-\frac{1}{2}$ used to find a value for r
	$r=-5$	(A1)		
	$p=-1\ ,\ q=3$	(A1A1)		

MPC4

Q	Solution	Marks	Total	Comments
(b)(ii)	$\left(\dfrac{4x^3+5x-2}{2x+1}=\right)2x^2+px+q+\dfrac{r}{2x+1}$	M1		
	$\dfrac{2}{3}x^3-\dfrac{1}{2}x^2+3x+k\ln(2x+1)\quad(+C)$	A1ft		ft on *p* and *q*
	$\dfrac{2}{3}x^3-\dfrac{1}{2}x^2+3x-\dfrac{5}{2}\ln(2x+1)\quad(+C)$	A1	3	CSO
	Total		**11**	
2(a)	$R=\sqrt{10}$	B1		Accept 3.2 or better. Can be earned in (b)
	$\tan\alpha=3$	M1		OE; M0 if $\tan\alpha=-3$ seen
	$\alpha=71.6$ or better	A1	3	$\alpha=71.56505...$
(b)	$\sin(x\pm\alpha)=\dfrac{-2}{R}$	M1		or their *R* and/or their α; PI
	$x(=-39.2+71.6)\quad=32(.333)$	A1		32 or better Condone 32.4
	or			
	$x-71.6=219.2$	m1		must see 219 and 72 or better PI by 291 or better as answer Condone extra solutions
	$x=$ 291	A1	4	Condone 290.8 or better CSO Withhold final A1 if more than two answers given within interval
	Total		**7**	

MPC4

Q	Solution	Marks	Total	Comments
3(a)	$(1+4x)^{\frac{1}{2}}=1+4\times\frac{1}{2}x+kx^2$	M1		
	$=1+2x-2x^2$	A1	2	
(b)(i)	$(4-x)^{-\frac{1}{2}}=4^{-\frac{1}{2}}\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$	B1		OE $\frac{1}{2}\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$
	$\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}=$ $1+\left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right)+\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{x}{4}\right)^2$	M1		Condone missing brackets and use of $\left(+\frac{x}{4}\right)$ instead of $\left(-\frac{x}{4}\right)$
	$=1+\frac{1}{8}x+\frac{3}{128}x^2$ $(4-x)^{-\frac{1}{2}}=\frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2$	A1	3	CSO $0.5+0.0625x+0.0117(1875)x^2$
	Alternative using formula from FB			
	$(4-x)^{-\frac{1}{2}}=4^{-\frac{1}{2}}+\left(-\frac{1}{2}\right)\times4^{-\frac{3}{2}}(-x)$ $+\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\times4^{-\frac{5}{2}}(-x)^2$	(M1)		Condone one error and missing brackets
	$=\frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2$	(A2)		CSO Must be fully correct
(b)(ii)	$-4<x<4$ or $x<4$ and $x>-4$	B1	1	Condone $\lvert x\rvert<4$ Must be **and**; not **or** not , (comma)
(c)	$\sqrt{\frac{1+4x}{4-x}}=(1+4x)^{\frac{1}{2}}(4-x)^{-\frac{1}{2}}$			
	$=\left(1+2x-2x^2\right)\left(\frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2\right)$	M1		product of their expansions
	$=\frac{1}{2}+\frac{17}{16}x-\frac{221}{256}x^2$	A1	2	CSO $0.5+1.0625x-0.8632(8...)x^2$
	Total		**8**	

MPC4

Q	Solution	Marks	Total	Comments
4(a)(i)	$1000 \times 1.03^5 \approx (£)1160$	B1	1	Condone missing £ sign;1160 only.
(ii)	$2000 < 1000\left(1+\frac{3}{100}\right)^n$	B1		Condone ‘=’ or ‘<’ used throughout
	$\ln 2 < n\ln 1.03$	M1		Take logs, any base, of their initial expression **correctly**
	$(n > 23.449....)\qquad (N =)24$	A1	3	Condone 23
(b)	$1000 \times \left(1+\frac{3}{100}\right)^n > 1500 \times \left(1+\frac{1.5}{100}\right)^n$	B1		Condone use of T for n Condone ‘=’ or ‘<’ used throughout
	$\ln 1000 + n\ln 1.03 > \ln 1500 + n\ln 1.015$	M1		Take logs, any base, of their initial expression **correctly**
	$n > \dfrac{\ln(1.5)}{\ln\left(\dfrac{1.03}{1.015}\right)}$	A1		Correct expression for n or T
	$(n > 27.63...)\qquad (T =)28$	A1	4	Condone 27
	Total		**8**	

MPC4

Q	Solution	Marks	Total	Comments
5 **(a)(i)**	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{\mathrm{d}y}{\mathrm{d}\theta}}{\frac{\mathrm{d}x}{\mathrm{d}\theta}}=\frac{6\cos 2\theta}{-2\sin\theta}$	M1 A1		condone coefficient errors
	$=\frac{6\left(1-2\sin^2\theta\right)}{-2\sin\theta}$	m1		Use $\cos 2\theta=1-2\sin^2\theta$
	$=6\sin\theta-3\operatorname{cosec}\theta$	A1	4	$a=6 \quad b=-3$
(a)(ii)	$\theta=\frac{\pi}{6} \quad \frac{\mathrm{d}y}{\mathrm{d}x}=6\times\frac{1}{2}-3\times 2=-3$	B1ft		$\theta=\frac{\pi}{6}$ substituted into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and evaluated
	gradient normal $=\frac{1}{3}$	B1ft	2	ft $\frac{\mathrm{d}y}{\mathrm{d}x}$, provided non-zero
(b)	$y=6\sin\theta\cos\theta$ $=(\pm)6\sqrt{1-\cos^2\theta}\times\cos\theta$	M1		Correct expansion of $\sin 2\theta$ and use $x=2\cos\theta$ to eliminate θ
	$=(\pm)6\sqrt{1-\left(\frac{x}{2}\right)^2}\times\left(\frac{x}{2}\right)$	A1		Correct elimination of θ
	$y^2=\frac{9}{4}x^2\left(4-x^2\right)$	A1	3	$p=\frac{9}{4}$ OE and $\left(4-x^2\right)$ shown
	Alternative using verification $y^2=9\sin^2 2\theta=36\sin^2\theta\cos^2\theta$	(M1)		must be squared
	$x^2\left(4-x^2\right)=4\cos^2\theta\times 4\sin^2\theta$	(A1)		
	$p=\frac{9}{4}$ OE	(A1)		or $y^2=\frac{9}{4}x^2\left(4-x^2\right)$
	Total		9	

MPC4

Q	Solution	Marks	Total	Comments
6	$9x^2 - 6xy + 4y^2 \qquad = 3$			
	$18x \qquad = 0$	B1		=0 PI
	$-6y - 6x\dfrac{dy}{dx}$	B1		or $\dfrac{d(6xy)}{dx} = 6y + 6x\dfrac{dy}{dx}$ seen separately
	$+8y\dfrac{dy}{dx}$	B1		$\dfrac{dy}{dx}(-6x+8y) = 6y - 18x$
	Use $\dfrac{dy}{dx} = 0$	M1		
	$\Rightarrow y = 3x$ or $x = \dfrac{y}{3}$	A1		CSO
	$y = 3x \Rightarrow 9x^2 - 6x \times 3x + 4(3x)^2 = 3$	m1		Substitute $y = ax$ into equation and solve for a value of x or y. Condone missing brackets.
	$27x^2 = 3 \Rightarrow x = \pm\dfrac{1}{3}$ OE	A1ft		Both values of x or y required. ft on their $y = 3x$
	$\left(\dfrac{1}{3}, 1\right) \quad \left(-\dfrac{1}{3}, -1\right)$	A1	8	CSO Correct corresponding simplified values of x and y. Withhold if additional answers given
	Total		**8**	

MPC4

Q	Solution	Marks	Total	Comments
7(a)	$2\lambda = 8 + 2\mu$ $-2 \quad = 3 + 5\mu$ $\lambda = 3$, $\mu = -1$	M1		Use the first two equations to set up and attempt to solve simultaneous equations for λ or μ. Must not assume q = 4.
	$q - \lambda = 5 + 4\mu$ $q = 5 + 3 - 4 = 4$	A1		Use 3rd equation to show $q = 4$ **AG**.
	P is at $(6,-2,1)$	B1	3	Condone as a column vector
(b)	$\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} \bullet \begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix} = 4 - 4 = 0 \Rightarrow$ perpendicular	B1	1	or $2 \times 2 + -1 \times 4 = 0$ seen **and** conclusion (condone $\theta = 90$)
(c)(i)	A is at (2, –2, 3) $AP^2 = (6-2)^2 + (-2--2)^2 + (1-3)^2$	M1		Candidate's $\left\lvert\overrightarrow{AP}\right\rvert^2$
	$= 20$	A1	2	CAO NMS $AP = \sqrt{20}$ M1A0
(ii)	$\left(\overrightarrow{PB} =\right)\begin{bmatrix} 8 \\ 3 \\ 5 \end{bmatrix} + \mu\begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix} - \begin{bmatrix} 6 \\ -2 \\ 1 \end{bmatrix} \quad \left(= \begin{bmatrix} 2+2\mu \\ 5+5\mu \\ 4+4\mu \end{bmatrix}\right)$	M1		Clear attempt to find $\overrightarrow{BP}$ or $\overrightarrow{PB}$ in terms of μ
	$\left(PB^2 =\right)(2+2\mu)^2 + (5+5\mu)^2 + (4+4\mu)^2$	m1		Find distance BP in terms of μ
	$45\mu^2 + 90\mu + 45 = 20$ $(5)(9\mu^2 + 18\mu + 5) = 0$	m1		Attempt to set up three-term quadratic in μ and equate to their AP^2
	$(3\mu+1)(3\mu+5) = 0$	m1		Solve quadratic equation to obtain **two** values of μ
	$\mu = -\dfrac{1}{3}$ and $\mu = -\dfrac{5}{3}$	A1		Both values correct.
	B is at $\left(\dfrac{22}{3}, \dfrac{4}{3}, \dfrac{11}{3}\right)$ and $\left(\dfrac{14}{3}, -\dfrac{16}{3}, -\dfrac{5}{3}\right)$	A1	6	Both sets of coordinates required. Condone column vectors. SC one value of μ correct and corresponding coordinates of B correct scores A1 A0.

