



**GCE**

# **Mathematics**

Advanced GCE

Unit **4734**: Probability and Statistics 3

## **Mark Scheme for January 2011**

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1(i)	Est $\mu$ = sample mean = 5.25	B1 1	
(ii)	Use (i) $\pm zSD$ SD = $0.19/\sqrt{5}$ $z = 1.96$ $5.083 < \mu < 5.417$	M1 B1 B1 A1 4 [5]	With $\sqrt{5}$ seen  Rounding to 5.08, 5.42
2	Use $G - M \sim N(-6.23, \sigma^2)$ $\sigma^2 = 6.87^2 + 10.25^2$ $z = (16.23)/\sigma$ $= 1.315$ Probability = 0.0942 or 0.0943	M1 A1 M1 A1 A1 [5]	<b>Or G-M-10~ N(-16.23, <math>\sigma^2</math>)</b>  Accept 0.094
3(i)	$\int_0^2 ae^{-t} dt + \int_2^\infty ae^{-t/2} dt = 1$ [ $ae^{-t}$ ] + [ $-2ae^{-t/2}$ ] $\Rightarrow a = 1/4e$ AG	M1  A1 A1 3	Properly obtained
(ii)	$\int_{q_3}^\infty \frac{1}{4} e^{-t/2} dt = \frac{1}{4}$ [ $-1/2 e^{-t/2}$ ] $-1/2 q_3 + 1 = -\ln 2$ $\Rightarrow q_3 = 2(\ln 2 + 1)$ or 3.39	M1  B1 M1 A1 4 [7]	OR $\int_0^2 \frac{1}{4} dt + \int_2^q \frac{1}{4} e^{-t/2} dt = \frac{3}{4}$ AEF For taking logs (not ln(-)) AEF
4	$\hat{p}_2 = 106/143, \hat{p}_1 = 61/107$ $= 0.7413 \quad = 0.5701$ Pooled est $p = 167/250$ Variance est = $(167/250)(83/250)(143^{-1} + 107^{-1})$ Test statistic $z = (0.7413 - 0.5701)/SD$ $= 2.84(35)$ Smallest significance level = 0.23% SR: No pe, B1B0B0M1A1(2.84)M1A1 Max 5/7	B1  B1 B1 M1 A1 M1 A1 $\sqrt{}$ [7]	For both  Only if used  ART 0.22 or 0.23 Accept 0.0023 $\sqrt{z}$ M1A0 if 0.25%
5(i)	$s^2 = 0.2 \times 0.8/90$ $p_s \pm z s$ $z = 1.645$ $0.1306 < p_y < 0.2693$	B1 M1 B1 A1 4	OR /89  Art (0.131, 0.269)
(ii)	$0.7306 < p_p < 0.8694$	B1ft 1	ft (i) Art (0.731, <b>0.869</b> )
(iii)	If a large number of such intervals were calculated from independent samples, approximately 90% of all such intervals would contain $p$	B2 2	Or: Probability that such an interval contains $p$ is 0.9 B1 for right idea
(iv)	(0.131, 0.269) encloses 0.25 so Mendel's theory is supported	M1 A1 $\sqrt{2}$ [9]	Or equivalent Ft CI(i)

