

Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Mechanics 3 (6679_01)



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- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

- 1. The total number of marks for the paper is 75.
- 2. The Edexcel Mathematics mark schemes use the following types of marks:

<u>'M' marks</u>

These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.

e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc. The following criteria are usually applied to the equation.

To earn the M mark, the equation

(i) should have the correct number of terms

(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.

For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned. e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

<u>'A' marks</u>

These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. MO A1 is impossible.

<u>'B' marks</u>

These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod benefit of doubt
- ft follow through
- the symbol $\sqrt{1}$ will be used for correct ft
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC: special case
- oe or equivalent (and appropriate)
- dep dependent
- indep independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- The second mark is dependent on gaining the first mark
- 4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
- 5. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
- 6. Ignore wrong working or incorrect statements following a correct answer.

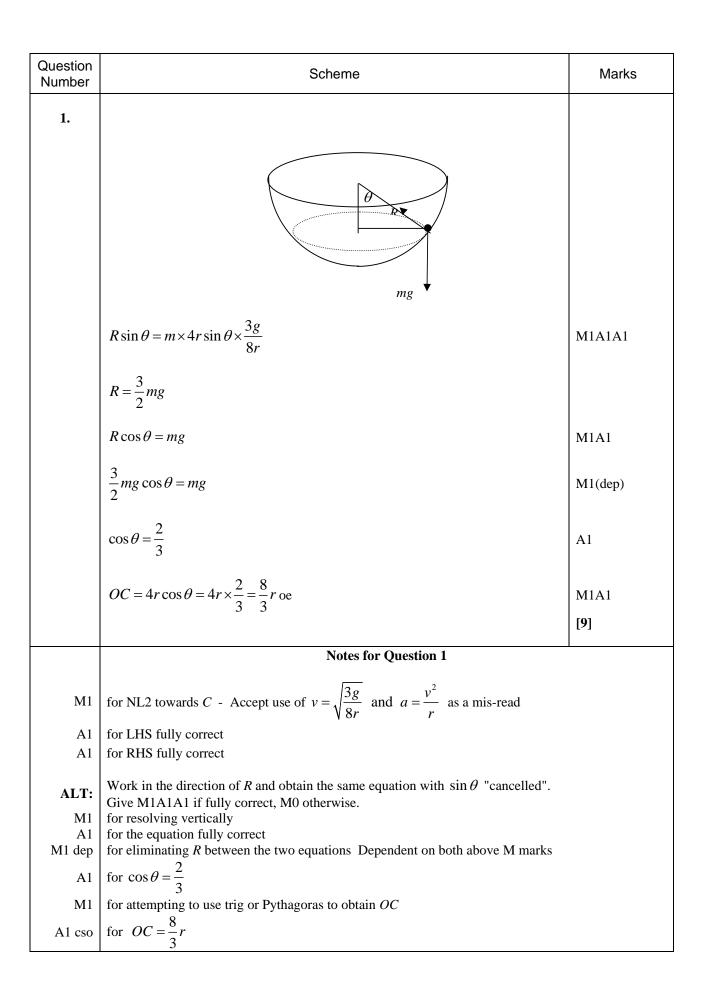
General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- dM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.
- Use of g = 9.81 should be penalised once per (complete) question.

N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),.....then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations
 - M(A) Taking moments about A.
 - N2L Newton's Second Law (Equation of Motion)
 - NEL Newton's Experimental Law (Newton's Law of Impact)
 - HL Hooke's Law
 - SHM Simple harmonic motion
 - PCLM Principle of conservation of linear momentum
 - RHS, LHS Right hand side, left hand side.