MPC4

Q	Solution	Marks	Total	Comments
	Alternative 1			
	$\left(\overrightarrow{AB}=\right)\begin{bmatrix}8\\3\\5\end{bmatrix}+\mu\begin{bmatrix}2\\5\\4\end{bmatrix}-\begin{bmatrix}2\\-2\\3\end{bmatrix}\quad\left(=\begin{bmatrix}6+2\mu\\5+5\mu\\2+4\mu\end{bmatrix}\right)$	(M1)		Clear attempt to find $\overrightarrow{AB}$ or $\overrightarrow{BA}$ in terms of μ
	$\left(AB^2=\right)(6+2\mu)^2+(5+5\mu)^2+(2+4\mu)^2$	(m1)		Find distance AB in terms of μ
	$45\mu^2+90\mu+65=40$ $(5)\left(9\mu^2+18\mu+5\right)=0$	(m1)		Attempt to set up three-term quadratic in μ and equate to their $2\times$ their AP^2
	As before			
	Alternative 2			
	$\overrightarrow{PB}=k\begin{bmatrix}2\\5\\4\end{bmatrix}$	(M1)		
	$k^2\left(2^2+5^2+4^2\right)=20$	(m1) (m1)		m1 for LHS m1 for equating to ‘their 20’ May score M1m0m1
	$k=\pm\frac{2}{3}$	(A1)		
	$\overrightarrow{OB}=\overrightarrow{OP}+(\pm)(\text{their value of } k)\begin{bmatrix}2\\5\\4\end{bmatrix}$	(m1)		
	B is at $\left(\frac{22}{3},\frac{4}{3},\frac{11}{3}\right)$ and $\left(\frac{14}{3},-\frac{16}{3},-\frac{5}{3}\right)$	(A1)		
	Total		**12**	

Q	Solution	Marks	Total	Comments
8(a)	$\frac{\mathrm{d}h}{\mathrm{d}t}$	B1		
	$derivative = * \times (2-h)$	M1		Use of $2-h$ or $h-2$; $*$ is a constant or expression in h and/or t.
	$\frac{\mathrm{d}h}{\mathrm{d}t} = k(2-h)$	A1	3	All correct; must be $(2-h)$
(b)(i)	$\int x\sqrt{2x-1}\ \mathrm{d}x = \int \frac{1}{15}\mathrm{d}t$	B1		Correct separation and notation; condone missing integral signs.
	$= \frac{1}{15}t$	B1		
	Substitute $u = 2x-1$ $\int x\sqrt{2x-1}\ \mathrm{d}x = \int \frac{1}{2}(u+1)\sqrt{u}\ \frac{1}{2}\mathrm{d}u$	M1		Suitable substitution and attempt to write integral in terms of *u* including d*x* replaced by $\frac{1}{2}\mathrm{d}u$ or 2 d*u*.
	$= \left(\frac{1}{4}\right)\int u^{\frac{3}{2}} + u^{\frac{1}{2}}\ \mathrm{d}u$	A1		$\frac{1}{4}$ need not be seen
	$= \frac{1}{4}\left(\frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}}\right)\ (+C)$	A1		Integration correct including $\frac{1}{4}$
	$x = 1,\ t = 0$ $u = 1,\ t = 0 \quad \frac{1}{4}\left(\frac{2}{5}+\frac{2}{3}\right) + C = 0$	M1		Use $x = 1,\ t = 0$ to find a value for constant *C* from equation in *x* and *t*.
	$C = -\frac{4}{15}$	A1		$C = -0.2666...$ $C = -0.267$ or better
	$t = \frac{1}{2}\left(3(2x-1)^{\frac{5}{2}} + 5(2x-1)^{\frac{3}{2}}\right) - 4$	A1	8	ISW $t = (2x-1)^{\frac{3}{2}}(3x+1) - 4$
	Alternative (Parts) As before	(B1B1)		
	$u = x\ ,\ \frac{\mathrm{d}v}{\mathrm{d}x} = (2x-1)^{\frac{1}{2}}$ $\mathrm{d}u = 1 \quad v = k(2x-1)^{\frac{3}{2}}$	(M1)		Attempt to use parts
	$\int x\sqrt{2x-1}\ \mathrm{d}x = x\frac{1}{3}(2x-1)^{\frac{3}{2}} - \int \frac{1}{3}(2x-1)^{\frac{3}{2}}\ \mathrm{d}x$	(A1)		Condone missing d*x*
	$= x\frac{1}{3}(2x-1)^{\frac{3}{2}} - \frac{1}{15}(2x-1)^{\frac{5}{2}}\ (+C)$	(A1)		
	$x = 1,\ t = 0 \quad \frac{1}{3} - \frac{1}{15} + C = 0$	(M1)		Use $x = 1,\ t = 0$ to find a value for constant *C* from equation in *x* and *t*
	$C = -\frac{4}{15}$	(A1)		$C = -0.2666...$ $C = -0.267$ or better
	$t = 5x(2x-1)^{\frac{3}{2}} - (2x-1)^{\frac{5}{2}} - 4$	(A1)		ISW $t = (2x-1)^{\frac{3}{2}}(3x+1) - 4$
(ii)	$x = 2 \quad t = 32.4 \quad$ (minutes)	B1	1	32.4 or better (32.373…)
	Total		**12**	
	TOTAL		**75**	

Version

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Mathematics MPC4

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Pure Core 4

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	$f\left(-\frac{1}{2}\right)=2\left(-\frac{1}{2}\right)^3+\left(-\frac{1}{2}\right)^2-8\left(-\frac{1}{2}\right)-7$	M1		Evaluate $f\left(-\frac{1}{2}\right)$, not long division.
	$=-3$	A1	2	
(b) **(i)**	$g\left(-\frac{1}{2}\right)=0 \Rightarrow -3+d=0$			Or $f\left(-\frac{1}{2}\right)+d=0$
	$d=3 \Rightarrow\ g(x)=2x^3+2x^2-8x-7+3$ $g(x)=2x^3+2x^2-8x-4$	B1	1	All steps seen with conclusion **AG** Allow verification with $-\frac{1}{4}+\frac{1}{4}+4-4=0$ seen, and conclusion ; therefore factor
(ii)	$g(x)=2x^3+x^2-8x-4=(2x+1)(x^2-4)$ $=(2x+1)(x+2)(x-2)$	B1	1	$a=-4$
(iii)	$2x^3-3x^2-2x=x(2x+1)(x-2)$	M1		Clear attempt to factorise denominator; 3 factors needed.
	$\frac{(2x+1)(x+2)(x-2)}{x(2x+1)(x-2)}=\frac{x+2}{x}$	m1		At least one correct factor cancelled
	$\frac{g(x)}{2x^3-3x^2-2x}=1+\frac{2}{x}$	A1	3	CSO part (a)(iii) NMS is 0/3
	Total		**7**	
(b)(iii)	Alternative $\frac{g(x)}{2x^3-3x^2-2x}=1+\frac{4x^2-6x-4}{2x^3-3x^2-2x}$	M1		$1+\frac{\text{quadratic}}{2x^3-3x^2-2x}$
	$=1+\frac{2(2x^2-3x-2)}{2x^3-3x^2-2x}$	A1		
	$=1+\frac{2}{x}$	A1	**3**	

Q	Solution	Marks	Total	Comments
2 (a)	$7x-1=A(1+3x)+B(3-x)$	M1		
	$x=3 \qquad x=-\frac{1}{3}$	m1		Use two values of *x* to find *A* **and** *B*. Or solve $A+3B=-1 \qquad 3A-B=7$ Or cover up rule
	$A=2 \qquad B=-1$	A1	3	
(b) (i)	$\frac{1}{1+3x}=(1+3x)^{-1}$			
	$=1+(-1)3x+\frac{1}{2}(-1)(-2)(3x)^2$	M1		Condone missing brackets
	$=1-3x+9x^2$	A1		
	$\frac{1}{3-x}=(3-x)^{-1}=\frac{1}{3}\left(1-\frac{x}{3}\right)^{-1}$	B1		
	$\left(1-\frac{x}{3}\right)^{-1}=1+(-1)\left(-\frac{x}{3}\right)+kx^2$	M1		Condone missing brackets
	$=1+\frac{x}{3}+\frac{x^2}{9}$	A1		
	$\frac{7x-1}{3+8x-3x^2}=$ $2\times\frac{1}{3}\times\left(1+\frac{x}{3}+\frac{x^2}{9}\right)-1\times\left(1-3x+9x^2\right)$	M1		Attempt to use PFs to combine expansions, or expand $(7x-1)(3-x)^{-1}(1+3x)^{-1}$ and simplify to $a+bx+cx^2$
	$=-\frac{1}{3}+\frac{29}{9}x-\frac{241}{27}x^2$	A1	7	
(ii)	0.4 is outside the range of validity, because $0.4>\frac{1}{3}$.	B1	1	OE Accept $0.4>\frac{1}{3}$
	Total		**11**	

Q	Solution	Marks	Total	Comments
3 **(a)(i)**	$R=\sqrt{13}$	B1		Accept 3.6 or better
	$\tan\alpha=\dfrac{2}{3}$	M1		OE
	$\alpha=33.7°$	A1	**3**	
(ii)	minimum value $=-\sqrt{13}$	B1ft		Accept -3.6 or better; ft R
	when $x-\alpha=\cos^{-1}\left(-1\right)$	M1		NMS 0/2
	$x=213.7°$	A1	**3**	Calculus used 0/2
(b)(i)	LHS $=\dfrac{\cos x}{\sin x}-2\sin x\cos x$	M1		Express $\cot x-\sin 2x$ in terms of $\sin x$ and $\cos x$; ACF
	$=\dfrac{\cos x}{\sin x}\left(1-2\sin^2 x\right)$	m1		Factor out $\frac{\cos x}{\sin x}$ and $1-2\sin^2 x$
	$=\cot x\cos 2x$	A1	**3**	All correct
(ii)	$\cot x-\sin 2x=0$			
	$\cot x\cos 2x=0$			
	$\cot x=0$ or $\cos 2x=0$	M1		Both equations correct
	$2x=90°\quad\left(270°\right)$	m1		Condone missing 270°
	$x=90°$, $45°$, $135°$	A1	**3**	All correct
	Total		**12**	
3 **(b)** **(i)**	Alternatives			
	$\text{RHS}=\cot x\cos 2x$			
	$=\dfrac{\cos x}{\sin x}\left(1-2\sin^2 x\right)$	M1		Express $\cot x\cos 2x$ in terms of $\cos x$ and $\sin x$, $\cos 2x$ ACF
	$=\dfrac{\cos x}{\sin x}-2\sin x\cos x$	m1		$\cos 2x=1-2\sin^2 x$ and multiply out and simplify.
	$=\cot x-\sin 2x$	A1	**3**	All correct.
	$\cot x\left(1-\cos 2x\right)-\sin 2x=0$			Rearrange to expression $=0$ and factor out $\cot x$;
	$\dfrac{\cos x}{\sin x}\left(1-\left(1-2\sin^2 x\right)\right)-2\sin x\cos x=0$	M1		Express $\cot x, \cos 2x$ and $\sin 2x$ in terms of $\sin x$ and $\cos x$, ACF
	$\dfrac{\cos x}{\sin x}\left(2\sin^2 x\right)-2\sin x\cos x=0$	m1		$\cos 2x=1-2\sin^2 x$ used
	$2\sin x\cos x-2\sin x\cos x=0$	A1	**3**	Simplified, with all correct

3 (b)(ii)	Alternative $\cot x - \sin 2x = \frac{\cos x}{\sin x} - 2\sin x\cos x = 0$ $\cos x\left(\frac{1}{\sin x} - 2\sin x\right) = 0$ $\cos x = 0 \quad \text{or} \quad 1 - 2\sin^2 x = 0$ $\sin x = (\pm)\frac{1}{\sqrt{2}}$ $x = 90° \ , \ 45° \ , \ 135°$	M1 m1 A1	**3**	Both equations

