



GCE

Mathematics

Unit **4723**: Core Mathematics 3

Advanced GCE

Mark Scheme for June 2015

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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4723

Mark Scheme

June 2015

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

4723

Mark Scheme

June 2015

Subject-specific Marking Instructions for GCE Mathematics Pure strand

- a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

4723

Mark Scheme

June 2015

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

- g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

4723**Mark Scheme****June 2015**

- h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

4723

Mark Scheme

June 2015

Question		Answer	Marks	Guidance	
1		<p>Attempt use of quotient rule or, after adjustment, product rule</p> <p>Obtain $\frac{5(3x-8)-3(5x+4)}{(3x-8)^2}$ or equiv</p> <p>Substitute 2 to obtain -13 or equiv</p> <p>Attempt to find equation of tangent</p> <p>Obtain $y = -13x + 19$ or $13x + y - 19 = 0$</p>	<p>*M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>For M1 allow one slip in numerator but must be minus sign in numerator and square of $3x - 8$ in denominator; allow M1 for numerator the wrong way round</p> <p>Allow if missing brackets implied by subsequent simplification or calculation</p> <p>Dep *M; equation of tangent not normal</p> <p>Or similarly simplified equiv with 3 non-zero terms</p>	<p>For product rule attempt, *M1 for $k_1(3x-8)^{-1} + k_2(5x+4)(3x-8)^{-2}$ form and A1 for correct constants 5 and -3;</p>
2	(i)	<p>State or imply $\tan \theta = \frac{1}{4}$</p> <p>State or imply use of $\frac{\tan \theta + 1}{1 - \tan \theta}$</p> <p>Obtain $\frac{5}{3}$ or $1\frac{2}{3}$ or $\frac{20}{12}$ or exact equiv</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>[3]</p>	<p>But not unsimplified equiv (such as $\frac{5}{4} / \frac{3}{4}$)</p>	<p>Note that both parts are to be answered without calculator so sufficient detail is needed</p>
	(ii)	<p>Attempt use of correct relevant identity or of right-angled triangle</p> <p>Obtain $\sqrt{17}$</p>	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>Such as $\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$, or $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$ with attempt at $\sin \theta$, or use of Pythagoras' theorem in right-angled triangle</p> <p>Final answer $\pm\sqrt{17}$ earns A0</p>	

4723

Mark Scheme

June 2015

Question	Answer	Marks	Guidance
3	Differentiate to obtain $kh^n(2 + \sqrt{h})^5$ Obtain $9h^{-\frac{1}{2}}(2 + \sqrt{h})^5$ or unsimplified equiv Divide 150 by their derivative, algebraic or numerical Substitute $h = 1.4$ and evaluate Obtain 0.06 or 0.060 or 0.0603	M1 A1 *M1 M1 A1 [5]	Any non-zero constants k, n ; condone presence of -192 here Without -192 now Using any recognisable attempt at first derivative Dep *M; assume appropriate substitution if calculation goes wrong But not greater accuracy in final answer; units not needed unless change made to metres and/or hours
4	Obtain $2a$ as one value of x Attempt to find second value of x Obtain $-8a$ Substitute each of at most two values of x (involving a) leading to one final answer in each case and showing correct application of modulus signs in at least one case Obtain $4a$ as final answer Obtain $-14a$ as final answer	B1 M1 A1 M1 A1 A1 [6]	Allow solution leading to $a = \frac{1}{2}x$ (B1) and $a = -\frac{1}{8}x$ (M1A1) If using quadratic formula to solve equation, substitution must be accurate By solving equation with signs of x and $5a$ different, or by squaring both sides and attempting solution of quadratic equation with three terms And no other values of x Obtained correctly from $x = 2a$ Obtained correctly from $x = -8a$