Alternative for Question 1				
M1A1A1	$R\sin\theta = m \times a \times \frac{3g}{8r}$			
M1 A1	$R\cos\theta = mg$			
M1 A1	$\tan\theta = \frac{3a}{8r}$			
M1	$\frac{a}{OC} = \frac{3a}{8r}$			
A1	$\frac{\overline{OC}}{\overline{OC}} = \frac{8r}{3}$ $OC = \frac{8r}{3}$			

Question Number	Scheme	Marks	6
2.			
(a)	(At surface) $\frac{k}{R^2} = mg \implies k = mgR^2$	M1A1	(2)
(b)	$m\ddot{x} = -\frac{mgR^2}{x^2}$ $v\frac{dv}{dx} = -\frac{gR^2}{x^2}$		
	$v\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{gR^2}{x^2}$	M1	
	$\int v \frac{\mathrm{d}v}{\mathrm{d}x} \mathrm{d}x = -gR^2 \int \frac{1}{x^2} \mathrm{d}x \text{or} \int \frac{\mathrm{d}\left(\frac{1}{2}v^2\right)}{\mathrm{d}x} \mathrm{d}x$ $1 2 gR^2 (\dots)$		
	$\frac{1}{2}v^2 = \frac{gR^2}{x} (+c)$	DM1A1	
	$x = \frac{5R}{4}, v = \sqrt{\frac{gR}{2}} \implies c = -\frac{11gR}{20}$	DM1A1	
	$v = 0 \ 0 = \frac{gR^2}{x} - \frac{11gR}{20}$	DM1	
	$x = \frac{20R}{11}$	A1	(7)
	11	[9]	

Notes for Question 2

(a) for $\frac{k}{R^2} = mg$. If not made clear that this applies at the surface of the Earth award M0 or $\frac{k}{r^2} = mg$ and x = R. M1 for $k = mgR^2 *$ A1 cso **(b)** for using accel = $v \frac{dv}{dr}$ or in NL2 with or w/o *m* Minus sign not required. M1 for attempting to integrate both sides - minus not needed M1 dep for fully correct integration, with or w/o the constant. Must have included the minus sign from the A1 start. for using $x = \frac{5R}{4}$, $v = \sqrt{\frac{gR}{2}}$ to obtain a value for the constant. Use of $x = \frac{R}{4}$ scores M0 Depends M1 dep on both previous M marks A1 for $c = -\frac{11gR}{20}$ for setting v = 0 and solving for x Depends on 1st and 2nd M marks, but not 3rd M1 dep for $x = \frac{20R}{11}$ A1 cso ALT: By definite integration First 3 marks as above, then Using limits $x = \frac{5R}{4}, v = \sqrt{\frac{gR}{2}}$ DM1 DM1 Using limit v = 0A1 Correct substitution for $x = \frac{20R}{11}$ A1 cso **NB**: The penultimate A mark has changed position, but must be entered on e-pen in its original position.

Alternative for Question 2

Qu 2 (a):	
Using $F = \frac{GM_1M_2}{x^2}$ with $x = R$ and one ma	ss as mass of Earth:
$mg = \frac{GmM_E}{R^2}$	
$GM_E = gR^2 \Longrightarrow F = \frac{mgR^2}{x^2} \Longrightarrow F = \frac{k}{x^2}$ with k	$x = mgR^2 *$
M1 Complete method A1 Correct answer	
Qu 2 (b):	
By conservation of energy:	
Work done against gravity $= \int_{\frac{5r}{4}}^{z} \frac{mgR^2}{x^2} dx = \int_{\frac{5}{4}}^{z}$	$mgR^2x^{-2} dx$ M1
$=\frac{4mgR}{5}-\frac{mgR^2}{z}$	DM1(integration)A1(correct)
Work-energy equation: $\frac{mgR}{4} = \frac{4mgR}{5} - \frac{mgR}{z}$	2 ² DM1A1
$z = \frac{20R}{11}$	DM1A1