Q	Solution	Marks	Total	Comments
4 **(a)(i)**	$2x-2y\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$	M1		Correct differentiation
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{x}{y}$			
	at (p,q) $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{p}{q}$	A1	**2**	(p,q) substituted into correct derivative or $x=p$ $y=q$ stated **AG**
(ii)	tangent at (p,q) $y-q=\dfrac{p}{q}(x-p)$	B1		ACF
	tangent at $(p,-q)$ $y-(-q)=\dfrac{-p}{q}(x-p)$	B1		ACF
	add $2y=0$	M1		Solve tangent equations for y .
	conclusion $y=0\Rightarrow$ intersect on Ox	A1	**4**	Conclusion required
(b)	$x^2=t^2+4+\dfrac{4}{t^2}$ $y^2=t^2-4+\dfrac{4}{t^2}$	M1		Attempt to square x and y and subtract.
	$x^2-y^2=8$	A1	**2**	All correct AG Allow $8=8$
	Total		**8**	

4(a)(i)	Alternative $y=\sqrt{x^2-8}$ $\quad \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}\times 2x\left(x^2-8\right)^{-\frac{1}{2}}=\frac{x}{y}$	M1		
	$=\frac{p}{q}$	A1	**2**	
(a)(i)	Alternative $\frac{\mathrm{d}y}{\mathrm{d}t}=1+\frac{2}{t^2} \qquad \frac{\mathrm{d}x}{\mathrm{d}t}=1-\frac{2}{t^2}$	M1		Attempt parametric derivatives and use chain rule.
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1+\frac{2}{t^2}}{1-\frac{2}{t^2}}=\frac{t+\frac{2}{t}}{t-\frac{2}{t}}=\frac{x}{y}$			
	at $\left(p,q\right)$ $\quad \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{p}{q}$	A1	**2**	$\left(p,q\right)$ substituted into correct derivative.
(ii)	tangent at $\left(p,q\right)$ $\quad y-q=\frac{p\left(x-p\right)}{q}$	B1		ACF
	tangent at $\left(p,-q\right)$ $\quad y-\left(-q\right)=\frac{-p\left(x-p\right)}{q}$	B1		ACF
	When $y=0$ $\quad \frac{-q^2}{p}=x-p$ and $\frac{q^2}{-p}=x-p$	M1		Substitute $y=0$ into both candidate's tangents and solve for x.
	$x=p-\frac{q^2}{p}$ is on both lines, so intersect on x axis	A1	**4**	Conclusion
	$x+y=2t \qquad x-y=\frac{4}{t}$			
(b)	$\left(x-y\right)\left(x+y\right)=2t\times\frac{4}{t}$	M1		Attempt to eliminate t
	$x^2-y^2=8$	A1	**2**	

Q	**Solution**	**Marks**	**Total**	**Comments**
5(a)	$\int x\left(x^2+3\right)^{\frac{1}{2}}\ \mathrm{d}x = p\left(x^2+3\right)^{\frac{3}{2}}$	M1		By inspection or substitution
	$=\frac{1}{3}\left(x^2+3\right)^{\frac{3}{2}}\quad(+C)$	A1	**2**	
(b)	$\int \mathrm{e}^{2y}\ \mathrm{d}y = \int x\sqrt{x^2+3}\ \mathrm{d}x$	B1		Correct separation and notation Condone missing integral signs
	$\frac{1}{2}\mathrm{e}^{2y}$	B1		
	$=\frac{1}{3}\left(x^2+3\right)^{\frac{3}{2}}+C$	M1		Equate to result from (a) with constant.
	$\frac{1}{2}=\frac{1}{3}\times 4^{\frac{3}{2}}+C$	m1		Use $(1,0)$ to find constant.
	$C=-\frac{13}{6}$	A1		CAO
	$2y=\ln\left(\frac{2}{3}\left(x^2+3\right)^{\frac{3}{2}}-\frac{13}{3}\right)$	m1		Solve for y , taking logs correctly.
	$y=\frac{1}{2}\ln\left(\frac{2}{3}\left(x^2+3\right)^{\frac{3}{2}}-\frac{13}{3}\right)$	A1	**7**	CSO
	Total		**9**	

Q	Solution	Marks	Total	Comments
6 **(a)(i)**	$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=\begin{bmatrix}8\\-4\\-6\end{bmatrix}-\begin{bmatrix}3\\1\\-6\end{bmatrix}=\begin{bmatrix}5\\-5\\0\end{bmatrix}=5\begin{bmatrix}1\\-1\\0\end{bmatrix}$	B1	**1**	Must see $\overrightarrow{OC}-\overrightarrow{OA}$ in correct components. $n=5$
(ii)	$\overrightarrow{BC}=\begin{bmatrix}3\\-2\\-6\end{bmatrix}$	B1		$\overrightarrow{BC}$ or $\overrightarrow{CB}$ correct
	$5\begin{bmatrix}1\\-1\\0\end{bmatrix}\bullet\begin{bmatrix}3\\-2\\-6\end{bmatrix}=5\sqrt{2}\sqrt{49}\cos ACB$	M1		Correct form of formula using consistent vectors; condone use of θ or a wrong angle and a missing multiple of 5
	$5(3+2)=5\sqrt{2}\sqrt{49}\cos ACB$	A1		Correct scalar product and moduli.
	$\cos ACB=\frac{5}{\sqrt{2}\times 7}=\frac{5\sqrt{2}}{2\times 7}=\frac{5\sqrt{2}}{14}$	A1	**4**	AG Must see, or rearrangement $\cos ACB=\frac{5}{\sqrt{2}\times 7}$ or $\frac{25}{35\sqrt{2}}$
(b)	vector equation $\mathbf{r}=\begin{bmatrix}3\\1\\-6\end{bmatrix}+\lambda\begin{bmatrix}5\\-5\\0\end{bmatrix}$	M1		$\mathbf{a}+\lambda\mathbf{d}$
		A1	**2**	OE
(c)(i)	$\begin{bmatrix}3\\1\\-6\end{bmatrix}+\lambda\begin{bmatrix}5\\-5\\0\end{bmatrix}=\begin{bmatrix}5\\-2\\0\end{bmatrix}+\mu\begin{bmatrix}1\\1\\p\end{bmatrix}$	M1		Equate vector equations for *AC* and *BD*. OE
	$3+5\lambda=5+\mu$ $1-5\lambda=-2+\mu$	M1		Set up equations and solve for μ; must find a value for μ
	$\mu=\frac{1}{2}$	A1		
	$-6=\mu p\Rightarrow p=-12$	A1	**4**	
(ii)	$\overrightarrow{AB}=\begin{bmatrix}2\\-3\\6\end{bmatrix}\quad \overrightarrow{CD}=\begin{bmatrix}-2\\3\\-6\end{bmatrix}$	M1		Clear attempt to find the vectors of the sides.
	$\overrightarrow{AD}=\begin{bmatrix}3\\-2\\-6\end{bmatrix}\quad \overrightarrow{BC}=\begin{bmatrix}3\\-2\\-6\end{bmatrix}$	A1		All vectors correct
		m1		Find the lengths of the sides, or state they all $=\sqrt{49}$ if all correct.
	All sides are of same length, 7; hence rhombus.	A1	**4**	Each side $=7$ and conclusion. Or adjacdnt sides $=7$ and opposite sides are parallel.
	Total		**15**	

(c)(ii)	Alternative $\overrightarrow{AC}\bullet\overrightarrow{BD} = 5 - 5$	M1		Calculate scalar product of $\overrightarrow{AC}$ and $\overrightarrow{BD}$
	$= 0 \Rightarrow$ $\overrightarrow{AC}$ and $\overrightarrow{BD}$ are perpendicular	A1		$= 0$ from correct $\overrightarrow{AC}$ and $\overrightarrow{BD}$ and conclusion
	$\mu = \frac{1}{2} \Rightarrow \lambda = \frac{1}{2} \Rightarrow$ intersection is at midpoint of AC and BD	M1		Find value of λ and attempt to use in argument about point of intersection
	Diagonals bisect each other at right angles; hence rhombus, with all sides equal to 7	A1		Fully correct conclusion. Must show diagonals bisect

Q	Solution	Marks	Total	Comments
7				
(a)(i)	$t=0 \quad N=50$	B1	1	
(ii)	$t=24 \quad N=345$	B1	1	Must be 345 (not 345.2534..)
(iii)	$1+9\mathrm{e}^{-\frac{t}{8}}=\frac{500}{400}\Rightarrow 9\mathrm{e}^{-\frac{t}{8}}=\frac{1}{4}$	M1		Correct algebra seen
	$\mathrm{e}^{\frac{t}{8}}=36$	m1		Or $e^{-\frac{t}{8}}=\frac{1}{36}$
	$t=8\ln 36$	A1	3	or $t=16\ln 6$
(b)				
(i)	$\frac{\mathrm{d}N}{\mathrm{d}t}=-500\left(1+9\mathrm{e}^{-\frac{t}{8}}\right)^{-2}\left(-\frac{9}{8}\mathrm{e}^{-\frac{t}{8}}\right)$	M1 A1		Clear attempt at chain rule or quotient rule.
	$=-500\left(-\frac{1}{8}\left(\frac{500}{N}-1\right)\right)\left(\frac{500}{N}\right)^{-2}$	m1		Use $\mathrm{e}^{-\frac{t}{8}}=\frac{1}{9}\left(\frac{500}{N}-1\right)$ to eliminate $\mathrm{e}^{-\frac{t}{8}}$.
	$=\frac{N^2}{500}\left(\frac{1}{8}\left(\frac{500}{N}-1\right)\right)$			
	$\frac{\mathrm{d}N}{\mathrm{d}t}=\frac{N}{4000}(500-N)$	A1	4	Correct algebra to AG
(ii)	$\frac{\mathrm{d}}{\mathrm{d}N}\left(500N-N^2\right)=500-2N$	M1		Differentiate and attempt to find N at max value
	$500-2N=0\Rightarrow N=250$	A1		Condone $\frac{\mathrm{d}^2}{\mathrm{d}t^2}$ for $\frac{\mathrm{d}}{\mathrm{d}N}$
	$9e^{-\frac{T}{8}}=1$	m1		
	$e^{\frac{T}{8}}=9$			
	$T=8\ln 9=17(.577)$	A1	4	$T=17$ or better CSO Accept 17, 18, 17.5, 17.6
	Total		**13**	
	TOTAL		**75**	
(b)(ii)	Alternative, by inspection Max of $N(500-N)$ occurs at $N=250$	B2		

(b)(i)	Alternatives			
	Alternative 1 implicit differentiation			
	$e^{-\frac{t}{8}}=\frac{500-N}{9N}$ $\frac{\mathrm{d}t}{\mathrm{d}N}\left(-\frac{1}{8}e^{-\frac{t}{8}}\right)=-\frac{500}{9N^2}$	M1 A1		Correct expressions for $e^{-\frac{t}{8}}$ and attempt to use implicit differentiation Fully correct
	use $e^{-\frac{t}{8}}=\frac{1}{9}\left(\frac{500}{N}-1\right)$	m1		Attempt to eliminate $e^{-\frac{t}{8}}$ using correct expression
	to get $\frac{\mathrm{d}t}{\mathrm{d}N}=\frac{4000}{9N^2}\times\frac{9N}{500-N}$ $\frac{\mathrm{d}N}{\mathrm{d}t}=\frac{N}{4000}(500-N)$	A1	**4**	
	Alternative 2 explicit differentiation			
	$t=-8\ln\left(\frac{500-N}{9N}\right)$ $\frac{\mathrm{d}t}{\mathrm{d}N}=-8\left(\frac{(500-N)\left(\frac{-1}{9N^2}\right)-\frac{1}{9N}}{\left(\frac{500-N}{9N}\right)}\right)$	M1 A1		Correct expression for t and attempt at differentiation with use of chain rule and product for ln derivative.
	$=\frac{8}{9N}\left(9+\frac{9N}{500-N}\right)$	m1		Clear fractions within fractions
	$=\frac{8}{9N}\left(\frac{4500}{500-N}\right)$ $\frac{\mathrm{d}N}{\mathrm{d}t}=\frac{N}{4000}(500-N)$	A1	**4**	All correct
	Or $t=-8\left(\ln(500-N)-\ln(9N)\right)$ $\frac{\mathrm{d}t}{\mathrm{d}N}=-8\left(\frac{-1}{500-N}-\frac{9}{9N}\right)$	M1 A1		Correct expression for t and ln derivatives, condone sign errors
	$=8\left(\frac{1}{500-N}+\frac{1}{N}\right)$ $=8\left(\frac{N+500-N}{N(500-N)}\right)$	m1		Common denominator to combine fractions
	$=\frac{4000}{N(500-N)}\Rightarrow\frac{\mathrm{d}N}{\mathrm{d}t}=\frac{4000}{N(500-N)}$	A1	**4**	All correct
	Alternative 3 solve differential equation			

