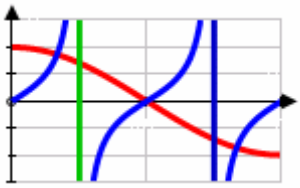
Mark Scheme 4722
June 2005

1	(i)	$u_1 = 2, u_2 = 5, u_3 = 8$ The sequence is an Arithmetic Progression	B1 B1 B1 3	For the correct value of u_1 For both correct values of u_2 and u_3 For a correct statement (any mention of arithmetic)
	(ii)	$\frac{1}{2} \times 100 \times (2 \times 2 + 99 \times 3) = 15050$	M1 M1 A1 3 **6**	For correct interpretation of Sigma notation – ie finding the sum of an AP or GP For use of correct $\frac{1}{2}n(2a + (n-1)d)$, or equiv, with n=100 and a & d not both =1 For correct value 15050
2	(i)	$r\theta = 12$, $\frac{1}{2}r^2\theta = 36$	B1 B1 2	For $r\theta = 12$ stated correctly at any point For $\frac{1}{2}r^2\theta = 36$ stated correctly at any point
	(ii)	$\frac{1}{2}r \times 12 = 36 \Rightarrow r = 6$ Hence $\theta = 2$	B1 B1 2	For showing given value correctly For correct value 2 (or 0.637π)
	(iii)	Segment area is $36 - \frac{1}{2} \times 6^2 \times \sin 2 = 19.6$ cm^2	M1* M1dep* A1 3 **7**	For use of $\Delta = \frac{1}{2}ab \sin C$, or equivalent For attempt at $36 - \Delta$ For correct value (rounding to) 19.6
3	(i)	$\int (2x^2 + 7x + 3)\,dx$ $= \frac{2}{3}x^3 + \frac{7}{2}x^2 + 3x + c$	M1 A1 A1 B1 4	For expanding and integration attempt For at least one term correct For all three terms correct For addition of arbitrary constant, and no $\int$ or dx
	(ii)	$\left[2x^{\frac{1}{2}}\right]_0^9$ = 6	M1 M1 A1 3 **7**	For integral of the form $kx^{\frac{1}{2}}$ For evaluating at least F(9) , following attempt at integration For final answer of 6 only
4	(i)	$\cos BCA = \frac{5^2+6^2-9^2}{2\times5\times6} = -\frac{1}{3}$ So $\sin BCA = \frac{2}{3}\sqrt{2} \approx 0.9428$ ...	M1 M1 A1 B1 M1 M1 A1 B1 4	For relevant use of the correct cosine formula For attempt to rearrange correct formula For obtaining the given value correctly For correct answer for sin *BCA* in any form OR For substituting cos BCA = $-1/3$ For attempt at evaluation For full verification For correct answer for sin *BCA* in any form
	(ii)	Angles *BCA* and *CAD* are equal So $\sin ADC = \frac{5}{15}\sin CAD = \frac{1}{3} \times \frac{1}{3}\sqrt{8} = \frac{2}{9}\sqrt{2}$ $\Rightarrow ADC = 18.3^o$	B1 M1 A1√ A1 4 **8**	For stating, using or implying the equal angles For correct use of the sine rule in Δ ADC (sides must be numerical, angles may still be in letters) For a correct equation from their value in (i) For correct answer, from correct working
5	(i)	f(-1) = 0 $\Rightarrow$ -1 – a + b = 0 f(3) = 16 $\Rightarrow$ 27 + 3a + b = 16 Hence a = -3, b = -2	M1 A1 M1 A1 A1 5	For equating their attempt at f(-1) to 0, or equiv For the correct (unsimplified) equation For equating their attempt at f(3) to 16, or equiv For the correct (unsimplified) equation For both correct values – must follow two correct equations
	(ii)	f(2) = 8 – 6 – 2 = 0	B1	For the correct verification (from correct a &

