OCR
Oxford Cambridge and RSA

Tuesday 16 June 2015 – Afternoon

A2 GCE MATHEMATICS

4724/01 Core Mathematics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4724/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 **(i)** Express $\dfrac{2}{3-x}+\dfrac{3}{1+x}$ as a single fraction in its simplest form. **[2]**

(ii) Hence express $\left(\dfrac{2}{3-x}+\dfrac{3}{1+x}\right)\times\dfrac{x^2+8x-33}{121-x^2}$ as a single fraction in its lowest terms. **[3]**

2 A triangle has vertices at A (1, 1, 3), B (5, 9, −5) and C (6, 5, −4). P is the point on AB such that $AP:PB = 3:1$.

(i) Show that $\overrightarrow{CP}$ is perpendicular to $\overrightarrow{AB}$. **[4]**

(ii) Find the area of the triangle ABC. **[2]**

3 The equation of a curve is $y = e^{2x}\cos x$. Find $\dfrac{dy}{dx}$ and hence find the coordinates of any stationary points for which $-\pi \leqslant x \leqslant \pi$. Give your answers correct to 3 significant figures. **[6]**

4 **(i)** Find the first three terms in the binomial expansion of $(8-9x)^{\frac{2}{3}}$ in ascending powers of x. **[4]**

(ii) State the set of values of x for which this expansion is valid. **[1]**

5 By first using the substitution $t=\sqrt{x+1}$, find $\int e^{2\sqrt{x+1}}\,dx$. **[6]**

6 **(i)** Use the quotient rule to show that the derivative of $\dfrac{\cos x}{\sin x}$ is $\dfrac{-1}{\sin^2 x}$. **[2]**

(ii) Show that $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi}\frac{\sqrt{1+\cos 2x}}{\sin x\sin 2x}\,dx = \frac{1}{2}\left(\sqrt{6}-\sqrt{2}\right)$. **[6]**

7 A curve has equation $(x+y)^2 = xy^2$. Find the gradient of the curve at the point where $x = 1$. **[7]**

8 In the year 2000 the population density, P, of a village was 100 people per km^2, and was increasing at the rate of 1 person per km^2 per year. The rate of increase of the population density is thought to be inversely proportional to the size of the population density. The time in years after the year 2000 is denoted by t.

(i) Write down a differential equation to model this situation, and solve it to express P in terms of t. **[6]**

(ii) In 2008 the population density of the village was 108 people per km^2 and in 2013 it was 128 people per km^2. Determine how well the model fits these figures. **[2]**

9 Two lines have equations

$$\mathbf{r} = 3\mathbf{i} + 5\mathbf{j} - \mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + \mathbf{k}) \text{ and } \mathbf{r} = 4\mathbf{i} + 10\mathbf{j} + 19\mathbf{k} + \mu(\mathbf{i} - \mathbf{j} + \alpha\mathbf{k}),$$

where α is a constant.

Find the value of α in each of the following cases.

(i) The lines intersect at the point (7, 7, 1). **[3]**

(ii) The angle between their directions is 60°. **[4]**

10 **(i)** Express $\frac{x+8}{x(x+2)}$ in partial fractions. **[3]**

(ii) By first using division, express $\frac{7x^2+16x+16}{x(x+2)}$ in the form $P+\frac{Q}{x}+\frac{R}{x+2}$. **[3]**

A curve has parametric equations $x=\frac{2t}{1-t}$, $y=3t+\frac{4}{t}$.

(iii) Show that the cartesian equation of the curve is $y=\frac{7x^2+16x+16}{x(x+2)}$. **[4]**

(iv) Find the area of the region bounded by the curve, the x-axis and the lines $x=1$ and $x=2$. Give your answer in the form $L + M\ln 2 + N\ln 3$. **[4]**

END OF QUESTION PAPER