	$\int \frac{\mathrm{d}N}{N\left(500-N\right)}=\int \frac{\mathrm{d}t}{4000}$	M1		Separate variables, and attempt to form partial fractions and integrate to ln terms $=kt+C$
	$\int \frac{1}{500}\left(\frac{1}{N}+\frac{1}{500-N}\right)\mathrm{d}N=\int \frac{\mathrm{d}t}{4000}$	A1		
	$\ln N-\ln\left(500-N\right)=\frac{500}{4000}t+C$			
	$\left(t=0\ \ N=50\right)\quad C=\ln\left(\frac{1}{9}\right)$	m1		Use $\left(50,0\right)$ to find C and obtain $\mathrm{e}^{\frac{1}{8}t}=\mathrm{f}\left(N\right)$
	$\ln\left(\frac{9N}{500-N}\right)=\frac{1}{8}t\Rightarrow\frac{9N}{500-N}=e^{\frac{1}{8}t}$			
	$N\left(9+e^{\frac{1}{8}t}\right)=500e^{\frac{1}{8}t}$	A1		Manipulate correctly to original given equation.
	$N=\frac{500e^{\frac{1}{8}t}}{9+e^{\frac{1}{8}t}}=\frac{500}{1+9e^{-\frac{1}{8}t}}$			

Version 1.0

General Certificate of Education (A-level)
June 2013

Mathematics **MPC4**

(Specification 6360)

Pure Core 4

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)(i)	$5-8x=A(1-3x)+B(2+x)$ $x=-2 \qquad x=\frac{1}{3}$ $A=3 \qquad B=1$	M1 m1 A1	3	Two values of x used to find values for A and B
(ii)	$\int_{-1}^{0} \frac{3}{2+x}+\frac{1}{1-3x}\ \mathrm{d}x$ $= 3\ln(2+x)-\frac{1}{3}\ln(1-3x)$ $=(3\ln 2-\frac{1}{3}\ln 1)-(3\ln 1-\frac{1}{3}\ln 4)$ $=3\ln 2+\frac{1}{3}\ln 4$ $=\frac{11}{3}\ln 2$	M1 m1 A1ft A1ft	4	$a\ln(2+x)+b\ln(1-3x)$ where a and b are constants $\mathrm{f}(0)-\mathrm{f}(-1)$ used ft A and B ft $\left(A+\frac{2}{3}B\right)\ln 2$
(b)(i)	$(C=)2$	B1	1	
(ii)	$\int \frac{9-18x-6x^2}{2-5x-3x^2}\ \mathrm{d}x=\int C\mathrm{d}x+\int \frac{5-8x}{2-5x-3x^2}\ \mathrm{d}x$ $\int_{-1}^{0} \frac{9-18x-6x^2}{2-5x-3x^2}\ \mathrm{d}x = 2+\frac{11}{3}\ln 2$	M1 A1ft	2	Seen or implied. Allow $\pm C+\int \frac{5-8x}{2-5x-3x^2}\mathrm{d}x$ Accept $2+3\ln 2+\frac{1}{3}\ln 4$ ft $2+$ candidate’s answer to part (a)(ii) if exact.
(a)(i)	**Alternative** $5-8x=A(1-3x)+B(2+x)$ $5=A+2B$ $-8=-3A+B$ $A=3 \quad B=1$	(M1) (m1) (A1)	(3)	Set up simultaneous equations and solve.
	Total		**10**	

Q	Solution	Marks	Total	Comments
2(a)(i)	$h^2=2^2+\sqrt{5}^2=9\Rightarrow h=3\Rightarrow \sin\alpha=\frac{2}{3}$	B1		Pythagoras used or all of $2,\ \sqrt{5},\ 3$ seen correctly on triangle **AG**
	$\cos\alpha=\frac{\sqrt{5}}{3}$	B1	2	$\frac{\sqrt{5}}{3}$ or $\sqrt{\frac{5}{9}}$ or $\frac{5}{3\sqrt{5}}$ seen
(ii)	$\sin 2\alpha=2\sin\alpha\cos\alpha$	M1		Correct formula seen or implied
	$=\left(2\times\frac{2}{3}\times\frac{\sqrt{5}}{3}\right)=\frac{4}{9}\sqrt{5}$	A1	2	Must see $\frac{\sqrt{5}}{3}$ here or in part (a)(i) Accept $\frac{4}{3}\sqrt{\frac{5}{9}}$
(b)	$\cos\beta=\frac{2}{\sqrt{5}}$ or $\sin\beta=\frac{1}{\sqrt{5}}$	B1		Either correct. Accept $\sqrt{\frac{4}{5}}$, $\frac{\sqrt{5}}{5}$
	$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$	M1		Correct formula seen or implied.
	$=\frac{\sqrt{5}}{3}\times\frac{2}{\sqrt{5}}+\frac{2}{3}\times\frac{1}{\sqrt{5}}$	A1		All correct
	$=\frac{2}{15}\left(5+\sqrt{5}\right)$	A1	4	$k=5$ with previous A mark awarded
(a)(i)	**Alternative**			
	$\text{cosec}^2\alpha=1+\cot^2\alpha=1+\frac{5}{4}=\frac{9}{4}$			
	$\text{cosec}\alpha=\frac{3}{2}\quad \sin\alpha=\frac{2}{3}$	(B1)		Must be positive
	$\sec^2\alpha=1+\tan^2\alpha=1+\frac{4}{5}=\frac{9}{5}$			
	$\sec\alpha=\frac{3}{\sqrt{5}}\quad \cos\alpha=\frac{\sqrt{5}}{3}$	(B1)		Must be posiitve
	Total		**8**	

Q	Solution	Marks	Total	Comments
3(a)	$(1+6x)^{-\frac{1}{3}}=1+\left(-\frac{1}{3}\right)6x+kx^2$	M1		
	$=1-2x+8x^2$	A1	2	
(b)(i)	$(27+6x)^{-\frac{1}{3}}=27^{-\frac{1}{3}}\left(1+\frac{6}{27}x\right)^{-\frac{1}{3}}$	B1		
	$\left(1+\frac{6}{27}x\right)^{-\frac{1}{3}}=1+\left(-\frac{1}{3}\times\frac{6}{27}x\right)+\left(-\frac{1}{3}\times-\frac{4}{3}\right)\frac{1}{2}\left(\frac{6}{27}x\right)^2$	M1		Condone missing brackets and one error
	$(27+6x)^{-\frac{1}{3}}=\frac{1}{3}-\frac{2}{81}x+\frac{8}{2187}x^2$	A1	3	
(ii)	$\left(\sqrt[3]{\frac{2}{7}}=\frac{2}{\sqrt[3]{28}}\Rightarrow 27+6x=28\Rightarrow x=\frac{1}{6}\right)$			
	$\sqrt[3]{\frac{1}{28}}=\frac{1}{3}-\frac{2}{81}\times\frac{1}{6}+\frac{8}{2187}\times\left(\frac{1}{6}\right)^2\ (\approx 0.3293..)$	M1		Substitute $x=\frac{1}{6}$ into expansion from (b)(i)
	$\left(\sqrt[3]{\frac{2}{7}}\approx 2\times 0.3293197..=0.6586394...\right)$			
	$=0.658639\quad(6\text{dp})$	A1	2	CSO
	Alternatives			
(b)(i)	$(27+6x)^{-\frac{1}{3}}=27^{-\frac{1}{3}}\left(1+\frac{6}{27}x\right)^{-\frac{1}{3}}$	(B1)		
	$\left(1+\frac{6}{27}x\right)^{-\frac{1}{3}}=1-2\times\frac{1}{27}x+8\times\left(\frac{1}{27}\right)^2x^2$	(M1)		Replace x with $\frac{1}{27}x$,**not** $\frac{6}{27}x$, in expansion from (a); condone missing brackets and one error
	$(27+6x)^{-\frac{1}{3}}=\frac{1}{3}-\frac{2}{81}x+\frac{8}{2187}x^2$	(A1)	(3)	
(b)(i)	$(27+6x)^{-\frac{1}{3}}=27^{-\frac{1}{3}}+\left(-\frac{1}{3}\right)27^{-\frac{4}{3}}\times 6x$ $+\left(-\frac{1}{3}\right)\times\left(-\frac{4}{3}\right)\frac{1}{2}27^{-\frac{7}{3}}\times(6x)^2$	(M1)		Use result from formula book; Condone missing brackets and one error
	$(27+6x)^{-\frac{1}{3}}=\frac{1}{3}-\frac{2}{81}x+\frac{8}{2187}x^2$	(A2)	(3)	A1 not available
	Total		**7**	

Q	**Solution**	**Marks**	**Total**	**Comments**
4(a)	$\left(\frac{\mathrm{d}x}{\mathrm{d}t}=\right)-16\mathrm{e}^{-2t}$ $\quad\left(\frac{\mathrm{d}y}{\mathrm{d}t}=\right)4\mathrm{e}^{2t}$	B1		Both derivatives correct
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\text{candidate's }\frac{\mathrm{d}y}{\mathrm{d}t}}{\text{candidate's }\frac{\mathrm{d}x}{\mathrm{d}t}}$	M1		chain rule used correctly
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{4\mathrm{e}^{2t}}{-16\mathrm{e}^{-2t}}$ $\quad\left(=-\frac{1}{4}\mathrm{e}^{4t}\right)$	A1	3	Simplification not required $4\mathrm{e}^{2t}$ and $-16\mathrm{e}^{-2t}$ must be seen. ISW.
(b) **(i)**	$t=\ln 2$ $\quad$ gradient at $P=-4$	B1ft	1	B0 if ISW result is used.
(ii)	coordinates of *P* $\quad x=-2$	B1		
	$y=12$	B1	2	
(iii)	gradient of normal $=\frac{1}{4}$	B1ft		ft gradient at *P*
	equation of normal $\quad\frac{y-12}{x--2}=\frac{1}{4}$	M1		Set up equation of normal
	at $y=0$ $\quad x=-50$	A1	3	$(-50,0)$ CSO
(c)	$xy+4y-4x=\left(8\mathrm{e}^{-2t}-4\right)\left(2\mathrm{e}^{2t}+4\right)$ $+4\left(2\mathrm{e}^{2t}+4\right)-4\left(8\mathrm{e}^{-2t}-4\right)$	M1		Write $xy+4y-4x$ in terms of *t*.
	$=16+32\mathrm{e}^{-2t}-8\mathrm{e}^{2t}-16$ $+8\mathrm{e}^{2t}+16-32\mathrm{e}^{-2t}+16$	m1		Multiply out and simplify using $\mathrm{e}^{-2t}\mathrm{e}^{2t}=1$ PI
	$(xy+4y-4x)=32$	A1	3	Correct working to $k=32$ $k=32$ NMS; SC1
(c)	**Alternative**			
	$\mathrm{e}^{-2t}=\frac{x+4}{8}$ or $\mathrm{e}^{2t}=\frac{y-4}{2}$	(M1)		Write e^{-2t} in terms of x or e^{2t} in terms of y. Condone sign errors
	$\mathrm{e}^{-2t}\mathrm{e}^{2t}=\left(\frac{x+4}{8}\right)\left(\frac{y-4}{2}\right)$ $=\frac{xy+4y-4x-16}{16}=1$	(m1)		Multiply out and use $\mathrm{e}^{-2t}\mathrm{e}^{2t}=1$
	$xy+4y-4x=32$	(A1)	(3)	All correct with $k=32$
	Other alternatives are possible			
	Total		**12**	

