

Mark Scheme (Results)

January 2011

GCE

GCE Further Pure Mathematics FP1 (6667) Paper 1

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General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod - benefit of doubt
- ft - follow through
- the symbol \checkmark will be used for correct ft
- cao - correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw - ignore subsequent working
- awrt - answers which round to
- SC: special case
- oe - or equivalent (and appropriate)
- dep - dependent
- indep - independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- \square The second mark is dependent on gaining the first mark

January 2011
Further Pure Mathematics FP1 6667
Mark Scheme

Question Number	Scheme	Marks
1.	$z = 5 - 3i, w = 2 + 2i$	
(a)	$z^2 = (5 - 3i)(5 - 3i)$ $= 25 - 15i - 15i + 9i^2$ $= 25 - 15i - 15i - 9$ $= 16 - 30i$	<p style="text-align: right;">An attempt to multiply out the brackets to give four terms (or four terms implied). zw is M0</p> <p style="text-align: right;">M1</p> <p style="text-align: right;">16 - 30i A1</p> <p style="text-align: right;">Answer only 2/2 (2)</p>
(b)	$\frac{z}{w} = \frac{(5 - 3i)}{(2 + 2i)}$ $= \frac{(5 - 3i)}{(2 + 2i)} \times \frac{(2 - 2i)}{(2 - 2i)}$ $= \frac{10 - 10i - 6i - 6}{4 + 4}$ $= \frac{4 - 16i}{8}$ $= \frac{1}{2} - 2i$	<p style="text-align: right;">Multiplies $\frac{z}{w}$ by $\frac{(2 - 2i)}{(2 - 2i)}$</p> <p style="text-align: right;">M1</p> <p style="text-align: right;">Simplifies realising that a real number is needed on the denominator and applies $i^2 = -1$ on their numerator expression and denominator expression.</p> <p style="text-align: right;">M1</p> <p style="text-align: right;">$\frac{1}{2} - 2i$ or $a = \frac{1}{2}$ and $b = -2$ or equivalent A1</p> <p style="text-align: right;">Answer as a single fraction A0</p> <p style="text-align: right;">(3) [5]</p>

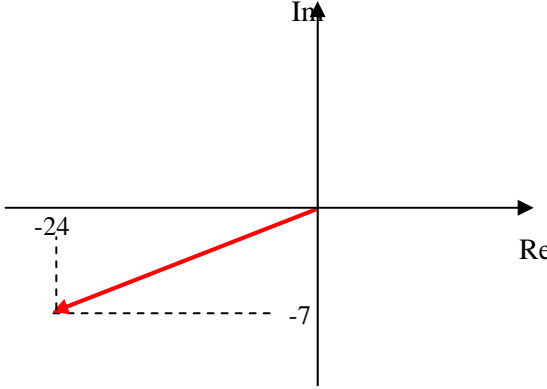
Question Number	Scheme	Marks
2. (a)	$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$ $\mathbf{AB} = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$ $= \begin{pmatrix} 2(-3) + 0(5) & 2(-1) + 0(2) \\ 5(-3) + 3(5) & 5(-1) + 3(2) \end{pmatrix}$ $= \begin{pmatrix} -6 & -2 \\ 0 & 1 \end{pmatrix}$	<p>A correct method to multiply out two matrices. Can be implied by two out of four correct elements. M1</p> <p>Any three elements correct A1</p> <p>Correct answer A1</p> <p>Correct answer only 3/3 (3)</p>
(b)	Reflection; about the y -axis.	<p><u>Reflection</u> <u>y-axis</u> (or $x = 0$.) M1</p> <p>A1</p> <p>(2)</p>
(c)	$\mathbf{C}^{100} = \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	<p>$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ or \mathbf{I} B1</p> <p>(1)</p> <p>[6]</p>

