

Cambridge  
International  
AS & A Level

**Cambridge International Examinations**  
Cambridge International Advanced Subsidiary and Advanced Level

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**MATHEMATICS**

**9709/13**

Paper 1

**May/June 2016**

MARK SCHEME

Maximum Mark: 75

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**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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### Mark Scheme Notes

Marks are of the following three types:

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep\*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol  $\nabla$  implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.  
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking  $g$  equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

### **Penalties**

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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1	$5C2 \left(\frac{1}{x}\right)^3 (3x^2)^2$ $10(\times 1) \times 3^2$ $90(x)$	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p>[3]</p>	<p>Can be seen in expansion</p> <p>Identified as leading to answer</p>
2	$(\pi) \int (x^3 + 1) dx$ $(\pi) \left[ \frac{x^4}{4} + x \right]$ $6\pi \text{ or } 18.8$	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>DM1A1</b></p> <p>[4]</p>	<p>Attempt to resolve <math>y^2</math> and attempt to integrate</p> <p>Applying limits 0 and 2. (Limits reversed: Allow M mark and allow A mark if final answer is <math>6\pi</math>)</p>
3 (i)	$6 + k = 2 \rightarrow k = -4$	<p><b>B1</b></p> <p>[1]</p>	
3 (ii)	$(y) = \frac{6x^3}{3} - \frac{4}{-2}x^{-2} (+c)$ $9 = 2 + 2 + c \quad c \text{ must be present}$ $(y) = 2x^3 + 2x^{-2} + 5$	<p><b>B1B1</b> ✓</p> <p><b>M1</b></p> <p><b>A1</b></p> <p>[4]</p>	<p>fit on <i>their</i> <math>k</math>. Accept <math>+\frac{k}{-2}x^{-2}</math></p> <p>Sub (1,9) with numerical <math>k</math>. Dep on attempt <math>\int</math></p> <p>Equation needs to be seen</p> <p>Sub (2, 3) <math>\rightarrow c = -13\frac{1}{2}</math> scores M1A0</p>
4	$r = \frac{3+2d}{3} \text{ or } \frac{3+12d}{3+2d} \text{ or } r^2 = \frac{3+12d}{3}$ $(3+2d)^2 = 3(3+12d) \text{ oe}$ <p>OR</p> $\text{sub } 2d = 3r - 3$ $(4)d(d-6) = 0$ <p>OR</p> $3r^2 = 18r - 15 \rightarrow (r-1)(r-5)$ $d = 6$ $r = 5$	<p><b>B1</b></p> <p><b>M1</b></p> <p><b>DM1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p>[5]</p>	<p>1 correct equation in <math>r</math> and <math>d</math> only is sufficient</p> <p>Eliminate <math>r</math> or <math>d</math> using valid method</p> <p>Attempt to simplify and solve quadratic</p> <p>Ignore <math>d = 0</math> or <math>r = 1</math></p> <p>Do not allow <math>-5</math> or <math>\pm 5</math></p>