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Question Number		Schem	e		Mark	S
3. (a)	She	l wax	filled shell			
(a)		n wax	inied shen			
	Mass ratio m	3 <i>m</i>	4 <i>m</i>			
	Dist. above vertex $\frac{2}{3} \times$	$6r \frac{3}{4} \times 2n$	\overline{x}		B1	
	$4mr + \frac{9}{2}mr = 4m\overline{x}$				M1A1ft	
	$\overline{x} = \frac{17}{8}r$				A1	(4)
(b)	$\tan\theta = \frac{r}{6r - \overline{x}} = \frac{r}{31r/8}$				M1A1ft	
	$\tan\theta = \frac{8}{31}$					
	θ=14.47°				A1 [7]	(3)
		Note	es for Question 3			
(a) B1 M1 A1 ft A1 cso	for correct distances from the ver for a dimensionally correct moments for a correct moments equation, for for $\overline{x} = \frac{17}{8}r$	ents equation	on with their distances and		t masses	
	NB: If $\frac{2}{3}$ and $\frac{3}{4}$ are interchange incorrect. Score: B0M1A1A0	d in the dis	tances, the correct answer	r is obtained bu	it the solut	ion is
(b)						
M1	for $\tan \theta = \frac{r}{6r - \overline{x}}$. Can be eithe	r way up, b	ut must include $6r - \overline{x}$.	Substitution for	\overline{x} not rec	quired
A1 ft	for $\tan \theta = \frac{r}{31r/8}$ or ft their		0.0000			
A1 cso	for $\theta = 14.47^{\circ}$ Accept 14°, 14 Accept 0.25 or better Obtuse angle accepted.	.5° or bette	r or $\theta = 0.2525$ rad			

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Question Number	Scheme	Marks
4		
(a)	$\frac{3mgx^2}{2l} = 2mgx\sin\alpha$	M1A1 B1(A1 on e- pen)
	$3x^{2} = 4xl \times \frac{3}{5}$ $5x^{2} = 4xl$	
	$5x^2 = 4xl$	
	$5x^2 = 4xl$ $x = \frac{4}{5}l$	DM1A1 (5)
(b)	$R = 2mg\cos\alpha \ \left(=\frac{8}{5}mg\right)$	B1
	$\frac{3mg}{2l} \times \frac{4}{25}l^2 = 2mg \times \frac{2}{5}l \times \frac{3}{5}, \mu \frac{8}{5}mg \times \frac{2}{5}l$	M1A1ft, B1ft (A1 on e- pen)
	$6 = 12 - 16\mu$	
	$16\mu = 6$ $\mu = \frac{3}{8}$	DM1A1 (6)
	-	[11]

Notes for Question 4 (a) for an energy equation with an EPE term of the form $\frac{kmgx^2}{l}$ and a GPE term. If a KE term is M1 included it must become 0 later. A1 for a correct EPE term for a correct GPE term. This can be in terms of the distance moved down the plane or the vertical **B**1 distance fallen for solving their equation to obtain the distance moved or using the vertical distance and obtaining M1 dep the distance moved along the plane. for $x = \frac{4}{5}l$ or $x = \frac{12}{15}l$ A1 **(b)** for resolving perpendicular to the plane to obtain $R = 2mg \cos \alpha$. May only be seen in an equation. B1 for an work-energy equation with an EPE term of the form $\frac{kmgx^2}{l}$, a GPE term and the work done M1 against friction. The work term must include a distance along the plane. for EPE and GPE terms correct and work subtracted from the GPE A1 B1 ft for the work term ft their R M1 dep for solving to obtain a value for μ for $\mu = \frac{3}{8}$ oe inc 0.375 but not 0.38 A1 cso If *m* used instead of 2*m*, assuming correct otherwise: **(a)** M1A1B0M1A0 (so 2 penalties for mis-read) **(b)** B1 $R = mg \cos \alpha$ Equation, with EPE correct and $mg \times \frac{2}{5}l \times \frac{3}{5}$ M1, A1 B1 ft $\left| \mu \frac{4mg}{5} \times \frac{2}{5}l \right|$

