edexcel

# June 2009
# 6681 Mechanics M5
# Mark Scheme

| Question Number | Scheme | Marks |
|---|---|---|
| Q1 | $\pm(8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k})$ | B1 |
| | $((4\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) + (8\mathbf{i} - 4\mathbf{j} + 7\mathbf{k})).(8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}) = \frac{1}{2}3v^2$ | M1 A1 f.t. |
| | $12 = v$ | A1 |
| | $\mathbf{v} = \frac{12}{\sqrt{8^2 + (-4)^2 + 8^2}}(8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k})$ | M1 |
| | $\mathbf{v} = (8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k})\,\text{ms}^{-1}$ | DM1 A1 |
| | | **[7]** |
| Q2 | C.F. is $\mathbf{r} = \mathbf{A}\cos 2t + \mathbf{B}\sin 2t$ | B1 |
| | P.I. is $\mathbf{r} = \mathbf{p}e^{2t}$ | B1 |
| | $\dot{\mathbf{r}} = 2\mathbf{p}e^{2t}$<br>$\ddot{\mathbf{r}} = 4\mathbf{p}e^{2t}$ | B1 ft |
| | $4\mathbf{p}e^{2t} + 4\mathbf{p}e^{2t} = \mathbf{j}e^{2t}$ | M1 |
| | so, (PI is) $\mathbf{r} = \frac{1}{8}\mathbf{j}e^{2t}$ | A1 |
| | GS is $\mathbf{r} = \mathbf{A}\cos 2t + \mathbf{B}\sin 2t + \frac{1}{8}\mathbf{j}e^{2t}$ | A1 ft |
| | $t = 0,\ \mathbf{r} = \mathbf{i} + \mathbf{j} \Rightarrow \mathbf{i} + \mathbf{j} = \mathbf{A} + \frac{1}{8}\mathbf{j} \Rightarrow \mathbf{i} + \frac{7}{8}\mathbf{j} = \mathbf{A}$ | DM1 A1 |
| | $\dot{\mathbf{r}} = -2\mathbf{A}\sin 2t + 2\mathbf{B}\cos 2t + \frac{1}{4}\mathbf{j}e^{2t}$ | M1A1 |
| | $t = 0,\ \dot{\mathbf{r}} = 2\mathbf{i} \Rightarrow 2\mathbf{i} = 2\mathbf{B} + \frac{1}{4}\mathbf{j} \Rightarrow \mathbf{i} - \frac{1}{8}\mathbf{j} = \mathbf{B}$ | A1 |
| | $\mathbf{r} = (\mathbf{i} + \frac{7}{8}\mathbf{j})\cos 2t + (\mathbf{i} - \frac{1}{8}\mathbf{j})\sin 2t + \frac{1}{8}\mathbf{j}e^{2t}$ | **[11]** |

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| Q3 (a) | $mv = (m+\delta m)(v+\delta v) - (-\delta m)(c-v)$<br>$mv = mv + m\delta v + v\delta m + c\delta m - v\delta m$<br>$-m\delta v = c\delta m$<br>$\frac{\mathrm{d}v}{\mathrm{d}m} = -\frac{c}{m}$ * | M1 A2<br><br>DM1 A1 (5) |
| (b) | $\frac{\mathrm{d}m}{\mathrm{d}t} = -m_0 k$ | B1 |
| | $\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{d}m} \times \frac{\mathrm{d}m}{\mathrm{d}t}$<br>$= -\frac{c}{m} \times -m_0 k$<br>$= \frac{cm_0 k}{m_0(1-kt)}$<br>$= \frac{ck}{(1-kt)}$ | M1<br>DM1<br>A1 (4)<br>**[9]** |

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| Q4 (a) | $\delta m = \dfrac{2Mx\delta x}{a^2}$ | M1 A1 |
| | $\delta I = \dfrac{1}{3}\dfrac{2Mx\delta x}{a^2}(2x)^2$ | M1 A1 |
| | $I = \int_0^a \dfrac{8Mx^3 \mathrm{d}x}{3a^2}$ | DM1 |
| | $= \dfrac{8M}{3a^2}\left[\dfrac{x^4}{4}\right]_0^a$ | |
| | $= \dfrac{2}{3}Ma^2$ * | A1 |
| | | (6) |
| (b) | $J.2a = \dfrac{2}{3}Ma^2\omega$ | M1 A1 |
| | $\dfrac{1}{2}\dfrac{2}{3}Ma^2\omega^2 = Mg\dfrac{2a}{3}(1+\cos 60^o)$ | M1 A2 |
| | solving for $J$ | DM1 |
| | $J = M\sqrt{\dfrac{ag}{3}}$ | A1 (7) |
| | | **[13]** |