Question Number	Scheme	Marks
3. (a)	$f(x) = 5x^2 - 4x^{\frac{3}{2}} - 6, \quad x \geq 0$ $f(1.6) = -1.29543081\dots$ $f(1.8) = 0.5401863372\dots$ $\frac{\alpha - 1.6}{\text{"1.29543081..."}} = \frac{1.8 - \alpha}{\text{"0.5401863372..."}}$ $\alpha = 1.6 + \left(\frac{\text{"1.29543081..."}}{\text{"0.5401863372..." + "1.29543081..."}} \right) 0.2$ $= 1.741143899\dots$	awrt -1.30 B1 awrt 0.54 B1 Correct linear interpolation method with signs correct. Can be implied by working below. M1 awrt 1.741 A1 Correct answer seen 4/4 (4)
(b)	$f'(x) = 10x - 6x^{\frac{1}{2}}$	At least one of $\pm ax$ or $\pm bx^{\frac{1}{2}}$ correct. M1 Correct differentiation. A1 (2)
(c)	$f(1.7) = -0.4161152711\dots$ $f'(1.7) = 9.176957114\dots$ $\alpha_2 = 1.7 - \left(\frac{\text{"-0.4161152711..."}}{\text{"9.176957114..."}} \right)$ $= 1.745343491\dots$ $= 1.745 \text{ (3dp)}$	$f(1.7) = \text{awrt } -0.42$ B1 $f'(1.7) = \text{awrt } 9.18$ B1 Correct application of Newton-Raphson formula using their values. M1 1.745 A1 cao Correct answer seen 4/4 (4) [10]

Question Number	Scheme	Marks
4. (a)	$z^2 + pz + q = 0, z_1 = 2 - 4i$ $z_2 = 2 + 4i$	$2 + 4i$ B1 (1)
(b)	$(z - 2 + 4i)(z - 2 - 4i) = 0$ $\Rightarrow z^2 - 2z - 4iz - 2z + 4 - 8i + 4iz - 8i + 16 = 0$ $\Rightarrow z^2 - 4z + 20 = 0$	An attempt to multiply out brackets of two complex factors and no i^2 . Any one of $p = -4, q = 20$. Both $p = -4, q = 20$. $\Rightarrow z^2 - 4z + 20 = 0$ only 3/3 M1 A1 A1 (3) [4]

Question Number	Scheme	Marks
5	<p>(a) $\sum_{r=1}^n r(r+1)(r+5)$</p> <p>$= \sum_{r=1}^n r^3 + 6r^2 + 5r$</p> <p>$= \frac{1}{4}n^2(n+1)^2 + 6 \cdot \frac{1}{6}n(n+1)(2n+1) + 5 \cdot \frac{1}{2}n(n+1)$</p> <hr/> <p>$= \frac{1}{4}n^2(n+1)^2 + n(n+1)(2n+1) + \frac{5}{2}n(n+1)$</p> <p>$= \frac{1}{4}n(n+1)(n(n+1) + 4(2n+1) + 10)$</p> <p>$= \frac{1}{4}n(n+1)(n^2 + n + 8n + 4 + 10)$</p> <p>$= \frac{1}{4}n(n+1)(n^2 + 9n + 14)$</p>	<p>Multiplying out brackets and an attempt to use at least one of the standard formulae correctly. M1</p> <p><u>Correct expression.</u> A1</p> <p>Factorising out at least $n(n+1)$ dM1</p> <p>Correct 3 term quadratic factor A1</p>
	<p>$= \frac{1}{4}n(n+1)(n+2)(n+7) *$</p>	<p>Correct proof. No errors seen. A1</p> <p>(5)</p>
	<p>(b) $S_n = \sum_{r=20}^{50} r(r+1)(r+5)$</p> <p>$= S_{50} - S_{19}$</p> <p>$= \frac{1}{4}(50)(51)(52)(57) - \frac{1}{4}(19)(20)(21)(26)$</p> <p>$= 1889550 - 51870$</p> <p>$= 1837680$</p>	<p>Use of $S_{50} - S_{19}$ M1</p> <p>1837680 A1</p> <p>Correct answer only 2/2</p> <p>(2) [7]</p>

Question Number	Scheme	Marks
6.	$C: y^2 = 36x \Rightarrow a = \frac{36}{4} = 9$	
(a)	$S(9, 0)$	$(9, 0)$ B1 (1)
(b)	$x + 9 = 0$ or $x = -9$	$x + 9 = 0$ or $x = -9$ or ft using their a from part (a). B1 $\sqrt{\quad}$ (1)
(c)	$PS = 25 \Rightarrow \underline{QP = 25}$	Either 25 by itself or $PQ = 25$. Do not award if just $PS = 25$ is seen. B1 (1)
(d)	x -coordinate of $P \Rightarrow x = 25 - 9 = 16$ $y^2 = 36(16)$ $\underline{y} = \sqrt{576} = \underline{24}$ Therefore $P(16, 24)$	$x = 16$ B1 $\sqrt{\quad}$ Substitutes their x -coordinate into equation of C . M1 $\underline{y} = 24$ A1 (3)
(e)	Area $OSPQ = \frac{1}{2}(9 + 25)24$ $= \underline{408} \text{ (units)}^2$	$\frac{1}{2}(\text{their } a + 25)(\text{their } y)$ or rectangle and 2 distinct triangles, correct for their values. 408 M1 A1 (2) [8]

