Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Statistics S4 (6686/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\surd$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper or ag- answer given
   - $\sqsubset$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

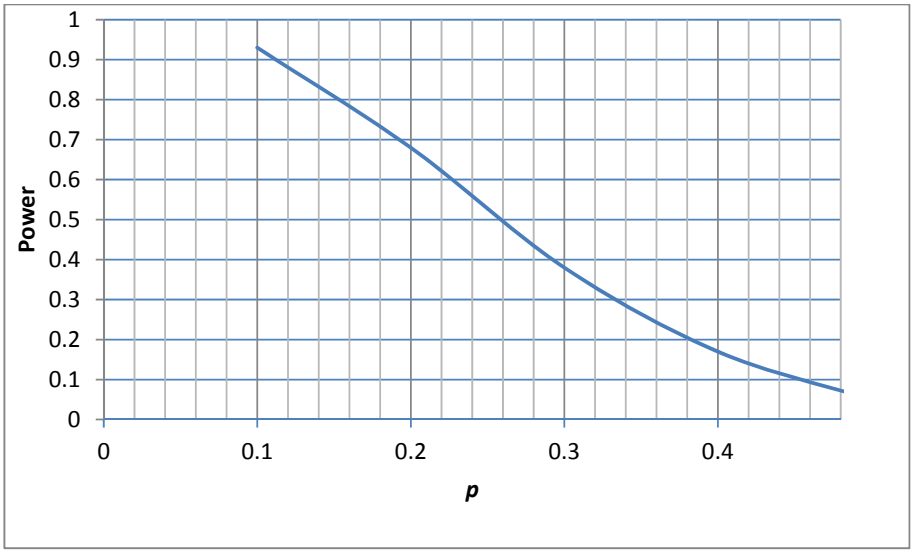
7. Ignore wrong working or incorrect statements following a correct answer.

Question Number	Scheme	Marks
1.	$H_0 : \mu = 100 \quad H_1 : \mu < 100$	B1
	$t = \frac{\lvert\bar{x}-\mu\rvert}{s/\sqrt{n}} = \frac{\lvert 92.875-100\rvert}{8.3055/\sqrt{8}} = 2.4264\ldots$ or $\frac{c-100}{8.3055/\sqrt{8}} = -1.895 \therefore$ CR $c < 94.435$	M1A1
	$t_7(5\%) = \pm\ 1.895$	B1
	There is evidence to reject H_0. Malcolm's belief is supported or there is evidence that the amount of oil placed in bottles is less than 100mm	A1ft (5)
	Notes	
	B1 both hypotheses M1 either $\frac{\lvert 92.875-100\rvert}{8.3055/\sqrt{8}}$ or $p = 0.0228$ or $\frac{c-100}{8.3055/\sqrt{8}} = -(\text{a } t \text{ value})$ A1 awrt 2.43 or awrt 94.4 or awrt 0.0228 B1 ± 1.895 or $0.0228 < 0.05$ (must have correct comparison for hypotheses) A1ft Do Not allow contradictions	

Question Number	Scheme	Marks
2(a)(i)	Type I - H_0 rejected when it is true	B1
(ii)	Type II - H_0 is accepted when it is false	B1
		(2)
(b)	$P(X < c \mid \lambda = 6) \approx 0.05$	M1
	$P(X \leq 2) = 0.0620$	
	$P(X \leq 1) = 0.0174$	
	Critical region = $X \leq 2$	A1
	P(Type 1 error) = $P(X \leq 2 \mid \lambda = 6) = 0.062$	A1cao
		(3)
(c)	P(Type 2 error) = $P(X \geq 3 \mid \lambda = 4)$	M1
	$= 1 - 0.2381$	
	$= 0.7619$	A1
		(2)
	Notes	
(b)	M1 use of Po(6)	
	A1 correct CR. May be implied by correct probability. Alow if written as part of a 2 tailed CR A1 awrt 0.062	
(c)	M1 using Po(4) and 1 - $P(X \leq 2)$,ft their CR in (b) if one tail A1awrt 0.762	