<p>6(i)</p>	<p> <math>G(y) = P(Y \leq y)</math>  <math>= P(X \geq 1/y)</math>  <math>= 1 - F(1/y)</math>  <math>= (2y - 1)/(y+1)</math>                      For <math>\frac{1}{2} \leq 1/y \leq 2 \Rightarrow \frac{1}{2} \leq y \leq 2</math>                      X and Y have identical distributions                       SR: CDF not used.                      y decreases with x                      Use <math>g(y) = f(x(y) dx/dy )</math>  <math>f(x) = 3/(x+1)^2</math>  <math> dx/dy  = 1/y^2</math>  <math>g(y) = [3/(y^{-1}+1)^2][1/y^2] = 3/(y+1)^2</math>; for <math>\frac{1}{2} \leq y \leq 2</math>                      So X and Y have identical distributions                 </p>	<p>                     M1                      A1                      M1                      A1                      B1                      B1 <b>6</b>                       M1                      M1A1                      B1                      M1A1B1                      B1 <b>8</b> </p>	<p>Seen</p>
<p>(ii)</p>	<p> <math>f(x) = F'(x) = 3/(x+1)^2, \frac{1}{2} \leq x \leq 2</math>  <math>E(X+1) = \int_{\frac{1}{2}}^2 \frac{3}{x+1} dx</math>  <math>= 3 \ln 2</math> (2.08)   <math>E(1/X) = E(X)</math>  <math>= 3 \ln 2 - 1</math> (1.08)                 </p>	<p>                     M1A1                      M1                      A1                      M1                      A1 <b>6</b>                      [12]                 </p>	<p>Must have range of x AEF Not if awarded in (i)</p>
<p>7(i)</p>	<p>In a 2x2 contingency table</p>	<p>B1 <b>1</b></p>	<p>Or equivalent Accept df=1</p>
<p>(ii)</p>	<p> <math>H_0</math>: Vaccine type and outcome are independent  <math>H_1</math>: They are not independent                      E-values: 10.81 12.19                      318.19 358.81  <math>\chi^2 = 7.69^2(10.81^{-1} + 12.19^{-1} + 318.19^{-1} + 358.81^{-1})</math>  <math>= 10.67</math>  <math>CV = 6.635</math>  <math>10.67 &gt; CV</math>                      Reject <math>H_0</math>, there is sufficient evidence at the 1% significance level that the outcome of the test depends on the vaccine used                       The results is significant at a level less than 1/2 %, so the evidence is very strong                 </p>	<p>                     B1M*dep                      M1                      A1                      M1                      M1                      A1                      B1                      M1                       A1√                      dep*M                      A1√ <b>10</b>                      [11]                 </p>	<p>                     Accept omission of <math>H_1</math>                       1 correct E value                      Accept 1 dp                      1 correct <math>\chi^2</math> value ft E values                      Using Yates' correctly                      Accept 10.7                       √ 10.67                       Sensible comment.√ 10.67                 </p>

8(i)	When independent samples are drawn from populations having a common variance	B1 1	For common variance
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(ii)	<p>(a) Lung capacities should have normal distributions with a common variance</p> <p>(b) <math>H_0: \mu_1 = \mu_2, H_1: \mu_1 &gt; \mu_2</math>  <math>s_1^2 = \frac{1}{19}(90.43 - 42.4^2 / 20)</math>  <math>s_2^2 = \frac{1}{21}(82.93 - 42.5^2 / 22)</math>  <math>\bar{x}_1 = 2.12 \quad \bar{x}_2 = 1.93(2)</math>  <math>PEV, s^2 = (19s_1^2 + 21s_2^2) / (20 + 22 - 2)</math>  <math>= 0.03424(3)</math></p> <p>Test statistic = <math>\frac{2.12 - 1.932}{\sqrt{s^2(20^{-1} + 22^{-1})}}</math>  <math>= 3.29(15)</math></p> <p>CV = 2.423                  TS &gt; CV</p> <p>There is sufficient evidence at the 1% SL that the mean lung capacity is greater for children whose parents do not smoke than for children whose parents do smoke                  SR1: For 2-tail test Lose 1<sup>st</sup> B1 and last 3. Max 8/11                  SR2: If <math>s^2 = s_1^2/20 + s_2^2/22</math>, B1M1A0A0M1A0A1(3.32)                  B1M1A1 Max 8/11</p> <p>(c) <math>t = 2.704</math>  <math>0.1882... \pm ts(20^{-1} + 22^{-1})^{1/2}</math>                  (0.0336, 0.3423)</p>	<p>B1 1</p> <p>B1</p> <p>M1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1A1</p> <p>A1√</p> <p>B1</p> <p>M1</p> <p>A1 11</p> <p>B1</p> <p>M1</p> <p>A1 3</p> <p>[16]</p>	<p>Normal distributions required In context here</p> <p>Or equivalent</p> <p>For 1 correct <math>s^2</math></p> <p>For both</p> <p>ft <math>s^2</math></p> <p>Accept answer rounding to 3.3                  If z used the B0M0A0                  Compare with CV</p> <p>Or equivalent, in context</p> <p>Accept 0.19                  (0.033-0.036, 0.342-0.346)</p>

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