Q	Solution	Marks	Total	Comments
5(a)	$\text{f}\left(-\frac{3}{2}\right)=4\left(-\frac{3}{2}\right)^3-11\left(-\frac{3}{2}\right)-3$	M1		$x=-\frac{3}{2}$ substituted
	$=-4\times\frac{27}{8}+\frac{33}{2}-3=0\Rightarrow$ factor	A1	2	Processing, $=0$ and conclusion
(b)	$2x^2-3x-1$	M1A1	2	M1 for any two of a,b,c correct
(c)(i)	$2\cos 2\theta\sin\theta+9\sin\theta+3$ $=2\left(1-2\sin^2\theta\right)\sin\theta+9\sin\theta+3$	M1		$\cos 2\theta$ expanded ; ACF and substituted
	$=2\sin\theta-4\sin^3\theta+9\sin\theta+3$	m1		All in terms of $\sin\theta$ or x and simplified to a cubic expression.
	$\sin\theta=x\Rightarrow 4x^3-11x-3=0$	A1	3	Reverse signs and express in x correctly **AG**
(c)(ii)	$2x^2-3x-1=0\Rightarrow x=\frac{3\pm\sqrt{17}}{4}$	M1		Use formula correctly to solve $ax^2+bx+c=0$ from part (b)
	$x=\frac{3-\sqrt{17}}{4}$ or $-0.28...$	A1		
	$\theta=196°$ and $344°$	A1		Both required and no others in range; condone greater accuracy Ignore solutions out of range.
	$x=\frac{3+\sqrt{17}}{4}$ no solutions for $\sin\theta$ $x=-\frac{3}{2}$ no solutions for $\sin\theta$	E1	4	Must have three correct roots and reject both other roots from cubic equation.
	Total		**11**	

Q	Solution	Marks	Total	Comments
6(a)	$\lambda=-1$	B1		$\lambda=-1$ seen or implied
	$\lambda=-1$ verified in all three components	B1	2	Shown
(b)	$\pm\begin{bmatrix}-2\\-3\\2\end{bmatrix}$	B1		$\overrightarrow{AB}$ or $\overrightarrow{BA}$ correct
	$\mathbf{r}=\overrightarrow{OA}+\mu\overrightarrow{AB}=\begin{bmatrix}3\\-2\\4\end{bmatrix}+\mu\begin{bmatrix}-2\\-3\\2\end{bmatrix}$	M1		$\mathbf{a}+\mu\mathbf{d}$
		A1ft	3	OE; ft on $\overrightarrow{AB}$ or $\overrightarrow{BA}$
(c)	$\overrightarrow{CD}=\overrightarrow{OD}-\overrightarrow{OC}$ $=\begin{bmatrix}3-2\mu\\-2-3\mu\\4+2\mu\end{bmatrix}-\begin{bmatrix}-4\\5\\-1\end{bmatrix}\quad\left(=\begin{bmatrix}7-2\mu\\-7-3\mu\\5+2\mu\end{bmatrix}\right)$	B1		$\pm\overrightarrow{CD}$ in terms of μ OE
	$\overrightarrow{CD}.\overrightarrow{AB}=0$ or $\overrightarrow{CD}.\overrightarrow{AD}=0$ $=\left(\begin{bmatrix}3-2\mu\\-2-3\mu\\4+2\mu\end{bmatrix}-\begin{bmatrix}-4\\5\\-1\end{bmatrix}\right).\begin{bmatrix}-2\\-3\\2\end{bmatrix}=0$	M1		Candidate's $\overrightarrow{CD}$ sp with candidate's $\overrightarrow{AB}$ or $\overrightarrow{AD}$ $=0$ PI by a solution for μ
	$-14+4\mu+21+9\mu+10+4\mu=0$ $17+17\mu=0$ $\mu=-1$	m1A1		Expand sp to an equation in μ and solve for μ
	D is at $(5,1,2)$	A1	5	Accept as a column vector
(d)	$\overrightarrow{OE}=\overrightarrow{OA}+\overrightarrow{AE}=\overrightarrow{OA}+3\overrightarrow{AD}$	M1		Accept $AE=3AD$
	$\overrightarrow{OE}=\begin{bmatrix}3\\-2\\4\end{bmatrix}+3\begin{bmatrix}2\\3\\-2\end{bmatrix}\qquad E$ is at $(9,7,-2)$	A1		Accept as a column vector
	Or $\overrightarrow{OE}=\overrightarrow{OA}+\overrightarrow{AE}=\overrightarrow{OA}+3\overrightarrow{DA}$	M1		Accept $AE=3DA$
	$\overrightarrow{OE}=\begin{bmatrix}3\\-2\\4\end{bmatrix}+3\begin{bmatrix}-2\\-3\\2\end{bmatrix}\qquad E$ is at $(-3,-11,10)$	A1	4	Accept as a column vector.

Q	Solution	Marks	Total	Comments
6(c)	**Alternative using Pythagoras** $\overrightarrow{CD}=\overrightarrow{OD}-\mu\overrightarrow{OC}$ $=\begin{bmatrix}3-2\mu\\-2-3\mu\\4+2\mu\end{bmatrix}-\begin{bmatrix}-4\\5\\-1\end{bmatrix}\quad\left(=\begin{bmatrix}7-2\mu\\-7-3\mu\\5+2\mu\end{bmatrix}\right)$	(B1)		$\pm\overrightarrow{CD}$ in terms of μ
	$AC^2=AD^2+CD^2$ $\left(7^2+7^2+5^2\right)=\mu^2\left(2^2+3^2+2^2\right)$ $+\left(\left(7-2\mu\right)^2+\left(7+3\mu\right)^2+\left(5+2\mu\right)^2\right)$	(M1)		$\overrightarrow{AC}=\begin{bmatrix}-7\\7\\-5\end{bmatrix}\quad\overrightarrow{AD}=\begin{bmatrix}-2\mu\\-3\mu\\2\mu\end{bmatrix}$ Correct Pythagoras expression in terms of μ;
	$123=17\mu^2+123+34\mu+17\mu^2$ $0=34\mu^2+34\mu$	(m1)		Multiply out and solve to find a value for μ
	$\mu=-1\qquad\left(\mu=0\text{ is point }A\right)$	(A1)		$\mu=-1$
	D is at $\left(5,1,2\right)$	(A1)	(5)	
6(d)	**Alternative** $\lvert\overrightarrow{DE}\rvert=2\lvert\overrightarrow{AD}\rvert\Rightarrow\overrightarrow{OE}=\overrightarrow{OD}+2\overrightarrow{AD}$	(M1)		
	$\overrightarrow{OE}=\begin{bmatrix}5\\1\\2\end{bmatrix}+2\begin{bmatrix}2\\3\\-2\end{bmatrix}\qquad E$ is at $\left(9,7,-2\right)$	(A1)		
	$\lvert DE\rvert=4\lvert DA\rvert\Rightarrow\overrightarrow{OE}=\overrightarrow{OD}+4\overrightarrow{DA}$	(M1)		
	$\overrightarrow{OE}=\begin{bmatrix}5\\1\\2\end{bmatrix}+4\begin{bmatrix}-2\\-3\\2\end{bmatrix}\qquad E$ is at $\left(-3,-11,10\right)$	(A1)	(4)	
	Total		**14**	

Q	Solution	Marks	Total	Comments
7	$\frac{\mathrm{d}h}{\mathrm{d}t}$	B1	1	$\frac{\mathrm{d}h}{\mathrm{d}t}$ seen
	$a=1.3$ or $a=-1.3$	B1	1	
	$k=\frac{\pi}{6}$ or $k=\frac{2\pi}{12}$	B1	1	
	Total		**3**	
8 (a)	$\int t\cos\left(\frac{\pi}{4}t\right)\mathrm{d}t$	M1		Clear attempt to use parts $u=t$ $\frac{\mathrm{d}v}{\mathrm{d}t}=\cos\left(\frac{\pi}{4}t\right)$ $\frac{\mathrm{d}u}{\mathrm{d}t}=1$ $v=k\sin\left(\frac{\pi}{4}t\right)$
	$=t\times\frac{4}{\pi}\sin\left(\frac{\pi}{4}t\right)-\frac{4}{\pi}\int\sin\left(\frac{\pi}{4}t\right)(\mathrm{d}t)$	A1		Must be in terms of π
	$=pt\sin\left(\frac{\pi}{4}t\right)+q\cos\left(\frac{\pi}{4}t\right)$	m1		Correct form, any non-zero values for p, q
	$=t\times\frac{4}{\pi}\sin\left(\frac{\pi}{4}t\right)+\frac{4}{\pi}\times\frac{4}{\pi}\cos\left(\frac{\pi}{4}t\right)$	A1	4	Any correct unsimplified form. Constant not required
(b)	$\int 32x\ \mathrm{d}x=\int t\cos\left(\frac{\pi}{4}t\right)\mathrm{d}t$	B1		Correct separation and notation.
	$16x^2=$	B1		$\frac{x^2}{2}$ if 32 not brought over; allow $32\times\frac{x^2}{2}$
	$t\times\frac{4}{\pi}\sin\left(\frac{\pi}{4}t\right)+\frac{16}{\pi^2}\cos\left(\frac{\pi}{4}t\right)+C$	M1		Equate to result from part (a) with constant and use $(0,4)$ to find a value for the constant
	$C=256-\frac{16}{\pi^2}$	A1		Accept $C=254$ or better $(254.37886...)$
	$t=45$ $16x^2=-40.514...-1.146...+254.378...$ $=212.718...$ $x^2=13.294...$ $x=3.646...=3.65$ m or $(\text{height}=)$ 365 cm	m1A1	6	Substitute $t=45$ into $kx^2=pt\sin\left(\frac{\pi}{4}t\right)+q\cos\left(\frac{\pi}{4}t\right)+C$ $p\neq0$, $q\neq0$ and calculate x. CSO
	Total		**10**	
	TOTAL		**75**	