 $\mu = 0$

DM1, A1

Alternative for Question 4 Qu 4: Using NL2: **(a)** $2ma = 2mg\sin\alpha - \frac{3mgx}{l}$ $2v\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{6g}{5} - \frac{3gx}{l}$ M1(equation and attempt integration) $v^2 = \frac{6gx}{5} - \frac{3gx^2}{2l}, +c$ A1, A1 (show c = 0) $v = 0 \quad 3gx \left(\frac{2}{5} - \frac{x}{2l}\right) = 0$ M1 (set v = 0 and solve) $x = \frac{4l}{5}$ A1 **(b)** $R = 2mg\cos\alpha$ **B**1 $2v\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{6g}{5} - \frac{3gx}{l} - \mu \frac{8g}{5}$ $v^2 = \frac{6gx}{5} - \frac{3gx^2}{2l} - \mu \frac{8gx}{5}, +c$ M1(eqn and int)A1, A1 (show c = 0) $v = 0 \ x = \frac{2l}{5} \ \mu \frac{8}{5} = \frac{6}{5} - \frac{3}{2l} \times \frac{2l}{5}$ M1 (set v = 0 and solve) $\mu = \frac{3}{8}$ A1 If SHM methods are used, SHM must be proved first.

PMT

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Question Number	Scheme	Marks
5.		
5.		
(a)	$Vol = \pi \int_0^{\frac{\pi}{2}} y^2 dx = \pi \int_0^{\frac{\pi}{2}} \cos^2 x dx$	M1
	$=\pi \int_{0}^{\frac{\pi}{2}} \frac{1}{2} (\cos 2x + 1) dx$	M1
	$=\frac{\pi}{2}\left[\frac{1}{2}\sin 2x + x\right]_{0}^{\frac{\pi}{2}} = \frac{\pi^{2}}{4}$	DM1A1 (4)
(b)	$\pi \int_0^{\frac{\pi}{2}} y^2 x \mathrm{d}x = \pi \int_0^{\frac{\pi}{2}} x \cos^2 x \mathrm{d}x$	M1
	$=\pi \int_{0}^{\frac{\pi}{2}} \frac{1}{2} x (\cos 2x + 1) dx$	
	$=\frac{\pi}{2}\int_{0}^{\frac{\pi}{2}}x\cos 2x\mathrm{d}x+\frac{\pi}{2}\left[\frac{x^{2}}{2}\right]_{0}^{\frac{\pi}{2}}$	
	$\frac{\pi}{2} \left[x \times \frac{1}{2} \sin 2x \right]_{0}^{\frac{\pi}{2}} - \frac{\pi}{2} \int_{0}^{\frac{\pi}{2}} \frac{1}{2} \sin 2x dx, + \frac{\pi^{3}}{16}$	M1,B1
	$=0+\frac{\pi}{2}\left[\frac{1}{4}\cos 2x\right]_{0}^{\frac{\pi}{2}}+\frac{\pi^{3}}{16}$	DM1
	$=\frac{\pi}{8}\left[-1-1\right]+\frac{\pi^3}{16}=\frac{\pi^3}{16}-\frac{\pi}{4}$	Alft
	$\overline{x} = \frac{\pi^3 - 4\pi}{16} \div \frac{\pi^2}{4} = \frac{\pi^2 - 4}{4\pi}$ or 0.467088	M1A1 (7)
		[11]

Notes for Question 5

(a)	
M1	for using Vol = $\pi \int_0^{\frac{\pi}{2}} \cos^2 x dx$. If π is missing here it must be included later to earn this mark.
M1	Limits not needed for using the double angle formula (correct) to prepare for integration. Formula must be correct. π and limits not needed for this mark. for attempting to integrate and substitute the correct limits (only sub of non-zero limit needed be to
M1 dep	seen) dependent on both M marks.
A1 cso	for $\frac{\pi^2}{4}$ * (check integration is correct, answer can be obtained by luck due to the limits)
(b)	NB : The first 5 marks can be earned with or without π
M1	for using $\pi \int_{0}^{\frac{\pi}{2}} x \cos^2 x dx = \pi$ not needed; limits not needed.
M1	for using the double angle formula (correct) and attempting the first stage of integration by parts
B1	for $\frac{\pi^3}{16}$ or $\frac{\pi^2}{16}$ if π not included. NB integration by parts not needed for this mark
M1 dep	for completing the integration by parts, limits not needed yet
A1 ft	for $=\frac{\pi}{8}[-1-1]+\frac{\pi^3}{16}=\frac{\pi^3}{16}-\frac{\pi}{4}$ or $=\frac{1}{8}[-1-1]+\frac{\pi^2}{16}=\frac{\pi^2}{16}-\frac{1}{4}$ ft on $\frac{\pi^3}{16}$
M1	for using $\overline{x} = \frac{\int \pi y^2 x dx}{\int \pi y^2 dx}$ The numerator integral need not be correct.
	π should be seen in both or neither integral
	for $\overline{x} = \frac{\pi^2 - 4}{4\pi}$ or $\frac{\pi}{4} - \frac{1}{\pi}$ or 0.467088
A1 cso	Accept 0.47 or better but no fractions within fractions
	(a) has a given answer, so the cso applies to the solution of (b) only.