4723

Mark Scheme

June 2015

Question		Answer	Marks	Guidance
5	(i)	State first derivative is $3e^{3x} - 12e^{2x}$ Equate first derivative to zero and attempt solution of equation of form $k_1e^{3x} - k_2e^{2x} = 0$ Obtain $\ln 4$ or exact equiv and no other Substitute $x = \ln 4$ or $e^x = 4$ to confirm $y = 0$	B1 M1 A1 A1 [4]	Or equiv At least as far as $e^x = c$; M0 for false method such as $\ln(3e^{3x}) - \ln(12e^{2x}) = 0$ Obtained by legitimate method AG; using exact working with all detail present: needs sight of $4^3 - 6 \times 4^2 + 32$ or similar equiv
	(ii)	Integrate to obtain $k_3e^{3x} + k_4e^{2x} + 32x$ Obtain $\frac{1}{3}e^{3x} - 3e^{2x} + 32x$ or equiv Apply limits correctly to expression of form $k_3e^{3x} + k_4e^{2x} + 32x$ Simplify to obtain $32\ln 4 - 24$ or $64\ln 2 - 24$	M1 A1 M1 A1 [4]	For non-zero constants Using limits 0 and their answer from part (i) Or suitably simplified equiv
6	(i)	State or clearly imply $a = \frac{1}{2}$ State or clearly imply $b = \frac{5}{2}$ (Implied by, for example, just $\frac{1}{2}$ and $\frac{5}{2}$ stated in that order)	B1 B1 [2]	$a = \frac{5}{2}$ and $b = \frac{1}{2}$ earn B0 B0 $\sin(-\frac{1}{2}\pi) + \frac{3}{2}$ and $\sin(\frac{1}{2}\pi) + \frac{3}{2}$ earn B0 B0

4723

Mark Scheme

June 2015

Question		Answer	Marks	Guidance	
	(ii) (a)	Carry out relevant calculations using radians Obtain 1.6 and 2.4 or -0.1 and 0.6 Conclude with reference to $1.6 < 1.7$ but $2.4 > 1.8$, or to sign change	M1 A1 A1 [3]	Involving $8\sin^{-1}(x - \frac{3}{2})$ or $8\sin^{-1}(x - \frac{3}{2}) - x$ or equiv; needs two explicit calculations Or equivs Or equiv	May carry out calculations in, for example, $\frac{3}{2} + \sin(\frac{1}{8}x) - x$
	(b)	State or imply $p = \frac{3}{2}$ and $q = \frac{1}{8}$ Obtain correct first iterate Carry out iteration process Obtain at least three correct iterates Conclude with clear statement that root is 1.712	B1 B1 M1 A1 A1 [5]	Implied by presence in iterative formula Having started with value x_1 such that $1.7 \leq x_1 \leq 1.8$; given to at least 4 s.f. Obtaining at least three iterates in all; having started with any non-negative value; implied by an apparently converging sequence of plausible values; all values to at least 4 s.f. Allowing recovery after error Final answer required to exactly 4 significant figures	Answer only can earn no more than the first B1 for values of p and q ; working in degrees can earn no more than the first B1 (for p and q) and M1
7	(i)	Integrate to obtain integral of form $k(7x+1)^{\frac{4}{3}}$ Obtain $\frac{3}{28}(7x+1)^{\frac{4}{3}}$ Apply limits correctly and attempt exact evaluation Obtain $\frac{180}{7}$	*M1 A1 M1 A1 [4]	Any non-zero constant k Or unsimplified equiv Dep *M; substitution of limits to be seen Or exact equiv such as $\frac{720}{28}$ or $25\frac{5}{7}$	
	(ii)	Attempt expression of form $k(y_0 + 4y_1 + y_2)$ Obtain $\frac{4}{3}(\sqrt[3]{8} + 4 \times \sqrt[3]{36} + \sqrt[3]{64})$ Obtain $8 + \frac{16}{3}\sqrt[3]{36}$	M1 A1 A1 [3]	Any constant k ; attempting exact y values corresponding to x values 1, 5, 9 No need for m and n to be stated separately	Missing brackets which are not implied by subsequent calculation and which lead to $ky_0 + 4y_1 + y_2$ earn M0