A-LEVEL
MATHEMATICS
Pure Core 4 – MPC4
Mark scheme

6360
June 2014

Version/Stage: Final V1.0

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Mark	Total	Comment
1 (a)	$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)=t \qquad \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)=-\frac{4}{t^2}$	**B1**		ACF **- Both** correct.
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-\frac{4}{t^2}}{t}$	**M1**		Attempt at their $\frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}$
	At $t=2 \quad \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2}$	**A1**	**3**	CSO
(b)	$t=\frac{4}{y+1}$ **and** $x=\mathrm{f}(y)$	**M1**		Attempt to isolate t **and** attempt to substitute
	$x=\frac{1}{2}\left(\frac{4}{y+1}\right)^2+1$	**A1**	**2**	ACF
	Total		**5**	
	Alternatives			
(b)	$x-1=\frac{t^2}{2} \qquad (y+1)^2=\left(\frac{4}{t}\right)^2$	**M1**		Solve for $\frac{t^2}{2}$ and $\left(\frac{4}{t}\right)^2$ and multiply
	$(x-1)(y+1)^2=8$	**A1**	**2**	ACF
(b)	$t^2=2x-2 \quad \& \quad y=f(x)$	**M1**		Attempt to find t^2 in terms of x **and** attempt to substitute.
	$y=\frac{4}{\pm\sqrt{2x-2}}-1$	**A1**	**2**	or $(y+1)^2=\frac{16}{2x-2}$ ACF

Q	Solution	Mark	Total	Comment
2(a)	$4x^3-2x^2+16x-3=$ $Ax\left(2x^2-x+2\right)+B\left(4x-1\right)$	**M1**		Attempt to multiply by $2x^2-x+2$ or long division with $2x$ seen or substitute two values of x
	$A=2$	**A1**		*A* stated or written in expression
	$B=3$	**A1**	**3**	*B* stated or written in expression
(b)	$\int 2x+\frac{3\left(4x-1\right)}{2x^2-x+2}\,\mathrm{d}x=$			
	x^2+	**B1ft**		ACF **ft** on their *A*
	$3\ln\left(2x^2-x+2\right)\quad\left(+C\right)$	**B1ft**		**ft** on their *B*
	$2=\left(-1\right)^2+3\ln\left(2\left(-1\right)^2-\left(-1\right)+2\right)+C$	**M1**		Substitute $\left(-1,2\right)$ into an expression of form $y=ax^2+b\ln\left(2x^2-x+2\right)+C$ **and** attempt to find the constant
	$y=x^2+3\ln\left(2x^2-x+2\right)+1-3\ln 5$	**A1**	**4**	CAO
	Total		**7**	

(a) If **M1** is not awarded then award **SC1** for **either** $A = 2$ (*or* $2x$) **or** $B = 3$.

NMS *A*= 2 **and** *B* =3 scores **SC3**; as the values of A and B can be found by inspection.

Q	Solution	Mark	Total	Comment
3(a)	$(1-4x)^{\frac{1}{4}}=1+\frac{1}{4}(-4x)+kx^2$	**M1**		k is any non-zero numerical expression
	$=1-x-\frac{3}{2}x^2$	**A1**	**2**	Simplified to this form , but allow -1.5
(b)	$(2+3x)^{-3}=2^{-3}\left(1+\frac{3}{2}x\right)^{-3}$	**B1**		OE e.g. $\frac{1}{8}\left(1+\frac{3}{2}x\right)^{-3}$
	$\left(1+\frac{3}{2}x\right)^{-3}=1-3\times\frac{3}{2}x+\frac{-3\times-4}{2}\left(\frac{3}{2}x\right)^2$	**M1**		Condone missing brackets and one sign error
	$(2+3x)^{-3}=\frac{1}{8}-\frac{9}{16}x+\frac{27}{16}x^2$	**A1**	**3**	or $\frac{1}{8}\left(1-\frac{9}{2}x+\frac{27}{2}x^2\right)$
	Alternative $(2+3x)^{-3}=$ $2^{-3}+(-3)2^{-4}(3x)+\frac{1}{2}(-3)(-4)2^{-5}(3x)^2$	**(M1)**		Condone missing brackets and one sign error.
	$=\frac{1}{8}-\frac{9}{16}x+\frac{27}{16}x^2$	**(A2)**	**(3)**	**A1 not** available
(c)	$\left(1-x-\frac{3}{2}x^2\right)\left(\frac{1}{8}-\frac{9}{16}x+\frac{27}{16}x^2\right)$	**M1**		Product of their expansions
	$=\frac{1}{8}-\frac{11}{16}x+\frac{33}{16}x^2$	**A1**	**2**	
	Total		**7**	

Q	Solution	Mark	Total	Comment
4 (a)	$A = 5000$	**B1**	**1**	
(b)(i)	$25000 = 5000p^{10} \Rightarrow p^{10} = 5$	**B1**	**1**	First equation seen and correct. **AG**
(ii)	$\ln p^t = t\ln p$	**B1**		PI
	$\ln\left(\frac{75000}{A}\right) = \ln p^t$	**M1**		**Correctly** taking logs of both sides. OE eg $\ln 75000 = \ln A + \ln p^t$
	$t = \frac{10\ln 15}{\ln 5}$ or $t = 16.8....$	**A1**		OE e.g. $t = \frac{\ln 15}{\ln 1.175}$ or 16.79… $t = \frac{\ln 15}{\ln 5^{\frac{1}{10}}}$ etc.
	2018	**B1**	**4**	
(c)(i)	$5000p^{T-10} = 2500q^T$	**B1**		Correct opening expression
	$\ln 2 + (T-10)\ln p = T\ln q$	**M1**		Use laws of logs correctly to obtain a linear equation in *T*. Powers must involve *T* and *T*±10.
	$T = \frac{10\ln p - \ln 2}{\ln p - \ln q}$	**m1**		Make T the subject of their expression correctly.
	$p^{10} = 5 \Rightarrow 10\ln p = \ln 5 \Rightarrow T = \frac{\ln\left(\frac{5}{2}\right)}{\ln\left(\frac{p}{q}\right)}$	**A1**	**4**	$p^{10} = 5 \Rightarrow 10\ln p = \ln 5$ used to get **AG**
(ii)	2023	**B1**	**1**	
	Total		**11**	

Q	Solution	Mark	Total	Comment
5 (a)(i)	$R=5$ $\tan\alpha=\frac{4}{3}$ $\alpha=53.1°$	**B1** **M1** **A1**	**3**	$R\sin\alpha=4$ or $R\cos\alpha=3$ using their R $\sin\alpha=4$ $\cos\alpha=3$ is **M0** 53.1° **only**
(ii)	$5\sin\left(2\theta+53.1\right)°=5$ $\left[\left(2\theta+53.1\right)°=90°\quad\text{and}\quad 450°\right]$ $\theta=18.4°$ $\theta=198.4°$	**M1** **A1** **A1ft**	**3**	Candidate's R and α but must use 2θ - PI. Accept $\theta=18.5°$ $180°+'their'\,18.4°$
(b)(i)	$\frac{2\tan\theta}{1-\tan^2\theta}\times\tan\theta=2$ $2\tan^2\theta=2\left(1-\tan^2\theta\right)$ $4\tan^2\theta=2$ $2\tan^2\theta=1$	**M1** **A1**	**2**	**Use** of correct form of $\tan 2\theta$ Correct derivation of **AG**.
(ii)	$\theta=35.3°$ $\theta=144.7°$	**B1** **B1**	**2**	
(c)(i)	$8\times\frac{1}{8}-4\times\frac{1}{2}+1=0\Rightarrow 2x-1$ is a factor	**B1**	**1**	Accept $1-2+1=0$ but need the conclusion
(ii)	$4\left(2\cos^2\theta-1\right)\cos\theta+1=8x^3-4x+1$	**B1**	**1**	$\cos 2\theta=2\cos^2\theta-1$ used correctly in deriving **AG**
(iii)	$8x^3-4x+1=\left(2x-1\right)\left(4x^2+2x-1\right)$ $x=\frac{-2\pm\sqrt{20}}{8}$ or $\frac{-2\pm2\sqrt{5}}{8}$ $\left(\cos72°>0\right)\Rightarrow\cos72°=\frac{\sqrt{5}-1}{4}$	**B1** **M1** **A1**	**3**	Award for quadratic factor Correct solution of their quadratic – ACF. **CSO**
	Total		**15**	

(a)(ii) Either $\theta=18.4°$ or $\theta=198.4°$ earns **A1** and any extras in the interval together with the two correct values earns **A1 A0ft**
Award **SC1** for both answers to greater degree of accuracy 18.43494 … and 198.43494561…

(b)(ii) Either $\theta=35.3°$ or $\theta=144.7°$ earns **B**1 and any extras in the interval together with the two correct values earns **B**1 **B0**
Award **SC1** for both answers to greater degree of accuracy 35.26413… and 144.735561…

Q	Solution	Mark	Total	Comment
6(a)	$\left(\overrightarrow{OP}\right)=\begin{bmatrix}5\\-8\\2\end{bmatrix}$ $\quad\left(\overrightarrow{OQ}\right)=\begin{bmatrix}11\\-14\\8\end{bmatrix}$	**B1**		PI by correct $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ below
	$\left(\overrightarrow{PQ}\right)=\begin{bmatrix}11\\-14\\8\end{bmatrix}-\begin{bmatrix}5\\-8\\2\end{bmatrix}$ or $\begin{bmatrix}6\\-6\\6\end{bmatrix}$	**M1**		$\overrightarrow{PQ}=\pm$ their $\left(\overrightarrow{OQ}-\overrightarrow{OP}\right)$
	$\overrightarrow{PQ}=6\begin{bmatrix}1\\-1\\1\end{bmatrix}$	**A1**	**3**	or $\begin{bmatrix}6\\-6\\6\end{bmatrix}$ stated to be parallel to $\begin{bmatrix}1\\-1\\1\end{bmatrix}$
(b)(i)	$\lambda=1$ **or** $\mu=-2$	**B1**		
	$b=-5+3$ or $b=-8+6,\ \left(\text{their } \lambda \text{ or } \mu\right)$ **or** $c=3+1$ or $c=6-2,\ \left(\text{their } \lambda \text{ or } \mu\right)$	**M1**		Attempt to find the value of *b* **or** *c*
	$b=-2$ **and** $c=4$	**A1**	**3**	$b=-2$ **shown** and $c=4$
(ii)	$\overrightarrow{RS}=\begin{bmatrix}5+2t\\-8-3t\\2+t\end{bmatrix}-\begin{bmatrix}3\\-2\\4\end{bmatrix}$	**M1**		**Clear** attempt to find $\pm\overrightarrow{RS}$
	$2+2t+6+3t-2+t=0$	**m1**		$\overrightarrow{RS}\bullet\begin{bmatrix}1\\-1\\1\end{bmatrix}=0$ or $\overrightarrow{RS}\bullet\begin{bmatrix}6\\-6\\6\end{bmatrix}=0$ $=0$ PI; correct direction vector
	$t=-1$	**A1**		
	S is at $(3,-5,1)$	**A1**	**4**	Accept as a column vector.
	Total		**10**	

