General Certificate of Education
Advanced Level Examination
January 2011

Mathematics MS2B

Unit Statistics 2B

Wednesday 26 January 2011 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 A factory produces bottles of brown sauce and bottles of tomato sauce.

(a) The content, Y grams, of a bottle of brown sauce is normally distributed with mean μ_Y and variance 4.

A quality control inspection found that the mean content, $\bar{y}$ grams, of a random sample of 16 bottles of brown sauce was 450.

Construct a 95% confidence interval for μ_Y. *(3 marks)*

(b) The content, X grams, of a bottle of tomato sauce is normally distributed with mean μ_X and variance σ^2.

A quality control inspection found that the content, x grams, of a random sample of 9 bottles of tomato sauce was summarised by

$$\sum x = 4950 \quad \text{and} \quad \sum (x - \bar{x})^2 = 334$$

(i) Construct a 90% confidence interval for μ_X. *(5 marks)*

(ii) Holly, the supervisor at the factory, claims that the mean content of a bottle of tomato sauce is 545 grams.

Comment, with a justification, on Holly's claim. State the level of significance on which your conclusion is based. *(3 marks)*

2 It is claimed that the way in which students voted at a particular general election was independent of their gender.

In order to investigate this claim, 480 male and 540 female students who voted at this general election were surveyed. These students may be regarded as a random sample.

The **percentages** of males and females who voted for the different parties are recorded in the table.

	Conservative	**Labour**	**Liberal Democrat**	**Other parties**
Male	32.5	30	25	12.5
Female	40	25	20	15

(a) Complete the contingency table below. *(2 marks)*

(b) Hence determine, at the 1% level of significance, whether the way in which students voted at this general election was independent of their gender. *(9 marks)*

	Conservative	**Labour**	**Liberal Democrat**	**Other parties**	**Total**
Male					480
Female					540
Total					1020

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3 Lucy is the captain of her school's cricket team.

The number of catches, X, taken by Lucy during any particular cricket match may be modelled by a Poisson distribution with mean 0.6 .

The number of run-outs, Y, effected by Lucy during any particular cricket match may be modelled by a Poisson distribution with mean 0.15 .

(a) Find:

(i) $\mathrm{P}(X \leqslant 1)$; *(1 mark)*

(ii) $\mathrm{P}(X \leqslant 1$ **and** $Y \geqslant 1)$. *(4 marks)*

(b) State the assumption that you made in answering part **(a)(ii)**. *(1 mark)*

(c) During a particular season, Lucy plays in 16 cricket matches.

(i) Calculate the probability that the number of catches taken by Lucy during this **season** is exactly 10. *(2 marks)*

(ii) Determine the probability that the **total** number of catches taken and run-outs effected by Lucy during this **season** is at least 15. *(3 marks)*

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4 (a) A red biased tetrahedral die is rolled. The number, X, on the face on which it lands has the probability distribution given by

x	1	2	3	4
$\mathrm{P}(X = x)$	0.2	0.1	0.4	0.3

(i) Calculate $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. *(3 marks)*

(ii) The red die is now rolled **three** times. The random variable S is the **sum** of the three numbers obtained.

Find $\mathrm{E}(S)$ and $\mathrm{Var}(S)$. *(2 marks)*

(b) A blue biased tetrahedral die is rolled. The number, Y, on the face on which it lands has the probability distribution given by

$$\mathrm{P}(Y = y) = \begin{cases} \dfrac{y}{20} & y = 1, 2 \text{ and } 3 \\[2ex] \dfrac{7}{10} & y = 4 \end{cases}$$

The random variable T is the value obtained when the number on the face on which it lands is **multiplied** by 3.

Calculate $\mathrm{E}(T)$ and $\mathrm{Var}(T)$. *(6 marks)*

(c) Calculate:

(i) $\mathrm{P}(X > 1)$; *(1 mark)*

(ii) $\mathrm{P}(X + T \leqslant 9$ **and** $X > 1)$; *(4 marks)*

(iii) $\mathrm{P}(X + T \leqslant 9 \mid X > 1)$. *(2 marks)*

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5 In 2001, the mean height of students at the end of their final year at Bright Hope Secondary School was 165 centimetres.

In 2010, David and James selected a random sample of 100 students who were at the end of their final year at this school. They recorded these students' heights, x centimetres, and found that $\bar{x} = 167.1$ and $s^2 = 101.2$.

To investigate the claim that the mean height had increased since 2001, David and James each correctly conducted a hypothesis test. They used the same null hypothesis and the same alternative hypothesis. However, David used a 5% level of significance whilst James used a 1% level of significance.

(a) (i) Write down the null and alternative hypotheses that both David and James used. *(1 mark)*

(ii) Determine the outcome of each of the two hypothesis tests, giving each conclusion in context. *(6 marks)*

(iii) State why both David and James made use of the Central Limit Theorem in their hypothesis tests. *(1 mark)*

(b) It was later found that, in 2010, the mean height of students at the end of their final year at Bright Hope Secondary School was actually 165 centimetres.

Giving a reason for your answer in each case, determine whether a Type I error or a Type II error or neither was made in the hypothesis test conducted by:

(i) David;

(ii) James. *(4 marks)*

Turn over ▶

6 The continuous random variable X has probability density function defined by

$$\mathrm{f}(x) = \begin{cases} \frac{3}{8}x^2 & 0 \leqslant x \leqslant \frac{1}{2} \\ \frac{3}{32} & \frac{1}{2} \leqslant x \leqslant 11 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the graph of f. *(3 marks)*

(b) Show that:

(i) $\mathrm{P}\left(X \geqslant 8\frac{1}{3}\right) = \frac{1}{4}$;

(ii) $\mathrm{P}(X \geqslant 3) = \frac{3}{4}$. *(3 marks)*

(c) Hence write down the **exact** value of:

(i) the interquartile range of X;

(ii) the median, m, of X. *(3 marks)*

(d) Find the **exact** value of $\mathrm{P}(X < m \mid X \geqslant 3)$. *(3 marks)*