Question Number	Scheme	Marks
7. (a)	 <p data-bbox="1002 562 1431 636">Correct quadrant with $(-24, -7)$ indicated.</p>	<p data-bbox="1453 577 1492 611">B1</p> <p data-bbox="1525 835 1562 869">(1)</p>
(b)	$\arg z = -\pi + \tan^{-1}\left(\frac{7}{24}\right)$ $= -2.857798544\dots = -2.86 \text{ (2 dp)}$	$\tan^{-1}\left(\frac{7}{24}\right) \text{ or } \tan^{-1}\left(\frac{24}{7}\right)$ <p data-bbox="1453 916 1492 949">M1</p> <p data-bbox="1453 996 1492 1030">A1</p> <p data-bbox="1525 1032 1562 1066">(2)</p>
(c)	$ w = 4, \arg w = \frac{5\pi}{6} \Rightarrow r = 4, \theta = \frac{5\pi}{6}$ $w = r \cos \theta + i r \sin \theta$ $w = 4 \cos\left(\frac{5\pi}{6}\right) + 4i \sin\left(\frac{5\pi}{6}\right)$ $= 4\left(\frac{-\sqrt{3}}{2}\right) + 4i\left(\frac{1}{2}\right)$ $= -2\sqrt{3} + 2i$ $a = -2\sqrt{3}, b = 2$	<p data-bbox="991 1279 1431 1352">Attempt to apply $r \cos \theta + i r \sin \theta$. Correct expression for w.</p> <p data-bbox="1453 1279 1492 1312">M1</p> <p data-bbox="1453 1319 1492 1352">A1</p> <p data-bbox="975 1420 1431 1453">either $-2\sqrt{3} + 2i$ or awrt $-3.5 + 2i$</p> <p data-bbox="1453 1420 1492 1453">A1</p> <p data-bbox="1525 1460 1562 1494">(3)</p>
(d)	$ z = \sqrt{(-24)^2 + (-7)^2} = \underline{25}$ $ zw = z \times w = (25)(4)$ $= \underline{100}$	$\frac{ z = 25 \text{ or } zw = (48\sqrt{3} + 14) + (14\sqrt{3} - 48)i \text{ or } \text{awrt } 97.1-23.8i$ <p data-bbox="1453 1637 1492 1671">B1</p> <p data-bbox="1142 1771 1431 1805">Applies $z \times w$ or zw</p> <p data-bbox="1453 1771 1492 1805">M1</p> <p data-bbox="1378 1854 1431 1888"><u>100</u></p> <p data-bbox="1453 1854 1492 1888">A1</p> <p data-bbox="1525 1895 1562 1928">(3)</p> <p data-bbox="1525 1935 1562 1968">[9]</p>

Question Number	Scheme	Marks
8.	$\mathbf{A} = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$	
(a)	$\det \mathbf{A} = 2(3) - (-1)(-2) = 6 - 2 = \underline{4}$	$\underline{4}$ B1 (1)
(b)	$\mathbf{A}^{-1} = \frac{1}{4} \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$	$\frac{1}{\det \mathbf{A}} \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$ $\frac{1}{4} \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$ M1 A1 (2)
(c)	$\text{Area}(R) = \frac{72}{4} = \underline{18} \text{ (units)}^2$	$\frac{72}{\text{their det } \mathbf{A}} \text{ or } 72 \text{ (their det } \mathbf{A})$ $\underline{18} \text{ or ft answer.}$ M1 A1 $\sqrt{\quad}$ (2)
(d)	$\mathbf{AR} = \mathbf{S} \Rightarrow \mathbf{A}^{-1} \mathbf{AR} = \mathbf{A}^{-1} \mathbf{S} \Rightarrow \mathbf{R} = \mathbf{A}^{-1} \mathbf{S}$ $\mathbf{R} = \frac{1}{4} \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 0 & 8 & 12 \\ 4 & 16 & 4 \end{pmatrix}$ $= \frac{1}{4} \begin{pmatrix} 8 & 56 & 44 \\ 8 & 40 & 20 \end{pmatrix}$ $= \begin{pmatrix} 2 & 14 & 11 \\ 2 & 10 & 5 \end{pmatrix}$ <p>Vertices are (2, 2), (14, 10) and (11, 5).</p>	<p>At least one attempt to apply \mathbf{A}^{-1} by any of the three vertices in \mathbf{S}.</p> <p>At least one correct column o.e.</p> <p>At least two correct columns o.e.</p> <p>All three coordinates correct.</p> M1 A1 $\sqrt{\quad}$ A1 A1 (4) [9]