Q	Solution	Mark	Total	Comment
7(a)(i)	$-2\sin 2y\dfrac{\mathrm{d}y}{\mathrm{d}x}$	**B1**		
	$+3y\,\mathrm{e}^{3x}+\mathrm{e}^{3x}\dfrac{\mathrm{d}y}{\mathrm{d}x}$	**M1**		$py\,\mathrm{e}^{3x}+q\mathrm{e}^{3x}\dfrac{\mathrm{d}y}{\mathrm{d}x}$
		A1		Product rule correct
	$=0$	**B1**		PI
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}(\mathrm{e}^{3x}-2\sin 2y)+3y\mathrm{e}^{3x}=0$	**m1**		Attempt to factorise.
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{-3y\mathrm{e}^{3x}}{\mathrm{e}^{3x}-2\sin 2y}$	**A1**	**6**	OE
(ii)	At *A* $\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\pi$	**B1**	**1**	Must have scored **all** 6 marks in (a)(i)
(b)	$\left(y-\dfrac{\pi}{4}\right)=\dfrac{1}{\pi}(x-\ln 2)$	**M1**		Finding the equation of **normal** with gradient $\frac{-1}{\text{their}(a)(\text{ii})}$.
	At *B* $y=\dfrac{\pi}{4}-\dfrac{\ln 2}{\pi}$	**A1**	**2**	
	Total		**9**	
	Alternative using $y=mx+c$			
(b)	$\dfrac{\pi}{4}=\dfrac{1}{\pi}\ln 2+c$ $\left(y=\dfrac{1}{\pi}x+c\right)$	**M1**		Use $y=mx+c$ and find c using their gradient.
	At *B* $y=\dfrac{\pi}{4}-\dfrac{\ln 2}{\pi}$	**A1**	**2**	Must see $y=\dfrac{\pi}{4}-\dfrac{\ln 2}{\pi}$ or a statement that c is the required y-coordinate

Q	**Solution**	**Mark**	**Total**	**Comment**
8 (a)	$16x = A(1+x)^2 + B(1-3x)(1+x) + C(1-3x)$	**B1**		OE
	$x = -1 \qquad -16 = 4C$ $x = \frac{1}{3} \qquad \frac{16}{3} = A\left(\frac{4}{3}\right)^2$	**M1**		Use $x = \frac{1}{3}$ or $x = -1$ to find a value for *A* or *C*.
	$A = 3 \quad B = 1 \quad C = -4$	**A1**		Any two correct
		A1	**4**	All three correct
(b)	$\int \frac{1}{e^{2y}}\,dy = \int \frac{16x}{(1-3x)(1+x)^2}\,dx$	**B1**		
	or $\int \frac{dy}{e^{2y}} = \int \frac{3}{1-3x} + \frac{1}{1+x} - \frac{4}{(1+x)^2}\;dx$			or correct **ft** separation on non-zero *A B C*
	$\frac{-e^{-2y}}{2}$	**B1**		**OE**
	$= -\ln(1-3x)$	**B1ft**		OE ft on $\frac{A}{-3}\ln(1-3x)$
	$+\ln(1+x)$	**B1ft**		OE ft on $B\ln(1+x)$
	$+\frac{4}{1+x}$	**B1ft**		OE ft on $\frac{C}{-1}(1+x)^{-1}$
	$-\frac{1}{2} = (-\ln 1 + \ln 1) + 4 + \text{constant}$	**M1**		Use $(0,0)$ **and** attempt to find a value for the constant.
	$-\frac{1}{2}e^{-2y} = -\ln(1-3x) + \ln(1+x) + \frac{4}{1+x} - \frac{9}{2}$	**A1**	**7**	ACF
	Total		**11**	
	TOTAL		**75**	

(b) For **M1** candidates must have a term of the form $ke^{\pm 2y}$ on one side and at least one ln term on the other, substitute $(0,0)$ **and** find a value for the constant.

A-LEVEL

Mathematics

Pure Core 4 – MPC4
Mark scheme

6360
June 2015

Version 1.1: Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

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Q1	Solution	Mark	Total	Comment
(a)	$19x-2=A(1+6x)+B(5-x)$	**M1**		Correct equation and attempt to find a value for *A* or *B*.
	$A=3$	**A1**		
	$B=-1$	**A1**	**3**	**NMS** or cover up rule; *A* or *B* correct **SC2** *A* **and** *B* correct **SC3**.
(b)	$\int \frac{3}{5-x}-\frac{1}{1+6x}\ \mathrm{d}x$			
	$=p\ln(5-x)+q\ln(1+6x)$	**M1**		Condone missing brackets **OE** Either term in a correct form
	$=-3\ln(5-x)$	**A1ft**		ft on their *A*
	$-\frac{1}{6}\ln(1+6x)$	**A1ft**		ft on their *B*
	$\int_0^4 =\left[-3\ln1-\frac{1}{6}\ln25\right]-\left[-3\ln5-\frac{1}{6}\ln1\right]$	**m1**		Substitute limits correctly in their integral; F(4) – F(0)
	$=-\frac{1}{6}\ln25+3\ln5$	**A1**		ACF. $\ln1=0$ PI
	$=\frac{8}{3}\ln5$	**A1**	**6**	CSO Condone equivalent fractions or recurring decimal
	Total		**9**	

Q2	Solution	Mark	Total	Comment
(a)	$R=\sqrt{29}$	**B1**		Allow 5.4 or better
	$\sqrt{29}\cos\alpha=2,\sqrt{29}\sin\alpha=5$ or $\tan\alpha=\frac{5}{2}$	**M1**		Their $\sqrt{29}$ Note $\cos\alpha=2$ or $\sin\alpha=5$ is M0
	$\alpha=1.19$	**A1**	**3**	Must be exactly this
(b)(i)	$R\cos(x+\alpha)=R$ or $\cos(x+\alpha)=1$ or $x+\alpha=2\pi$ or $x+\alpha=0$ or $x=-\alpha$	**M1**		Candidate's R and α
	$(x=)$ 5.09	**A1**	**2**	Must be exactly this
(ii)	$\cos(x+\alpha)=-\frac{1}{R}$	**M1**		Candidate's R and α; PI
	$(x+\alpha=)$ 1.75757... and 4.52560...	**A1**		Rounded or truncated to at least 2 dp; Ignore 'extra' solutions
	$x=0.567$ and $x=3.34$	**A1**	**3**	Condone $x=0.568$; $x=3.34$ must be correct NMS is 0/3 **A0** if extra values in interval $0<x<2\pi$
	Total		**8**	

Q3	Solution	Mark	Total	Comment
(a)	$\text{f}\left(-\frac{1}{2}\right)=-1-3+1+d=-2$ $d=1$	**M1** **A1**	2	Attempt to evaluate $\text{f}\left(-\frac{1}{2}\right)$ **and** equated to -2 NMS is 0/2
(b)(i)	$(2x+1)$ is a factor $\text{g}(x)=(2x+1)\left(4x^2+bx+3\right)$ $\text{g}(x)=(2x+1)\left(4x^2-8x+3\right)$ $\text{g}(x)=(2x+1)(2x-1)(2x-3)$	**B1** **M1** **A1**	3	**OE** $\left(x+\frac{1}{2}\right)$ Attempt to find quadratic factor or a second linear factor using Factor Theorem **OE** if $\left(x+\frac{1}{2}\right)$ is used **OE**; must be a product **NMS** : **SC3** if product is correct **SC1** if one or two factors are correct
(ii)	$\frac{4x^2-1}{\text{g}(x)}=\frac{1}{2x-3}$ $\frac{\text{d}}{\text{d}x}\left(\frac{1}{2x-3}\right)=\frac{k}{(2x-3)^2}$ $=-\frac{2}{(2x-3)^2}$ (Derivative is) negative, or <0 hence decreasing	**B1** **M1** **A1** **E1**	4	Attempt to differentiate simplified h Correct derivative Explanation and conclusion required Derivative must be correct
	Total		**9**	
(b)(ii)	**Special case** $\text{h}(x)=\frac{1}{2x-3}$ $2x-3$ is an increasing function, so $\frac{1}{2x-3}$ is a decreasing function	**B1** **E1**	2	Award only if $\text{h}(x)=\frac{1}{2x-3}$ is correct

Q4	Solution	Mark	Total	Comment
(a)	$1+\frac{1}{5}\times 5x+kx^2$	M1		k any non-zero numerical expression
	$1+x-2x^2$	A1	2	Simplified to this
(b) (i)	$(8+3x)^{-\frac{2}{3}}=8^{-\frac{2}{3}}(1+\frac{3}{8}x)^{-\frac{2}{3}}$	B1		ACF for $8^{-\frac{2}{3}}=\frac{1}{4}$
	$\left(1+\frac{3}{8}x\right)^{-\frac{2}{3}}$ $=1+\left(-\frac{2}{3}\right)\left(\frac{3}{8}x\right)+\frac{1}{2}\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)\left(\frac{3}{8}x\right)^2$	M1		Expand correctly using their $\frac{3}{8}x$ Condone poor use of or missing brackets
	$\frac{1}{4}-\frac{1}{16}x+\frac{5}{256}x^2$	A1	3	Accept $=\frac{1}{4}\left(1-\frac{1}{4}x+\frac{5}{64}x^2\right)$
(ii)	$x=\frac{1}{3}$	M1		$x=\frac{1}{3}$ **used** in their expansion from (b)(i)
	0.2313 (4dp)	A1	2	Note **3** in **4th** decimal place
	Total		7	

Q5	Solution	Mark	Total	Comment
(a)	$\left(\frac{\mathrm{d}x}{\mathrm{d}t}=\right)-2\sin 2t \qquad \left(\frac{\mathrm{d}y}{\mathrm{d}t}=\right)\cos t$	**B1**		Both correct
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{\cos t}{-2\sin 2t}$	**M1**		Correct use of chain rule with their derivatives of form $a\sin 2t,\ b\cos t$
	At $t=\frac{\pi}{6}$ gradient $m_{\mathrm{T}}=-\frac{1}{2}$	**A1**	**3**	
(b)	Gradient of normal $m_{\mathrm{N}}=2$	**B1ft**		ft gradient of tangent; $m_{\mathrm{N}}=\frac{-1}{m_{\mathrm{T}}}$
	$\left(y-\cos\left(\frac{2\pi}{6}\right)\right)=m_{\mathrm{N}}\left(x-\sin\left(\frac{\pi}{6}\right)\right)$	**M1**		For m_{N}, allow their m_{T} with a change of sign or the reciprocal at $\left(\sin\frac{\pi}{6},\cos\frac{2\pi}{6}\right)$ or $\left(\frac{1}{2},\frac{1}{2}\right)$
	$y=2x-\frac{1}{2}$	**A1**	**3**	Must be in this $y=mx+c$ form
	Alternative for M1 $\sin\left(\frac{\pi}{6}\right)=2\cos\left(\frac{2\pi}{6}\right)+c$			Use $y=mx+c$ to find c with their gradient m_{N} at $\left(\sin\frac{\pi}{6},\cos\frac{2\pi}{6}\right)$ or $\left(\frac{1}{2},\frac{1}{2}\right)$
(c)	$\cos 2q=1-2\sin^2 q$	**B1**		Seen or used in this form
	$\sin q=2\left(1-2\sin^2 q\right)-\frac{1}{2}$	**M1**		Use parametric equations and candidate's $\cos 2q$ in the form $\pm 1+k\sin^2 q$
	$8\sin^2 q+2\sin q-3=0$ **OE**	**A1**		Collect like terms; must be a quadratic equation
	$\left(\sin q=\frac{1}{2}\right) \qquad \sin q=-\frac{3}{4}$	**A1**		Must come from a correct quadratic equation with the previous 3 marks awarded
	$(x=)\ -\frac{1}{8}$	**A1**	**5**	Previous 4 marks must have been awarded
	Total		**11**	