		Hence f(x) = $(x + 1)^2(x – 2)$	M1 A1 3 **8**		b) For recognition or use of two linear factors, or full division attempt by either $(x + 1)$ or $(x – 2)$ For correct third factor (repeated) of $(x + 1)$, and full linear factorisation stated
6	(i)	$x^6 + 3x^3 + 3 + \frac{1}{x^3}$	M1 A1 A1 A1 4		For 4 term binomial attempt or equiv For any one (unsimplified) term correct For any other (unsimplified) term correct For full, simplified, expansion correct
	(ii)	$\frac{1}{7}x^7 + \frac{3}{4}x^4 + 3x - \frac{1}{2}x^{-2} + c$	M1 A1√ M1 A1√ 4 **8**		For any correct use of $\frac{x^{n+1}}{n+1}$ For any two terms integrated correctly For any correct use of x^{n+1} using a negative index For all terms integrated correctly (must have at least 4 terms, including at least 1 negative index) [No penalty for omission of +c in this part]
7	(i)	$\log_5\left(\frac{15\times20}{12}\right) = \log_5 25 = 2$	M1 A1 A1 3		For any relevant combination of log a ± log b For log 25 – must follow correct working only For correct answer 2
	(ii)	Method A $\frac{1}{3}y = 10^{2x}$ Hence $2x = \log_{10}\left(\frac{1}{3}y\right)$ i.e. $x = \frac{1}{2}\log_{10}\left(\frac{1}{3}y\right)$	M1 M1 A1 A1 4		For correct division of both sides by 3 For relevant use of $a = b^c \Leftrightarrow c = \log_b a$ For correct equation involving logs to base 10 For correct answer for x
		Method B $\frac{1}{3}y = 10^{2x}$ $\log\frac{1}{3}y = \log 10^{2x}$ $\log\frac{1}{3}y = 2x\log 10$ i.e. $x = \frac{1}{2}\log_{10}\left(\frac{1}{3}y\right)$	M1 M1 A1 A1 4		For correct division of both sides by 3 For taking logs of both sides For correct linear equation involving logs For correct answer for x
		Method C $y = 3\times10^{2x} \Rightarrow \log y = \log 3\times10^{2x}$ $\log y = \log 3 + \log 10^{2x}$ $\log y = \log 3 + 2x\log 10$ i.e. $x = \frac{1}{2}\log_{10}\left(\frac{1}{3}y\right)$	M1 A1 M1 A1 4		For introducing logs throughout For correct RHS log 3 + log 10^{2x} For correct use of log a^b = b log a For correct answer for x
		Method D $x = a\log(b\times3\times10^{2x})$ $x = a\log 3b + a\log 10^{2x}$ $x = 2ax\log 10 \Rightarrow 2a = 1 \Rightarrow a = ½$ $a\log 3b = 0 \Rightarrow 3b = 1 \Rightarrow b = ⅓$	M1 M1 A1 A1 4 **7**		For substituting for y, and separating RHS into at least 2 terms For attempting values for a and b For obtaining a = $^1/_2$ For obtaining b = $^1/_3$
8	(i)	100 000 x 0.9^3 = 72900	M1 A1 2		For relevant use of ar^3 or equiv For the correct answer 72900
	(ii)	100 000 x 0.9^x = 5000 Hence $x$log 0.9 = log 0.05 So x = 28.4, 28 or 29; or n = 29.4, 29 or 30 i.e. 30th year / 30 years / year is 2030	B1 M1 A1 A1√ 4		For a correct equation or inequality For complete solution method by logs or trial For correct solution for **their** index – allow integer values either side For correctly linking **their** index to date or

						number of years
	(iii)	Total is $\frac{100000\left(1-0.9^{30}\right)}{1-0.9} = 957609$		M1 A1√ A1 **9**	3	For relevant use of $\frac{a\left(1-r^n\right)}{1-r}$ For correct (unsimplified) statement for their integer n (if no n stated then use their year – 2000) For answer 958000 or better, including decimal
9	(a)	(i)	$\cos \frac{1}{6}\pi = \frac{1}{2}\sqrt{3}$ $\tan \frac{1}{3}\pi = \sqrt{3}$ Hence $2\cos \frac{1}{6}\pi = 2 \times \frac{1}{2}\sqrt{3} = \tan \frac{1}{3}\pi$		B1 B1 B1	3	For any correct exact value For any correct exact value For correct verification (allow via decimals)
		(ii)	 Other roots are π/2 and 5π/6		B1 B1 B1 B1	4	For correct sketch of either $y = \tan 2x$ or $y = 2\cos x$ For second correct sketch, with both graphs in proportion (ie 3 points of intersection) For one of π/2 or 5π/6 (or equiv in degrees) For second correct value, and no others in range $0 \le x \le \pi$
	(b)	(i)	0.05(0.1003 + 2(0.2027 + 0.3093) + 0.4228) = 0.0774		M1 M1 A1 A1	4	State at least three of tan 0.1, tan 0.2, tan 0.3, tan 0.4 Substitute numerical values (must be attempt at y-coords, not x-coords) into correct trapezium rule, with h consistent with number of strips Obtain 0.05(tan 0.1 + 2(tan 0.2 + tan 0.3)+ tan 0.4) or equiv in decimals (SC – award A1 if values are now decimals from using degrees – gives final answer of 0.00131) Obtain 0.077 or better
		(ii)	Overestimate; tops of trapezia above the curve or equiv		B1 **12**	1	For correct statement and justification