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<p>5</p>	$\frac{dy}{dx} = [8] + [-2] [(2x-1)^{-2}]$ $= 0 \rightarrow 4(2x-1)^2 = 1 \text{ oe eg } 16x^2 - 16x + 3 = 0$ $x = \frac{1}{4} \text{ and } \frac{3}{4}$ $\frac{d^2y}{dx^2} = 8(2x-1)^{-3}$ <p>When <math>x = \frac{1}{4}</math>, <math>\frac{d^2y}{dx^2} (= -64)</math> and/or <math>&lt; 0</math> MAX</p> <p>When <math>x = \frac{3}{4}</math>, <math>\frac{d^2y}{dx^2} (= 64)</math> and/or <math>&gt; 0</math> MIN</p>	<p><b>B2,1,0</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>B1</b><sup>*/</sup></p> <p><b>DB1</b></p> <p><b>DB1</b></p> <p>[7]</p>	<p>Set to zero, simplify and attempt to solve soi</p> <p>Needs both <math>x</math> values. Ignore <math>y</math> values</p> <p>fit to <math>k(2x-1)^{-3}</math> where <math>k &gt; 0</math></p> <p>Alt. methods for last 3 marks (values either side of 1/4 &amp; 3/4) must indicate <u>which</u> <math>x</math>-values and cannot use <math>x = 1/2</math>. (M1A1A1)</p>
<p>6</p>	$BAC = \sin^{-1}(3/5) \text{ or } \cos^{-1}(4/5) \text{ or } \tan^{-1}(3/4)$ $ABC = \sin^{-1}(4/5) \text{ or } \cos^{-1}(3/5) \text{ or } \tan^{-1}(4/3)$ <p><math>ACB = \pi/2</math> (Allow <math>90^\circ</math>)</p> <p>Shaded area = <math>\Delta ABC</math> – sectors (<math>AEF + BEG + CFG</math>)</p> $\Delta ABC = \frac{1}{2} \times 4 \times 3 \text{ oe}$ <p>Sum sectors = <math>\frac{1}{2} [3^2 \cdot 0.6435] +</math>  <math>2^2 \cdot 0.9273 + 1^2 \cdot 1.5708]</math></p> <p><b>OR</b> <math>\frac{\pi}{360} [3^2 \cdot 36.8(7) + 2^2 \cdot 53.1(3) + 1^2 \cdot 90]</math></p> $6 - 5.536 = 0.464$	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p>[7]</p>	<p>Accept <math>36.8(7)^\circ</math></p> <p>Accept <math>53.1(3)^\circ</math></p>
<p>7</p>	$\frac{dy}{dx} = 2x - 5x^{1/2} + 5$ $\frac{dy}{dx} = 2$ $2x - 5x^{1/2} + 5 = 2$ $2x - 5x^{1/2} + 3 (= 0) \text{ or equivalent 3-term quadratic}$ <p>Attempt to solve for <math>x^{1/2}</math> e.g.</p> $(2x^{1/2} - 3)(x^{1/2} - 1) = 0$ $x^{1/2} = 3/2 \text{ and } 1$ $x = 9/4 \text{ and } 1$	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>DM1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p>[7]</p>	<p>Equate their <math>dy/dx</math> to <i>their</i> 2 or <math>1/2</math>.</p> <p>Dep. on 3-term quadratic</p> <p>ALT</p> $5x^{1/2} = 2x + 3 \rightarrow 25x = (2x + 3)^2$ $4x^2 - 13x + 9 (= 0)$ $x = 9/4 \text{ and } 1$