Question Number	Scheme	Marks
3(a) (i)	$\bar{x} = \frac{181}{9} = 20.111\ldots$	B1
	$s_x^2 = \left(\frac{3913 - 9\times\bar{x}^2}{8} =\right) 34.1111 \quad (s_x = 5.84)$	B1
	$t_8(0.025)$ cv = 2.306	B1
	95% CI for μ is $= 20.111 \pm 2.306 \times \frac{5.84}{\sqrt{9}}$	M1
	= (15.6, 24.6) awrt **(15.6, 24.6)**	A1, A1
(ii)	$\chi_8^2(0.025) = 2.18(0)$, $\chi_8^2(0.975) = 17.535$	B1B1
	95% CI for σ^2 is given by $2.180 < \frac{8s_x^2}{\sigma^2} < 17.535$	M1
	So 95% CI for σ^2 is = **awrt (15.6, 125)**	A1
		(10)
(b)	Require $P(X < 16) = P\left(Z < \frac{16-\mu}{\sigma}\right)$ to be as small as possible OR	
	$\frac{16-\mu}{\sigma}$ to be as large as possible but negative; **imply lowest σ and largest μ.**	M1
	$P(Z < \frac{16-24.6}{\sqrt{15.6}})$; = 1 – 0.9854 = **0.0146 or 0.0147**	M1A1ft;A1
		(4)
	Notes	
(a)(i)	1st M1 ' their $\bar{x}$ ' $\pm$ *t value* $\times \frac{\text{'their } s\text{'}}{\sqrt{9}}$	
	1st A1 awrt 15.6	
	2nd A1 awrt 24.6	
(ii)	2nd M1 $\chi^2 < \frac{8s^2}{\sigma^2} < \chi^2$	
	A1 awrt 15.6 and 125	
(b)	M1 Identify must use **lowest σ and largest μ**	
	M1 standardising and finding correct area use either limit for μ and σ	
	A1 ft their **lowest σ and largest μ**	
	A1 awrt 0.0146 or 0.0147	

Question Number	Scheme	Marks
4(a)	The <u>differences</u> in the mean number of hours sleep are <u>normally</u> distributed	B1 (1)
(b)	Differences are 0.8, 0.7, -0.3, 1.2, 0.7, 2.9, 1.3, 0.8	M1
	$\bar{d} = = \frac{8.1}{8} = 1.0125$	M1
	$s_d = \sqrt{\frac{13.89 - 8 \times 1.0125^2}{7}} = 0.901\ldots$ both $\bar{d}$ and s	M1
	$H_0 : \mu_D = 1/6 \quad H_1 : \mu_D > 1/6$	B1
	$t = \frac{1.0125 - 1/6}{0.901/\sqrt{8}} =$ awrt 2.65 or $\frac{c - 1/6}{0.901/\sqrt{8}} = 2.988 \therefore$ CR $c >$ awrt 1.12	M1A1
	$t_7(1\%) = 2.998$ (or prob. = awrt 0.0164)	B1
	There is insufficient evidence to suggest the <u>drug</u> <u>increases</u> the mean number of hours <u>slept</u> by <u>more</u> than <u>10</u> minutes.	A1ft (8)
	Notes	
(a)	B1 for a comment that mentions "differences" and "normal" distribution	
(b)	1st M1 for attempting the *d*s	
	2nd M1 for attempting $\bar{d}$	
	1st M1 for s_d or s_d^2	
	1st B1 for both hypotheses correct in terms of μ or μ_d .(allow a defined symbol) Do not allow 10 instead of 1/6(awrt 0.167) unless working in minutes throughout	
	3rd M1 for attempting the correct test statistic $\frac{\bar{d} - 1/6}{s_d/\sqrt{8}}$ or p = awrt 0.016 or $\frac{c - 1/6}{0.901/\sqrt{8}} = t$ value	
	2nd A1 awrt 2.65 /2.655 or awrt 1.12 or awrt 0.016	
	2nd B1 2.988 or 0.0164	
	3rd A1ft for a correct comment in context based on their test statistic and their cv. Do not allow contradictions.	

Question Number	Scheme	Marks
5 (a)	$X \sim B(10, 0.5)$ Size = P(reject $H_0 \mid p = 0.5$) = P($X < 3 \mid p = 0.5$) = 0.0547	B1 (1)
(b)	Power = P($X = 2$) + P($X = 1$) + P($X = 0$) $= 45p^2(1 - p)^8 + 10p(1 - p)^9 + (1 - p)^{10}$ $= (1 - p)^8(45p^2 + 10p(1 - p) + (1 - p)^2)$ $= (1 - p)^8(36p^2 + 8p + 1)$	M1 A1 A1cso (3)
(c)	$r = 0.68$ $s = 0.17$	B1 B1 (2)
(d)		B1 points B1 curve (2)
(e)	P(Type II error) $\leq$ 0.4 1 – power ≤ 0.4 Power ≥ 0.6 $p <$ 0.23	M1 A1 A1 (3)
	Notes	
(b) (c) (e)	M1 for a correct expression/selection of probabilities A1 for a fully correct expression SC B1 B0 both correct but not given to 2 dp M1 may be implied by Power ≥ 0.6 or correct value or by correct answer A1 may be implied by correct answer A1 allow number between 0.22 and 0.23 inclusive and either < or $\leq$	

