

1.

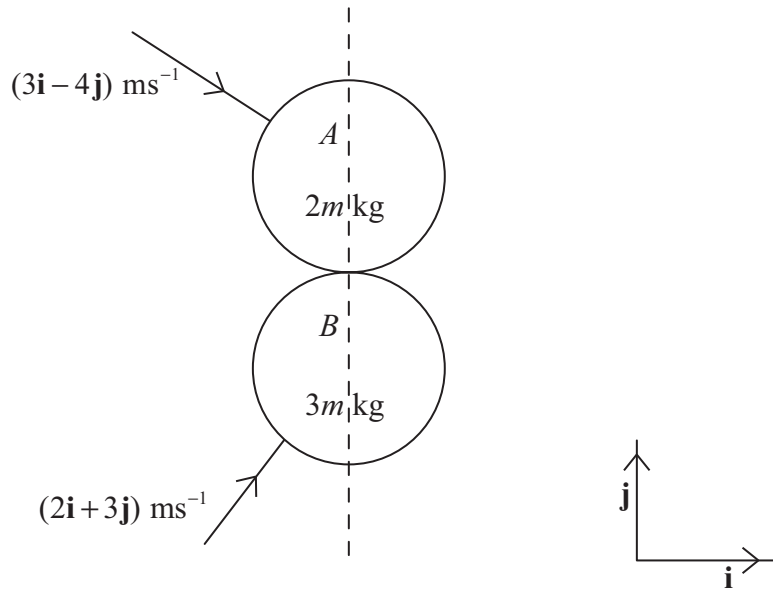


Figure 1

Two smooth uniform spheres A and B have masses $2m$ kg and $3m$ kg respectively and equal radii. The spheres are moving on a smooth horizontal surface. Initially, sphere A has velocity $(3\mathbf{i} - 4\mathbf{j}) \text{ m s}^{-1}$ and sphere B has velocity $(2\mathbf{i} - 3\mathbf{j}) \text{ m s}^{-1}$. When the spheres collide, the line joining their centres is parallel to \mathbf{j} , as shown in Figure 1. The coefficient of restitution between the spheres is $\frac{3}{7}$. Find, in terms of m , the total kinetic energy lost in the collision.

(10)

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Question 1 continued

Lined area for writing answers to Question 1 continued.

(Total 10 marks)

Q1

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2.

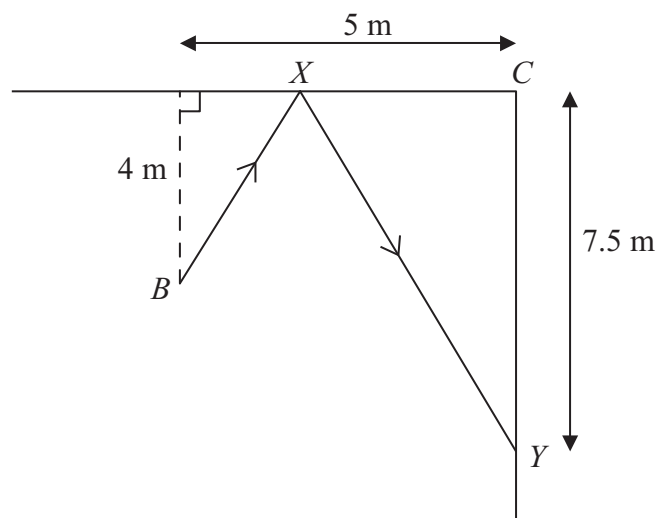


Figure 2

Figure 2 represents part of the smooth rectangular floor of a sports hall. A ball is at B , 4 m from one wall of the hall and 5 m from an adjacent wall. These two walls are smooth and meet at the corner C . The ball is kicked so that it travels along the floor, bounces off the first wall at the point X and hits the second wall at the point Y . The point Y is 7.5 m from the corner C .

The coefficient of restitution between the ball and the first wall is $\frac{3}{4}$.

Modelling the ball as a particle, find the distance CX .

(9)



3. [In this question the unit vectors \mathbf{i} and \mathbf{j} are due east and due north respectively.]