Mark scheme Alternative

Q5	Solution	Mark	Total	Comment
(a)	$x=1-2y^2 \quad 1=-4y\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}=-4y$	**B1**		Find a correct Cartesian equation and differentiate implicitly correctly
	$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{4\sin\frac{\pi}{6}}$	**M1**		Use $y=\sin\frac{\pi}{6}$ or $y=\frac{1}{2}$ in their $\frac{\mathrm{d}y}{\mathrm{d}x}$; PI
	At $t=\frac{\pi}{6}$ gradient $m_{\mathrm{T}}=-\frac{1}{2}$	**A1**	3	CSO
(b)	Gradient of normal $=2$	**B1ft**		ft gradient of tangent, $m_{\mathrm{N}}=\frac{-1}{m_{\mathrm{T}}}$
	$\left(y-\cos\left(\frac{2\pi}{6}\right)\right)=m_{\mathrm{N}}\left(x-\sin\left(\frac{\pi}{6}\right)\right)$	**M1**		For m_{N}, allow their m_{T} with a change of sign or the reciprocal at $\left(\sin\frac{\pi}{6},\cos\frac{2\pi}{6}\right)$ or $\left(\frac{1}{2},\frac{1}{2}\right)$
	$y=2x-\frac{1}{2}$	**A1**	**3**	CSO
	Alternative for M1 $\sin\left(\frac{\pi}{6}\right)=2\cos\left(\frac{2\pi}{6}\right)+c$			Use $y=mx+c$ to find c with candidate's gradient m_{N} at $\left(\sin\frac{\pi}{6},\cos\frac{2\pi}{6}\right)$ or $\left(\frac{1}{2},\frac{1}{2}\right)$
(c)	$x=1-2y^2$	**B1**		PI by $x=1-2\left(2x-\frac{1}{2}\right)^2$
	$1-2y^2=\frac{y+\frac{1}{2}}{2}$	**M1**		Use their Cartesian equation and normal to eliminate x
	$4y^2+y-\frac{3}{2}=0\Rightarrow$			Collect like terms; must be a quadratic equation
	$8\sin^2 q+2\sin q-3=0$	**A1**		
	$\left(\sin q=\frac{1}{2}\right) \qquad \sin q=-\frac{3}{4}$	**A1**		Must come from a correct quadratic equation with the previous 3 marks awarded
	$(x=)\ -\frac{1}{8}$	**A1**	**5**	Previous 4 marks must have been awarded
	Total		**11**	

Q6	Solution	Mark	Total	Comment
(a)	$\left(\overrightarrow{AB}=\right)\begin{bmatrix}2\\-4\\-6\end{bmatrix}$ $\overrightarrow{AB}\bullet\begin{bmatrix}3\\1\\-2\end{bmatrix}=(2\times3)+(-4\times1)+(6\times-2)$ $\sqrt{56}\sqrt{14}\cos BAC=14$ angle $BAC=60°$	**B1** **M1** **m1** **A1**	**4**	Or $\left(\overrightarrow{BA}=\right)\begin{bmatrix}-2\\4\\6\end{bmatrix}$ Correctly ft on "their" $\overrightarrow{AB}$ Correct use of formula with consistent vectors; ACF or $\pi/3$; **NMS** $60°$ scores **0/4**
(b)	$\left(\overrightarrow{BC}=\right)\begin{bmatrix}3\\2\\10\end{bmatrix}+\lambda\begin{bmatrix}3\\1\\-2\end{bmatrix}-\begin{bmatrix}5\\-2\\4\end{bmatrix}$ $\overrightarrow{AB}\bullet\overrightarrow{BC}=$ $2(3\lambda-2)-4(\lambda+4)-6(-2\lambda+6)=0$ $14\lambda-56=0 \Rightarrow \lambda=4$ C is at $(15,6,2)$	**B1** **M1** **A1** **A1**	**4**	$\pm\overrightarrow{BC}$ **ACF** Correct scalar product with their $\overrightarrow{AB}$, their $\overrightarrow{BC}$, equate to 0 and solve for λ Accept as a column vector **NMS** (15,6,2) scores **0/4**
(c)	E_1 is at $(11,0,0)$ $\overrightarrow{OD}=\overrightarrow{OC}+\overrightarrow{AB}=\begin{bmatrix}15\\6\\2\end{bmatrix}+\begin{bmatrix}2\\-4\\-6\end{bmatrix}=\begin{bmatrix}17\\2\\-4\end{bmatrix}$ $\overrightarrow{OE}_2=\begin{bmatrix}17\\2\\-4\end{bmatrix}+\frac{1}{2}\times4\begin{bmatrix}3\\1\\-2\end{bmatrix}$ E_2 is at $(23,4,-8)$	**B1** **B1** **M1** **A1**	**4**	Accept as a column vector Correct vector expression with their λ and their $\overrightarrow{OD}$ Accept as a column vector
	Total		**12**	
(b)	**Alternative by Pythagoras**			
	$\left(\overrightarrow{BC}=\right)\begin{bmatrix}3\\2\\10\end{bmatrix}+\lambda\begin{bmatrix}3\\1\\-2\end{bmatrix}-\begin{bmatrix}5\\-2\\4\end{bmatrix}$ $(3\lambda)^2+(\lambda)^2+(-2\lambda)^2$ $=56+(-2+3\lambda)^2+(4+\lambda)^2+(6-2\lambda)^2$ $112-28\lambda=0 \quad \lambda=4$ C is at $(15,6,2)$	**B1** **M1** **A1** **A1**	**4**	$\pm\overrightarrow{BC}$ **ACF** $AC^2=AB^2+BC^2$ Correct Pythagoras expression, attempt to expand and solve for λ Accept as a column vector

(b)	**Alternative by** $\cos 60 = \frac{1}{2}$			
	$\frac{1}{2} = \frac{\lvert\overrightarrow{AB}\rvert}{\lvert\overrightarrow{AC}\rvert} = \frac{\sqrt{56}}{\sqrt{(3\lambda)^2+(\lambda)^2+(-2\lambda)^2}}$	**B1**		
	$\frac{1}{4} = \frac{56}{14\lambda^2}$	**M1**		Square and simplify
	$\lambda^2 = 16 \Rightarrow \lambda = 4 \quad (\text{or } \lambda = -4)$	**A1**		
	C is at $(15,6,2)$	**A1**	**4**	Accept as a column vector

(c)	**Alternatives**			
Alt (i)				
	$\overrightarrow{OE_1} = \overrightarrow{OB} + \tfrac{1}{2}\overrightarrow{AC} = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 12 \\ 4 \\ -8 \end{bmatrix}$			
	E_1 is at $(11,0,0)$	**B1**		
	$\overrightarrow{OE_2} = \overrightarrow{OB} + 3\overrightarrow{BE_1} = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix} + 3\begin{bmatrix} 6 \\ 2 \\ -4 \end{bmatrix}$	**M1** **B1`**		Correct vector expression with their $\overrightarrow{BE_1}$ All correct
	E_2 is at $(23,4,-8)$	**A1**	**4**	
Alt (ii)				
	$\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{AC} = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix} + \begin{bmatrix} 12 \\ 4 \\ -8 \end{bmatrix}$			
	D is at $(17,2,-4)$	**B1**		
	$\overrightarrow{OE_2} = \overrightarrow{OD} + \frac{1}{2}\overrightarrow{AC} = \begin{bmatrix} 17 \\ 2 \\ -4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 12 \\ 4 \\ -8 \end{bmatrix}$	**M1**		Correct vector expression with their $\overrightarrow{OD}$ and their $\overrightarrow{AC}$
	E_2 is at $(23,4,-8)$	**A1**		
	$\overrightarrow{OE_1} = \overrightarrow{OB} + \tfrac{1}{2}\overrightarrow{AC} = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 12 \\ 4 \\ -8 \end{bmatrix}$			
	E_1 is at $(11,0,0)$	**B1**	**4**	

Q7	Solution	Mark	Total	Comment
(a)	$k=\left(\frac{1}{2}\right)^3+2\mathrm{e}^{-3\ln 2}\times\frac{1}{2}-\ln 2$ $=\frac{1}{8}+\frac{1}{8}-\ln 2=\frac{1}{4}-\ln 2$	B1	1	Clear use of $\left(\frac{1}{2}\right)^3=\frac{1}{8}$ and $\mathrm{e}^{-3\ln 2}=\frac{1}{8}$ Accept $\frac{2}{8}-\ln 2$
(b)	$3y^2\frac{\mathrm{d}y}{\mathrm{d}x}$	B1		
	$pye^{-3x}+qe^{-3x}\frac{\mathrm{d}y}{\mathrm{d}x}$	M1		
	$-6ye^{-3x}+2e^{-3x}\frac{\mathrm{d}y}{\mathrm{d}x}$	A1		
	$-1=0$	B1		Both required -1 and no other terms
	$\frac{3}{4}\frac{\mathrm{d}y}{\mathrm{d}x}-6\times\frac{1}{8}\times\frac{1}{2}+2\times\frac{1}{8}\frac{\mathrm{d}y}{\mathrm{d}x}-1\quad(=0)$	m1		Substitute $x=\ln 2$ or $\mathrm{e}^{-3x}=\frac{1}{8}$ and $y=\frac{1}{2}$ into their expression
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{11}{8}$ or 1.375	A1	6	
	Total		7	

Q8	Solution	Mark	Total	Comment
(a)(i)	$\int \frac{1}{\sqrt{4+5x}}\,\mathrm{d}x = \int \frac{1}{5(1+t)^2}\,\mathrm{d}t$	**B1**		Correct separation and notation seen on a single line somewhere in their solution
	$a(4+5x)^{\frac{1}{2}}$ or $b(1+t)^{-1}$	**M1**		OE $a\sqrt{4+5x}$ or $b\left(\frac{1}{1+t}\right)$
	$\frac{2}{5}(4+5x)^{\frac{1}{2}}$	**A1**		OE $\frac{2}{5}\sqrt{4+5x}$
	$-\frac{1}{5}(1+t)^{-1} \quad (+C)$	**A1**		OE $-\frac{1}{5(1+t)}$
	$x=0$, $t=0 \quad \Rightarrow \quad C=1$	**m1**		Use $(0,0)$ to find a constant
	$\frac{2}{5}(4+5x)^{\frac{1}{2}} = 1-\frac{1}{5}(1+t)^{-1}$	**A1**		**OE**
	$x=\frac{5}{4}\left(1-\frac{(1+t)^{-1}}{5}\right)^2-\frac{4}{5}$	**A1**	**7**	ACF eg $x=\frac{1}{20}\left(\frac{4+5t}{1+t}\right)^2-\frac{4}{5}$
(b)(i)	$\frac{\mathrm{d}r}{\mathrm{d}t}$	**B1**		Seen; allow *R* for *r*
	$\frac{1}{r^2}$	**M1**		$\frac{1}{r^2}$ seen **;** allow *R* for *r*
	$\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{k}{r^2}$	**A1**	**3**	Any constant k including $\frac{c}{\pi}$ but **not** including variable *t* Must use *R* or *r* consistently
(ii)	$\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)=4.5=\frac{k}{1^2}$ or $4.5=\frac{c}{\pi\times 1^2}$	**M1**		**Use** $\frac{\mathrm{d}r}{\mathrm{d}t}=4.5$ with $r=1$ to find a value for the constant
	$0.5=\frac{4.5}{r^2}\Rightarrow r=3$ (metres)	**A1**	**2**	
	Total		**12**	