Question Number	Scheme	Marks
9.	<p>$u_{n+1} = 4u_n + 2$, $u_1 = 2$ and $u_n = \frac{2}{3}(4^n - 1)$</p> <p>$n = 1$; $u_1 = \frac{2}{3}(4^1 - 1) = \frac{2}{3}(3) = 2$</p> <p>So u_n is true when $n = 1$.</p> <p>Assume that for $n = k$ that, $u_k = \frac{2}{3}(4^k - 1)$ is true for $k \in \mathbb{Z}^+$.</p> <p>Then $u_{k+1} = 4u_k + 2$</p> $= 4\left(\frac{2}{3}(4^k - 1)\right) + 2$ $= \frac{8}{3}(4)^k - \frac{8}{3} + 2$ $= \frac{2}{3}(4)(4)^k - \frac{2}{3}$ $= \frac{2}{3}4^{k+1} - \frac{2}{3}$ $= \frac{2}{3}(4^{k+1} - 1)$ <p>Therefore, the general statement, $u_n = \frac{2}{3}(4^n - 1)$ is true when $n = k + 1$. (As u_n is true for $n = 1$,) then u_n is true for all positive integers by mathematical induction</p>	<p>Check that $u_n = \frac{2}{3}(4^n - 1)$ yields 2 when $n = 1$.</p> <p>B1</p> <p>Substituting $u_k = \frac{2}{3}(4^k - 1)$ into $u_{n+1} = 4u_n + 2$.</p> <p>M1</p> <p>An attempt to multiply out the brackets by 4 or $\frac{8}{3}$</p> <p>M1</p> <p>$\frac{2}{3}(4^{k+1} - 1)$</p> <p>A1</p> <p>Require 'True when $n=1$', 'Assume true when $n=k$' and 'True when $n = k + 1$' then true for all n o.e.</p> <p>A1</p> <p>(5) [5]</p>

Question Number	Scheme	Marks
10. (a)	$xy = 36$ at $(6t, \frac{6}{t})$. $y = \frac{36}{x} = 36x^{-1} \Rightarrow \frac{dy}{dx} = -36x^{-2} = -\frac{36}{x^2}$ At $(6t, \frac{6}{t})$, $\frac{dy}{dx} = -\frac{36}{(6t)^2}$ So, $m_T = \frac{dy}{dx} = -\frac{1}{t^2}$ T: $y - \frac{6}{t} = -\frac{1}{t^2}(x - 6t)$ T: $y - \frac{6}{t} = -\frac{1}{t^2}x + \frac{6}{t}$ T: $y = -\frac{1}{t^2}x + \frac{6}{t} + \frac{6}{t}$ T: $y = -\frac{1}{t^2}x + \frac{12}{t}$ *	An attempt at $\frac{dy}{dx}$. or $\frac{dy}{dt}$ and $\frac{dx}{dt}$ M1 An attempt at $\frac{dy}{dx}$ in terms of t M1 $\frac{dy}{dx} = -\frac{1}{t^2}$ * A1 Must see working to award here Applies $y - \frac{6}{t} = \text{their } m_T(x - 6t)$ M1 Correct solution . A1 cso (5)
(b)	Both T meet at $(-9, 12)$ gives $12 = -\frac{1}{t^2}(-9) + \frac{12}{t}$ $12 = \frac{9}{t^2} + \frac{12}{t} \quad (\times t^2)$ $12t^2 = 9 + 12t$ $12t^2 - 12t - 9 = 0$ $4t^2 - 4t - 3 = 0$ $(2t - 3)(2t + 1) = 0$ $t = \frac{3}{2}, -\frac{1}{2}$ $t = \frac{3}{2} \Rightarrow x = 6(\frac{3}{2}) = 9, y = \frac{6}{(\frac{3}{2})} = 4 \Rightarrow (9, 4)$ $t = -\frac{1}{2} \Rightarrow x = 6(-\frac{1}{2}) = -3,$ $y = \frac{6}{(-\frac{1}{2})} = -12 \Rightarrow (-3, -12)$	Substituting $(-9, 12)$ into T . M1 An attempt to form a "3 term quadratic" M1 An attempt to factorise. M1 $t = \frac{3}{2}, -\frac{1}{2}$ A1 An attempt to substitute either their $t = \frac{3}{2}$ or their $t = -\frac{1}{2}$ into x and y . M1 At least one of $(9, 4)$ or $(-3, -12)$. A1 Both $(9, 4)$ and $(-3, -12)$. A1 (7) [12]