A coastguard patrol boat C is moving with constant velocity $(8\mathbf{i} + u\mathbf{j})$ km h^{-1} . Another ship S is moving with constant velocity $(12\mathbf{i} + 16\mathbf{j})$ km h^{-1} .

- (a) Find, in terms of u , the velocity of C relative to S . (2)

At noon, S is 10 km due west of C .
If C is to intercept S ,

- (b) (i) find the value of u .
(ii) Using this value of u , find the time at which C would intercept S . (4)

If instead, at noon, C is moving with velocity $(8\mathbf{i} + 8\mathbf{j})$ km h^{-1} and continues at this constant velocity,

- (c) find the distance of closest approach of C to S . (5)



5. A particle Q of mass 6 kg is moving along the x -axis. At time t seconds the displacement of Q from the origin O is x metres and the speed of Q is v m s $^{-1}$. The particle moves under the action of a retarding force of magnitude $(a + bv^2)$ N, where a and b are positive constants. At time $t = 0$, Q is at O and moving with speed U m s $^{-1}$ in the positive x -direction. The particle Q comes to instantaneous rest at the point X .

(a) Show that the distance OX is

$$\frac{3}{b} \ln \left(1 + \frac{bU^2}{a} \right) \text{ m} \tag{6}$$

Given that $a = 12$ and $b = 3$,

(b) find, in terms of U , the time taken to move from O to X . (5)



6. A particle P of mass 4 kg moves along a horizontal straight line under the action of a force directed towards a fixed point O on the line. At time t seconds, P is x metres from O and the force towards O has magnitude $9x$ newtons. The particle P is also subject to air resistance, which has magnitude $12v$ newtons when P is moving with speed v m s $^{-1}$.

(a) Show that the equation of motion of P is

$$4 \frac{d^2x}{dt^2} + 12 \frac{dx}{dt} + 9x = 0 \tag{4}$$

It is given that the solution of this differential equation is of the form

$$x = e^{-\lambda t} (At + B)$$

When $t = 0$ the particle is released from rest at the point R , where $OR = 4$ m.

Find,

(b) the values of the constants λ , A and B , (4)

(c) the greatest speed of P in the subsequent motion. (5)



7.

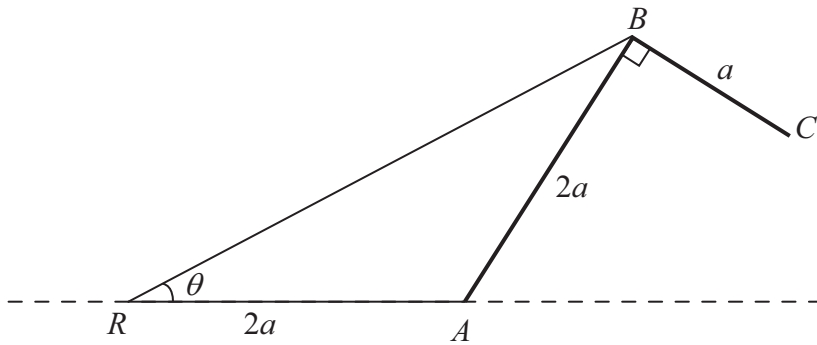


Figure 3

Figure 3 shows a framework ABC , consisting of two uniform rods rigidly joined together at B so that $\angle ABC = 90^\circ$. The rod AB has length $2a$ and mass $4m$, and the rod BC has length a and mass $2m$. The framework is smoothly hinged at A to a fixed point, so that the framework can rotate in a fixed vertical plane. One end of a light elastic string, of natural length $2a$ and modulus of elasticity $3mg$, is attached to A . The string passes through a small smooth ring R fixed at a distance $2a$ from A , on the same horizontal level as A and in the same vertical plane as the framework. The other end of the string is attached to B . The angle ARB is θ , where $0 < \theta < \frac{\pi}{2}$.

(a) Show that the potential energy V of the system is given by

$$V = 8amg \sin 2\theta + 5amg \cos 2\theta + \text{constant} \quad (7)$$

(b) Find the value of θ for which the system is in equilibrium. (4)

(c) Determine the stability of this position of equilibrium. (3)