4723

Mark Scheme

June 2015

Question		Answer	Marks	Guidance	
	(iii)	<p>Equate answers to parts (i) and (ii) and carry out complete correct relevant rearrangement</p> <p>Obtain $\frac{93}{28}$ or $\frac{372}{112}$</p>	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>Provided $\sqrt[3]{36}$ is involved</p> <p>Or equiv of requested form</p>	<p>Correct answer only seen: M1A1</p> <p>answer only seen: if follows correctly from their parts (i) and (ii): M1A0</p>
8	(i)	<p>Obtain 6 or 2 + 4 at any stage for application of f</p> <p>Attempt composition of functions the right way round</p> <p>Obtain $a = \frac{1}{4}$ or $\frac{9}{36}$ or equiv</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p>		
	(ii)	<p>Obtain expression involving e^{y-2} or e^{x-2}</p> <p>Obtain $e^{x-2} - 3$</p> <p>State $x \geq 2 + \ln 3$ or equiv</p>	<p>M1</p> <p>A1</p> <p>B1</p> <p>[3]</p>	<p>Not for >; not for decimal equiv ; using x</p>	
	(iii)	<p><u>Either:</u></p> <p>Apply f once to obtain $2 + N$</p> <p>Apply f to their expression involving N</p> <p>Obtain $2 + \ln(N + 5)$ or $2 + \ln(2 + N + 3)$</p> <p>Attempt solution of equation of form $2 + \ln(pN + q) = \ln(53e^2)$</p> <p>Obtain 48 from correct work</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>Involving manipulation so that value of N is apparent</p>	

4723

Mark Scheme

June 2015

Question		Answer	Marks	Guidance
		<p><u>Or 1:</u> Obtain ff(x) of form $k_1 + \ln[k_2 + \ln(x + 3)]$ Obtain correct $2 + \ln[5 + \ln(x + 3)]$ Substitute for x to obtain $2 + \ln(N + 5)$</p> <p>Attempt solution of equation of form $2 + \ln(pN + q) = \ln(53e^2)$ Obtain 48 from correct work</p> <p><u>Or 2:</u> Apply f^{-1} to obtain $e^{\ln(53e^2)-2} - 3$</p> <p>Attempt simplification of expression involving ln and e Obtain $f(e^N - 3) = 50$ Apply f, or apply f^{-1} to right-hand side Obtain 48</p>	<p>M1 A1 A1</p> <p>M1 A1</p> <p>B1</p> <p>M1 A1 M1 A1 A1 [5]</p>	<p>Or equiv with immediate substitution for x; missing bracket(s) may be implied by subsequent work</p> <p>Involving manipulation so that value of N is apparent</p>
9	(i)	<p>Use at least one addition formula accurately</p> <p>Obtain $\cos \theta$ State $\cos 4\theta = 2\cos^2 2\theta - 1$ Attempt correct use of relevant formulae to express in terms of $\cos \theta$ Obtain correct unsimplified expression in terms of $\cos \theta$ only Simplify to confirm $8\cos^4 \theta - 3$</p>	<p>M1 A1 B1 M1 A1 A1 [6]</p>	<p>Without substituting values for $\cos 30^\circ$, etc. yet AG; necessary detail needed Or $\cos 4\theta = \cos^2 2\theta - \sin^2 2\theta$ Or in terms of $\cos \theta$ and $\sin \theta$ e.g. $2(2c^2 - 1)^2 - 1 + 4(2c^2 - 1)$ AG; necessary detail needed</p>

4723

Mark Scheme

June 2015

Question		Answer	Marks	Guidance	
	(ii) (a)	Obtain $\frac{1}{12}$ Substitute 0 for $\cos \theta$ in correct expression Obtain $\frac{1}{4}$	B1 M1 A1 [3]	No need to specify greatest and least	
	(b)	State or imply $8\cos^4(3\alpha) - 3 = 1$ Attempt correct method to obtain at least one value of α Obtain 10.9 Obtain 49.1	B1 M1 A1 A1 [4]	Or $2\cos^2 6\alpha + 4\cos 6\alpha - 2 = 0$ Allow for equation of form $\cos^4(3\alpha) = k$ where $0 < k < 1$ or for three-term quadratic equation in $\cos 6\alpha$ Or greater accuracy 10.921... Or greater accuracy 49.078...; and no others between 0 and 60	Answer(s) only: 0/4

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