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Question number	Scheme	Marks
1.	$\begin{vmatrix} (7-\lambda) & 6 \\ 6 & (2-\lambda) \end{vmatrix} = 0$	
	$(7-\lambda)(2-\lambda) - 36 = 0$	M1 A1
	$\lambda^2 - 9\lambda + 14 - 36 = 0$	
	$\lambda^2 - 9\lambda - 22 = 0$	
	$(\lambda - 11)(\lambda + 2) = 0 \Rightarrow \lambda_1 = -2, \lambda_2 = 11$	M1 A1 (4)
2.	$9\left(\frac{e^x + e^{-x}}{2}\right) - 6\left(\frac{e^x - e^{-x}}{2}\right) = 7$	B1
	$3e^{2x} - 14e^x + 15 = 0$	M1 A1
	$(3e^x - 5)(e^x - 3) = 0 \qquad e^x = \frac{5}{3}, \quad e^x = 3$	M1 A1
	$x = \ln\frac{5}{3} \qquad x = \ln 3$	A1 (6)
3.	$s = \int_0^{2\pi} \left[\dot{x}^2 + \dot{y}^2\right]^{\frac{1}{2}} dt$	
	$\frac{dx}{dt} = \dot{x} = a(1 - \cos t); \quad \frac{dy}{dt} = \dot{y} = a\sin t$	M1 A1; A1
	$s = \int_0^{2\pi} a\left[(1-\cos t)^2 + \sin^2 t\right]^{\frac{1}{2}} dt = a\int_0^{2\pi} [2 - 2\cos t]^{\frac{1}{2}} dt$	
	$= 2a\int_0^{\frac{\pi}{2}} \sin\left(\frac{t}{2}\right) dt, \quad = -4a\left[\cos\left(\frac{t}{2}\right)\right]_0^{2\pi} = 8a$	M1 A1 A1ft (6)

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4.	$x = 2\sinh t$	B1
	$\sqrt{(x^2+4)} = (4\sinh^2 t + 4)^{\frac{1}{2}} = 2\cosh t$	
	$dx = 2\cosh t\, dt$	
	$I = \int \sqrt{(x^2+4)}\,dx = 4\int \cosh^2 t\, dt$	M1A1
	$= 2\int (\cosh 2t + 1)\,dt$	
	$= \sinh 2t + 2t + c$	M1 A1
	$= \frac{1}{2}x\sqrt{(x^2+4)} + 2\operatorname{arsinh}\left(\frac{x}{2}\right) + c$	M1 A1ft (7)
5. (a)	$y = \arcsin x$	
	$\Rightarrow \sin y = x$	M1
	$\cos y \frac{dy}{dx} = 1$	
	$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$	M1 A1 (3)
(b)	$\frac{d^2y}{dx^2} = -\frac{1}{2}(1-x^2)^{\frac{-3}{2}}(-2x)$	
	$= x(1-x^2)^{\frac{-3}{2}}$	M1 A1
	$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = (1-x^2)x(1-x^2)^{\frac{-3}{2}} - x(1-x^2)^{-\frac{1}{2}} = 0$	M1 A1 (4)

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6. (a)	$I_n = \int_0^{\frac{\pi}{2}} x^n \sin x \, dx$	
	$= \left[x^n(-\cos x)\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} nx^{n-1}(-\cos x)dx$	M1 A1
	$= 0 + n\left\{\left[x^{n-1}\sin x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}}(n-1)x^{n-2}\sin x \, dx\right\}$	A1
	$= n\left[\left(\frac{\pi}{2}\right)^{n-1} - (n-1)I_{n-2}\right]$	
	So $I_n = n\left(\frac{\pi}{2}\right)^{n-1} - n(n-1)I_{n-2}$	A1 (4)
(b)	$I_3 = 3\left(\frac{\pi}{2}\right)^2 - 3.2I_1$	
	$I_1 = \int_0^{\frac{\pi}{2}} x\sin x \, dx \quad = \left[x(-\cos x)\right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x \, dx$	M1
	$= \left[\sin x\right]_0^{\frac{\pi}{2}} = 1$	A1
	$I_3 = (3)\left(\frac{\pi}{2}\right)^2 - 6 = \frac{3\pi^2}{4} - 6$	M1 A1 (4)