Other Possible Solutions

Question Number	Scheme	Marks
<p>4.</p> <p>(a) (i) <i>Aliter</i> (ii) Way 2</p>	<p>$z^2 + pz + q = 0, z_1 = 2 - 4i$</p> <p>$z_2 = 2 + 4i$</p> <p>Product of roots = $(2 - 4i)(2 + 4i)$</p> <p style="text-align: center;">$= 4 + 16 = 20$</p> <p>or $b^2 - 4ac = (8i)^2$</p> <p>Sum of roots = $(2 - 4i) + (2 + 4i) = 4$</p> <p>$= z^2 - 4z + 20 = 0$</p>	<p style="text-align: right;">$2 + 4i$</p> <p>B1</p> <p>M1</p> <p>No i^2. Attempt Sum and Product of roots or Sum and discriminant</p> <p>Any one of $p = -4, q = 20$. A1</p> <p>Both $p = -4, q = 20$. A1</p> <p style="text-align: right;">(4)</p>
<p>4.</p> <p>(a) (i) <i>Aliter</i> (ii) Way 3</p>	<p>$z^2 + pz + q = 0, z_1 = 2 - 4i$</p> <p>$z_2 = 2 + 4i$</p> <p>$(2 - 4i)^2 + p(2 - 4i) + q = 0$</p> <p>$-12 - 16i + p(2 - 4i) + q = 0$</p> <p>Imaginary part: $-16 - 4p = 0$</p> <p>Real part: $-12 + 2p + q = 0$</p> <p>$4p = -16 \Rightarrow p = -4$</p> <p>$q = 12 - 2p \Rightarrow q = 12 - 2(-4) = 20$</p>	<p style="text-align: right;">$2 + 4i$</p> <p>B1</p> <p>M1</p> <p>An attempt to substitute either z_1 or z_2 into $z^2 + pz + q = 0$ and no i^2.</p> <p>Any one of $p = -4, q = 20$. A1</p> <p>Both $p = -4, q = 20$. A1</p> <p style="text-align: right;">(4)</p>

Question Number	Scheme	Marks
<p><i>Aliter</i> 7. (c) Way 2</p>	<p>$w = 4, \arg w = \frac{5\pi}{6}$ and $w = a + ib$</p> <p>$w = 4 \Rightarrow a^2 + b^2 = 16$</p> <p>$\arg w = \frac{5\pi}{6} \Rightarrow \arctan\left(\frac{b}{a}\right) = \frac{5\pi}{6} \Rightarrow \frac{b}{a} = -\frac{1}{\sqrt{3}}$</p> <p>$a = -\sqrt{3}b \Rightarrow a^2 = 3b^2$</p> <p>So, $3b^2 + b^2 = 16 \Rightarrow b^2 = 4$</p> <p>$\Rightarrow b = \pm 2$ and $a = \mp 2\sqrt{3}$</p> <p>As w is in the second quadrant</p> <p>$w = -2\sqrt{3} + 2i$</p> <p>$a = -2\sqrt{3}, b = 2$</p>	<p>Attempts to write down an equation in terms of a and b for either the modulus or the argument of w. Either $a^2 + b^2 = 16$ or $\frac{b}{a} = -\frac{1}{\sqrt{3}}$</p> <p>M1</p> <p>A1</p> <p>either $-2\sqrt{3} + 2i$ or awrt $-3.5 + 2i$</p> <p>A1</p> <p>(3)</p>

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