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**General Certificate of Education**

**Mathematics 6360**

**MFP2 Further Pure 2**

**Mark Scheme**

*2009 examination - January series*

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

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### Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
-x EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

### No Method Shown

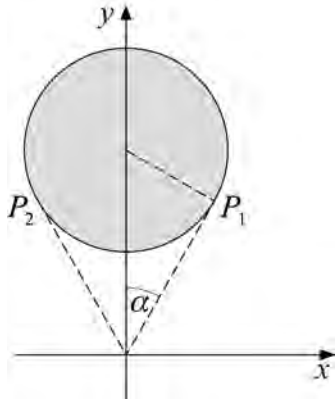
Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

<b>MFP2</b>				
<b>Q</b>	<b>Solution</b>	<b>Marks</b>	<b>Total</b>	<b>Comments</b>
<b>1(a)</b>	$\text{LHS} = 1 + \frac{1}{2}(e^{2\theta} - 2 + e^{-2\theta})$ $= \frac{1}{2}(e^{2\theta} + e^{-2\theta}) = \cosh 2\theta$	M1 A1 A1	3	Expansion of $\frac{1}{2}(e^\theta - e^{-\theta})^2$ correctly Any form AG
<b>(b)</b>	$3 + 6\sinh^2 \theta = 2\sinh \theta + 11$ $3\sinh^2 \theta - \sinh \theta - 4 = 0$ $(3\sinh \theta - 4)(\sinh \theta + 1) = 0$ $\sinh \theta = \frac{4}{3} \text{ or } -1$ $\theta = \ln 3$ $\theta = \ln(\sqrt{2} - 1)$	M1 A1 M1 A1F A1F A1F	6	OE Attempt to factorise or formula ft if factorises or real roots found
<b>Total</b>			<b>9</b>	
<b>2(a)</b>		B1 B1 B1 B1F	4	Circle Correct centre Correct radius Inside shading
<b>(b)</b>	<p>Correct points <math>P_1</math> and <math>P_2</math> indicated</p> $\sin \alpha = \frac{2}{4}$ $\alpha = \frac{\pi}{6}$ <p>Range is <math>\frac{\pi}{3} \leq \arg z \leq \frac{2\pi}{3}</math></p>	B1F M1 A1 A1	4	Possibly by tangents drawn ft mirror image of circle in $x$ -axis Deduct 1 for angles in degrees
<b>Total</b>			<b>8</b>	

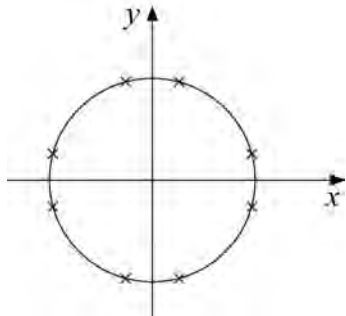
**MFP2 (cont)**

Q	Solution	Marks	Total	Comments
3(a)	$f(r) - f(r-1)$ $= \frac{1}{4}r^2(r+1)^2 - \frac{1}{4}(r-1)^2r^2$ $= \frac{1}{4}r^2(r^2 + 2r + 1 - r^2 + 2r - 1)$ $= r^3$	M1 A1 A1	3	Correct expansions of $(r+1)^2$ and $(r-1)^2$ AG
(b)	$r = n: n^3 = \frac{1}{4}n^2(n+1)^2 - \frac{1}{4}(n-1)^2n^2$ $r = 2n:$ $(2n)^3 = \frac{1}{4}(2n)^2(2n+1)^2 - \frac{1}{4}(2n-1)^2(2n)^2$ $\sum_{r=n}^{2n} r^3 = \frac{1}{4} \cdot 4n^2(2n+1)^2 - \frac{1}{4}(n-1)^2n^2$ $= \frac{3}{4}n^2(5n+1)(n+1)$	M1 A1 A1 M1 A1	5	For either $r = n$ or $r = 2n$ . PI AG <b>Alternatively</b> $\sum_{r=1}^{2n} r^3$ and $\sum_{r=1}^{n-1} r^3$ stated M1A1A1 (M1 for either) Difference M1 Answer A1
<b>Total</b>			<b>8</b>	
4(a)	Use of $(\sum \alpha)^2 = \sum \alpha^2 + 2\sum \alpha\beta$ $1 = -5 + 2\sum \alpha\beta$ $\sum \alpha\beta = 3$	M1 A1 A1	3	AG
(b)	$1(-5-3) = -23 - 3\alpha\beta\gamma$ $\alpha\beta\gamma = -5$	M1 A1	2	For use of identity
(c)	$z^3 - z^2 + 3z + 5 = 0$	M1 A1F	2	For correct signs and “= 0”
(d)	$\alpha^2 + \beta^2 + \gamma^2 < 0 \Rightarrow$ non real roots Coefficients real $\therefore$ conjugate pair	B1 B1	2	
(e)	$f(-1) = 0 \Rightarrow z+1$ is a factor $(z+1)(z^2 - 2z + 5) = 0$ $z = -1, 1 \pm 2i$	M1A1 A1 A1	4	
<b>Total</b>			<b>13</b>	