Question number	Scheme	Marks
7. (a)	$\mathbf{A}(x) = \begin{pmatrix} 1 & x & -1 \\ 3 & 0 & 2 \\ 1 & 1 & 0 \end{pmatrix}$	
	Cofactors $\begin{pmatrix} -2 & 2 & 3 \\ -1 & 1 & x-1 \\ 2x & -5 & -3x \end{pmatrix}$	M1 A1 A1 A1
	Determinant $= 2x-3-2 = 2x-5$	M1 A1
	$\mathrm{A}^{-1}(x) = \dfrac{1}{2x-5}\begin{pmatrix} -2 & -1 & 2x \\ 2 & 1 & -5 \\ 3 & (x-1) & -3x \end{pmatrix}$	M1 A1ft (8)
(b)	$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \mathrm{B}^{-1}\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = \dfrac{1}{1}\begin{pmatrix} -2 & -1 & 6 \\ 2 & 1 & -5 \\ 3 & 2 & -9 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$	M1 A1ft
	$= (17, -13, -24)$	M1 A1 (4)

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Question number	Scheme	Marks
8. (a)	$\overrightarrow{AB} = (-1,\ 3, -1);\ \overrightarrow{AC} = (-1, 3, 1)$.	M1 A1
	$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 3 & -1 \\ -1 & 3 & 1 \end{vmatrix}$	
	$= \mathbf{i}(3+3) + \mathbf{j}(1+1) + \mathbf{k}(-3+3)$	
	$= 6\mathbf{i} + 2\mathbf{j}$	M1 A1 A1
	Area of $\Delta\, ABC = \frac{1}{2}\left\| \overrightarrow{AB} \times \overrightarrow{AC} \right\|$	
	$= \frac{1}{2}\sqrt{36+4} = \sqrt{10}$ square units	M1 A1ft (7)
(b)	Volume of tetrahedron $= \frac{1}{6}\left\| \overrightarrow{AD} \,.\, \left(\overrightarrow{AB} \times \overrightarrow{AC}\right)\right\|$	
	$= \frac{1}{6}\left\| -12+8 \right\|$	
	$= \frac{2}{3}$ cubic units	M1 A1 (2)
(c)	Unit vector in direction $\overrightarrow{AB} \times \overrightarrow{AC}$ i.e. perpendicular to plane containing A, B, and C is	
	$\mathbf{n} = \frac{1}{\sqrt{40}}(6\mathbf{i} + 2\mathbf{j}) = \frac{1}{\sqrt{10}}(3\mathbf{i} + \mathbf{j})$	M1
	$p = \left\|\mathbf{n} \cdot \overrightarrow{AD}\right\| = \frac{1}{\sqrt{10}}\left\|(3\mathbf{i} + \mathbf{j}) \cdot (-2\mathbf{i} + 4\mathbf{j})\right\|$	
	$= \frac{1}{\sqrt{10}}\left\|-6+4\right\| = \frac{2}{\sqrt{10}}$ units.	M1 A1 (3)

Question number	Scheme	Marks
9. (a)	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	
	$\frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0$	M1 A1
	$\frac{dy}{dx} = \frac{2x}{a^2}\frac{b^2}{2y} = \frac{b^2}{a^2}\frac{a\sec\theta}{b\tan\theta} = \frac{b}{a\sin\theta}$	M1 A1
	Gradient of normal is then $-\frac{a}{b}\sin\theta$	
	Equation of normal: $(y - b\tan\theta) = -\frac{a}{b}\sin\theta(x - a\sec\theta)$	
	$ax\sin\theta + by = (a^2 + b^2)\tan\theta$	M1 A1 (6)
(b)	M: A normal cuts $x = 0$ at $y = \frac{(a^2 + b^2)}{b}\tan\theta$	M1 A1
	B normal cuts $y = 0$ at $x = \frac{a^2 + b^2}{a\sin\theta}\tan\theta$	
	$= \frac{(a^2 + b^2)}{a\cos\theta}$	A1
	Hence M is $\left[\frac{(a^2+b^2)}{2a}\sec\theta, \frac{(a^2+b^2)}{2b}\tan\theta\right]$	M1
	Eliminating θ	M1
	$\sec^2\theta = 1 + \tan^2\theta$	
	$\left[\frac{2aX}{a^2+b^2}\right]^2 = 1 + \left[\frac{2bY}{a^2+b^2}\right]^2$	A1
	$4a^2X^2 - 4b^2Y^2 = [a^2 + b^2]^2$	A1 (7)