| Question Number | Scheme | Marks |
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| Q5 (a) | $(2\mathbf{i} + \mathbf{j}) + (-2\mathbf{j} - \mathbf{k}) + \mathbf{F}_3 = \mathbf{0}$ | M1 |
| | $\mathbf{F}_3 = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$ | A1 |
| | $\lvert\mathbf{F}_3\rvert = \sqrt{(-2)^2 + 1^2 + 1^2} = \sqrt{6}$ N | M1 A1 (4) |
| (b) | $(3\mathbf{i} + \mathbf{j} + \mathbf{k}) \times (2\mathbf{i} + \mathbf{j}) + (\mathbf{i} - 2\mathbf{j}) \times (-2\mathbf{j} - \mathbf{k}) + (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \times (-2\mathbf{i} + \mathbf{j} + \mathbf{k})$ | M1 |
| | $(-\mathbf{i} + 2\mathbf{j} + \mathbf{k}) + (2\mathbf{i} + \mathbf{j} - 2\mathbf{k}) + ((y - z)\mathbf{i} + (-2z - x)\mathbf{j} + (x + 2y)\mathbf{k})$ | A3 |
| | $y - z = -1,\ -x - 2z = -3,\ x + 2y = 1$ | DM1 |
| | $x = 1,\ y = 0,\ z = 1$ is a solution | DM1 |
| | so, $\mathbf{r} = (\mathbf{i} + \mathbf{k}) + \lambda(-2\mathbf{i} + \mathbf{j} + \mathbf{k})$ is a vector equn of line of action of $\mathbf{F}_3$ | M1 A1 (8) |
| (c) | $(3\mathbf{i} + \mathbf{j} + \mathbf{k}) \times (2\mathbf{i} + \mathbf{j}) + (\mathbf{i} - 2\mathbf{j}) \times (-2\mathbf{j} - \mathbf{k}) = \mathbf{G}$ | M1 |
| | $(-\mathbf{i} + 2\mathbf{j} + \mathbf{k}) + (2\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = (\mathbf{i} + 3\mathbf{j} - \mathbf{k}) = \mathbf{G}$ | A1 |
| | $\lvert\mathbf{G}\rvert = \sqrt{1^2 + 3^2 + (-1)^2} = \sqrt{11}$ N m | M1 A1 (4) |
| | | **[16]** |

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| Q6 (a) | $\frac{1}{3}2m(4a)^2+\frac{1}{12}4ma^2+4m(4a)^2$ | B1 M1 A1 |
| | $=\frac{32}{3}ma^2+\frac{1}{3}ma^2+64ma^2$ | |
| | $=75ma^2$ * | A1 (4) |
| (b) | $\frac{1}{2}75ma^2\omega^2=2mg2a(\cos\theta-\cos\alpha)+4mg4a(\cos\theta-\cos\alpha)$ | M1 A2 |
| | $a\omega^2=\frac{8}{15}g(\cos\theta-\frac{24}{25})=\frac{8}{375}g(25\cos\theta-24)$ | A1 |
| | $X-6mg\cos\theta=2m2a\omega^2+4m4a\omega^2=20ma\omega^2$ | M1 A2 |
| | $X=6mg\cos\theta+20m\frac{8}{375}g(25\cos\theta-24)$ | D M1 |
| | $=\frac{50mg\cos\theta}{3}-\frac{256mg}{25}$ | A1 (9) |
| (c) | $-2mg2a\sin\theta-4mg4a\sin\theta=75ma^2\ddot{\theta}$ | M1 A1 |
| | $\ddot{\theta}=-\frac{4g}{15a}\sin\theta$ | A1 |
| | $\approx-\frac{4g}{15a}\theta$, SHM | M1 |
| | Time $=\frac{1}{4}2\pi\sqrt{\frac{15a}{4g}}$ | M1 |
| | $=\frac{\pi}{4}\sqrt{\frac{15a}{g}}$ | A1 (6) |
| | | **[19]** |