Question Number	Scheme	Marks
6(a)	It is the probability distribution of *T*.	B1 (1)
(b)	An estimator is biased if E(T) $\neq \theta$	B1 (1)
(c)	E($\hat{\mu}_1$) = $\frac{E(X_3)+E(X_5)+E(X_7)}{3} = \frac{\mu+\mu+\mu}{3} = \mu$ ∴ Bias = 0	M1A1
	E($\hat{\mu}_2$) = $\frac{5E(X_1)+2E(X_2)+E(X_9)}{6} = \frac{5\mu+2\mu+\mu}{6} = \frac{4\mu}{3}$ ∴ Bias = $\frac{\mu}{3}$	A1
	E($\hat{\mu}_3$) = $\frac{3E(X_{10})-E(X_1)}{3} = \frac{3\mu-\mu}{3} = \frac{2\mu}{3}$ ∴ Bias = $-\frac{\mu}{3}$	A1 (4)
(d)	Var ($\hat{\mu}_1$) = $\frac{1}{9}(\text{Var}(X_3) + \text{Var}(X_5) + \text{Var}(X_7))$	M1
	= $\frac{1}{9}(\sigma^2 + \sigma^2 + \sigma^2)$	
	= $\frac{\sigma^2}{3}$	A1
	Var ($\hat{\mu}_2$) = $\frac{1}{36}(25\text{Var}(X_1) + 4\text{Var}(X_2) + \text{Var}(X_9))$	M1
	= $\frac{1}{36}(25\sigma^2 + 4\sigma^2 + \sigma^2)$	
	= $\frac{5}{6}\sigma^2$	A1
	Var($\hat{\mu}_3$) = $\frac{1}{9}(9\text{Var}(X_{10}) + \text{Var}(X_1))$	M1
	= $\frac{1}{9}(9\sigma^2 + \sigma^2)$	
	= $\frac{10\sigma^2}{9}$	A1 (6)
(e)(i)	$\hat{\mu}_1$ is the best estimator. It has no bias	B1
(ii)	It has <u>same magnitude of bias as $\hat{\mu}_2$</u> but it has the <u>largest variance</u> $\hat{\mu}_3$ is the worst estimator.	B1ft B1dcao (3)
	Notes	
(c)	M1 finding E($\hat{\mu}$) A1 bias 0 A1 $\pm\frac{\mu}{3}$ A1 $\pm\frac{\mu}{3}$	
(d)	For method marks allow an incorrect variance, M1 squaring 9, M1 Squaring 5 and 2, M1 adding variances. Do not penalise same mistake twice.	
(e)(ii)	Must have idea that its bias is the same as another ($\hat{\mu}_2$) and state it has largest variance for first B1 . ft their values of Var. Second B1 dependent on first B1cao SC $\hat{\mu}_3$ because <u>largest variance</u> B1 B0	

Question Number	Scheme	Marks
7(a)	The variance of the two group's marks must be the same.	B1
	$H_0 : \sigma_1^2 = \sigma_2^2 \quad H_1 : \sigma_1^2 \neq \sigma_2^2$	B1
	$s_1^2 = 16.25$	B1
	$\left(F_{8,6} =\right)\frac{16.25}{12.9} = (1.2597...) \qquad \left(\frac{1}{F_{8,6}} = \frac{12.9}{16.25} = 0.7938...\right)$	M1A1
	$F_{8,6}$ 5% (two-tail) cv = **4.15** (0.241) (or prob. = awrt 0.39)	B1
	Not significant so can accept the assumption that variances are equal.	A1
		(7)
(b)	$H_0 : \mu_1 = \mu_2 \quad H_1 : \mu_1 \neq \mu_2$	B1
	$s_p^2 = \frac{8\times16.25 + 6\times12.9}{14}, = 14.814...$ or $s_p = 3.8489...$	M1
	$\left(t_{14} =\right)(\pm)\frac{30.33 - 31.29}{s_p\sqrt{\frac{1}{9}+\frac{1}{7}}} = (\pm)0.494927..$ = awrt **<u>0.49</u>**	B1 M1A1
	$t_{14}(0.025)$ two-tail cv = **2.145**	B1
	There is insufficient evidence to reject H_0. There is no evidence of a significant difference between the **<u>mean marks</u>** of the two groups	A1
		(7)
	Notes	
(a)	2nd B1allow σ or σ^2 3rd B1 allow awrt 16.3 or s_1 = awrt 4.03 M1 for use of the correct test statistic 5th B1 allow "assumption is correct"	
(b)	1st M1 for attempting s_p or s_p^2 1st B1 for 30.33 2nd M1 for use of a correct test statistic 2nd A1 for awrt 0.49 (accept $\pm$) or 0.495 2nd B1 for 2.145 (allow $\pm$ 1.761 for one-tailed H_1)	