**MFP2 (cont)**

Q	Solution	Marks	Total	Comments
<p><b>5(a)</b></p> <p><b>(b)</b></p>	$\frac{du}{dx} = 2 \cosh x \sinh x$ $= \sinh 2x$ $I = \int_{x=0}^{x=1} \frac{du}{1+u^2}$ $= \left[ \tan^{-1} u \right]_{x=0}^{x=1}$ $= \left[ \tan^{-1} (\cosh^2 x) \right]_0^1$ $= \tan^{-1} (\cosh^2 1) - \tan^{-1} (\cosh^2 0)$ $= \tan^{-1} (\cosh^2 1) - \frac{\pi}{4}$	<p>M1</p> <p>A1</p> <p>M1A1</p> <p>A1</p> <p>A1</p> <p>A1</p>	<p>2</p> <p>5</p>	<p>Any correct method</p> <p>AG</p> <p>Ignore limits here</p> <p>Or A1 for change of limits</p> <p>AG</p>
<b>Total</b>			<b>7</b>	
<p><b>6</b></p>	<p>Assume result true for <math>n = k</math></p> <p>Then <math>\sum_{r=1}^{k+1} \frac{2^r \times r}{(r+1)(r+2)}</math></p> $= \frac{2^{k+1}}{k+2} + \frac{2^{k+1}(k+1)}{(k+2)(k+3)} - 1$ $= \frac{2^{k+1}(k+3+k+1)}{(k+2)(k+3)} - 1$ $= \frac{2^{k+1}2(k+2)}{(k+2)(k+3)} - 1$ $= \frac{2^{k+2}}{k+3} - 1$ <p><math>k = 1</math>: LHS = <math>\frac{1}{3}</math>, RHS = <math>\frac{2^2}{3} - 1</math></p> <p><math>P_k \Rightarrow P_{k+1}</math> and <math>P_1</math> true</p>	<p>M1A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>E1</p>	<p>7</p> <p>7</p>	<p>SC If no series at all indicated on LHS, deduct 1 and give E0 at end</p> <p>Putting over common denominator (not including the <math>-1</math>, unless separated later)</p> <p>Must be completely correct</p>
<b>Total</b>			<b>7</b>	

**MFP2 (cont)**

Q	Solution	Marks	Total	Comments
7(a)	$\frac{d}{dx} \left( \cosh^{-1} \frac{1}{x} \right) = \frac{1}{\sqrt{\frac{1}{x^2} - 1}} \left( -\frac{1}{x^2} \right)$ $= \frac{-1}{x\sqrt{1-x^2}}$	M1A1 A1	3	M0 if $\frac{dy}{dx} = f(y)$ and no attempt to substitute back to $x$ AG
(b)(i)	$\frac{d}{dx} (\sqrt{1-x^2}) = \frac{-2x}{2\sqrt{1-x^2}}$ $\frac{dy}{dx} = \frac{-x}{\sqrt{1-x^2}} + \frac{1}{x\sqrt{1-x^2}}$ $= \frac{1-x^2}{x\sqrt{1-x^2}} = \frac{\sqrt{1-x^2}}{x}$	B1 B1 M1 A1	4	For numerator For denominator (not $(1-x^2)^{\frac{1}{2}}$ ) For attempt to put over a common denominator AG
(ii)	$s = \int_{\frac{1}{4}}^{\frac{3}{4}} \sqrt{1 + \frac{1-x^2}{x^2}} dx = \int_{\frac{1}{4}}^{\frac{3}{4}} \frac{1}{x} dx$ $= [\ln x]_{\frac{1}{4}}^{\frac{3}{4}}$ $= \ln \frac{3}{4} - \ln \frac{1}{4} = \ln 3$	M1 A1A1 M1 A1	5	For use of $\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ AG
<b>Total</b>			<b>12</b>	
8(a)	Correct multiplication of brackets $e^{i\theta} + e^{-i\theta} = 2\cos\theta$	M1 A1	2	Clearly shown
(b)	$2\cos\theta = 1$ $\theta = \frac{\pi}{3}$ $z^4 = e^{\frac{\pi i}{3}} \text{ or } e^{-\frac{\pi i}{3}}$ $z = e^{i\frac{\pi}{12} + \frac{2k\pi i}{4}} \text{ or } e^{-i\frac{\pi}{12} + \frac{2k\pi i}{4}}$ $e^{\pm \frac{\pi i}{12}}, e^{\pm \frac{7\pi i}{12}}, e^{\pm \frac{5\pi i}{12}}, e^{\pm \frac{11\pi i}{12}}$	M1 A1 M1 m1 A2, 1, 0F	6	SC If 'hence' not used and, say, $z^8 - z^4 + 1 = 0$ is solved by formula, lose M1A1, but then continue M1m1 etc if $\frac{\pi}{3}$ is obtained A1 if 3 roots correct
(c)	 <p>Indication that <math>r = 1</math></p>	B2,1,0 B1	3	B1 for 4 roots indicated correctly on a circle. CAO
<b>Total</b>			<b>11</b>	
<b>TOTAL</b>			<b>75</b>	