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<p><b>8 (i)</b></p> <p><math>3\sin^2 x - \cos^2 x + \cos x = 0</math> Use <math>s^2 = 1 - c^2</math> and simplify to 3-term quad <math>\cos x = -3/4</math> and 1</p> <p><math>x = 2.42</math> (allow <math>0.77\pi</math>) or 0 (extra in range max 1)</p> <p><b>(ii)</b></p> <p><math>2x = 2\pi - \text{their } 2.42</math> or <math>360 - 138.6</math></p> <p><math>x = 1.21</math> (<math>0.385\pi</math>), <math>1.93</math> (<math>0.614/5\pi</math>), 0, <math>\pi</math> (<math>3.14</math>) (extra max 1)</p>	<p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1A1</b> [5]</p> <p><b>B1</b><sup>h</sup></p> <p><b>B1B1</b> [3]</p>	<p>Multiply by <math>\cos x</math></p> <p>Expect <math>4c^2 - c - 3 = 0</math></p> <p>SC1 for 0.723 (or <math>0.23\pi</math>), <math>\pi</math> following <math>4c^2 + c - 3 = 0</math></p> <p>Expect <math>2x = 3.86</math></p> <p>Any 2 correct B1. Remaining 2 correct B1. SCB1 for all 69.3, 110.7, 0, 180 (degrees) SCB1 for .361, <math>\pi/2</math>, 2.78 after <math>4c^2 + c - 3 = 0</math></p>
<p><b>9 (i)</b></p> <p><math>\mathbf{AB} = \mathbf{OB} - \mathbf{OA} = \begin{pmatrix} -1 \\ 2 \\ p+4 \end{pmatrix}</math></p> <p><math>\mathbf{CB} = \mathbf{OB} - \mathbf{OC} = \begin{pmatrix} -4 \\ 5 \\ p-2 \end{pmatrix}</math></p> <p><math>1+4+(p+4)^2 = 16+25+(p-2)^2</math> <math>p=2</math></p> <p><b>(ii)</b></p> <p><math>\mathbf{AB} \cdot \mathbf{CB} = 4+10-5 = 9</math> <math> \mathbf{AB}  = \sqrt{1+4+25} = \sqrt{30}</math>, <math> \mathbf{CB}  = \sqrt{16+25+1} = \sqrt{42}</math> <math>\cos ABC = \frac{9}{\sqrt{30}\sqrt{42}}</math> or <math>\frac{9}{6\sqrt{35}}</math> <math>ABC = 75.3^\circ</math> or 1.31rads (ignore reflex angle <math>285^\circ</math>)</p>	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b> [4]</p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b> [4]</p>	<p>Ignore labels. Allow <b>BA</b> or <b>BC</b></p> <p>Use of <math>x_1x_2 + y_1y_2 + z_1z_2</math></p> <p>Product of moduli</p> <p>Allow one of <b>AB</b>, <b>CB</b> reversed - but award <b>A0</b></p>
<p><b>10 (i)</b></p> <p><math>2(ax^2 + b) + 3 = 6x^2 - 21</math> <math>a = 3</math>, <math>b = -12</math></p> <p><b>(ii)</b></p> <p><math>3x^2 - 12 \geq 0</math> or <math>6x^2 - 21 \geq 3</math> <math>x \leq -2</math> i.e. (max) <math>q = -2</math></p> <p><b>(iii)</b></p> <p><math>y \geq 6(-3)^2 - 21 \Rightarrow</math> range is <math>(y) \geq 33</math></p>	<p><b>M1</b></p> <p><b>A1A1</b> [3]</p> <p><b>M1</b></p> <p><b>A1</b> [2]</p> <p><b>B1</b> [1]</p>	<p>Allow = or <math>\leq</math> or <math>&gt;</math> or <math>&lt;</math>. Ft from <i>their a, b</i> Must be in terms of <math>q</math> (eg <math>q \leq -2</math>)</p> <p>Do not allow <math>y &gt; 33</math>. Accept all other notations e.g. <math>[33, \infty)</math> or <math>[33, \infty]</math></p>

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<b>(iv)</b>	$y = 6x^2 - 21 \Rightarrow x = (\pm)\sqrt{\frac{y+21}{6}}$ $(fg)^{-1}(x) = -\sqrt{\frac{x+21}{6}}$ <p>Domain is <math>x \geq 33</math></p>	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>B1</b><sup>h</sup></p> <p>[3]</p>	<p>Allow <math>y = \dots</math>. Must be a function of <math>x</math></p> <p>ft from <i>their</i> part (iii) but <math>x</math> essential</p>
<b>11 (i)</b>	$AB^2 = 6^2 + 7^2 = 85, BC^2 = 2^2 + 9^2 = 85$ <p>(<math>\rightarrow</math> isosceles)</p> $AC^2 = 8^2 + 2^2 = 68$ $M = (2, -2) \text{ or } BM^2 = (\sqrt{85})^2 - (\frac{1}{2}\sqrt{68})^2$ $BM = \sqrt{2^2 + 8^2} = \sqrt{68} \text{ or } \sqrt{85 - 17} = \sqrt{68}$ $\text{Area } \triangle ABC = \frac{1}{2}\sqrt{68}\sqrt{68} = 34$	<p><b>B1B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p>[6]</p>	<p>Or <math>AB = BC = \sqrt{85}</math> etc</p> <p>Where <math>M</math> is mid-point of <math>AC</math></p>
<b>(ii)</b>	<p>Gradient of <math>AB = 7/6</math></p> <p>Equation of <math>AB</math> is <math>y + 1 = \frac{7}{6}(x + 2)</math></p> <p>Gradient of <math>CD = -6/7</math></p> <p>Equation of <math>CD</math> is <math>y + 3 = \frac{-6}{7}(x - 6)</math></p> <p>Sim Eqns <math>2 = \frac{-6}{7}x + \frac{36}{7} - \frac{7}{6}x - \frac{14}{6}</math></p> $x = \frac{34}{85} = \frac{2}{5} \text{ oe}$	<p><b>B1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p>[6]</p>	<p>Or <math>y - 6 = \frac{7}{6}(x - 4)</math></p>