PMT

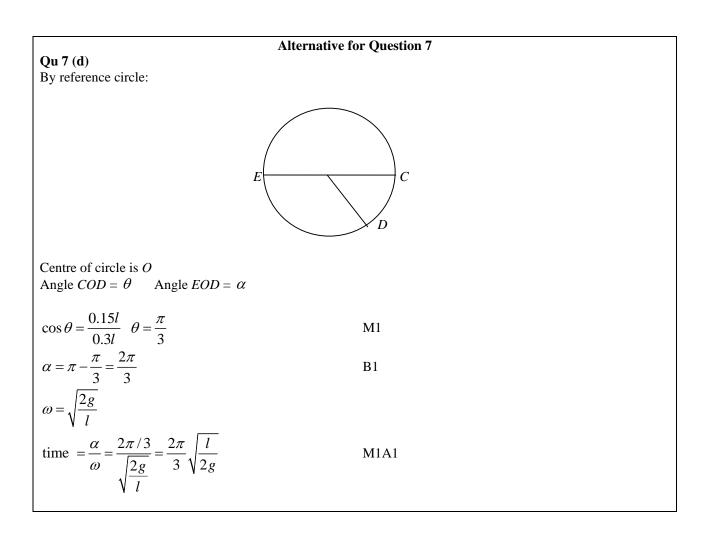
Question	Scheme	Marks
Number 6.	Scheme	INIAL KS
	$\frac{1}{2}mU^2 - \frac{1}{2}mv^2 = 2mga$	M1A1
	$T + mg = m\frac{v^2}{a}$	M1A1
	$T = \frac{\left(mU^2 - 4mga\right)}{a} - mg$	DM1
	$T = \frac{mU^2 - 5mga}{a}$	A1
	$T \ge 0 \implies U^2 \ge 5ga$ $U \ge \sqrt{5ag} \qquad \qquad *$	DM1
	$U \geqslant \sqrt{5ag}$ *	A1 (8)
(b)	At top: $T = \frac{9mga - 5mga}{a} = 4mg$	M1(either tension)A1
	At bottom: $T' - mg = \frac{mU^2}{a}$	A1
	$kT = mg + \frac{9mag}{a} = 10mg$	DM1
	$k = \frac{10mg}{4mg} = \frac{5}{2}$	A1 (5) [13]

Notes for Question 6

(a)	
	for an energy equation, from the bottom to the top. A difference of KE terms and a PE term needed.
M1	From bottom to a general point gets M0 until a value for θ at the top is used. $v^2 = u^2 + 2as$ scores M0
A1	for all terms correct (inc signs)
M1	for NL2 along the radius at the top. Two forces and mass x acceleration needed. Accel can be in either form here. But see NB at end of (a)
A1	for a fully correct equation. Acceleration should be $\frac{v^2}{a}$ now.
M1 dep	for eliminating v (vel at top) between the two equations. Dependent on both previous M marks. If v is set = 0, award M0
A1	for a correct expression for T
M1 dep	for using $T \ge 0$ to obtain an inequality for U^2 or U. Allow with > Dependent on all previous M marks.
A1 cso	for $U \ge \sqrt{5ag}$ * Watch square root! Give A0 if > seen on previous line.
	NB: The second and fourth M marks (and their As if earned) can be given together $\frac{2}{3}$
	if $mg \le m \frac{v^2}{a}$ is seen
(b)	
M1	for obtaining an expression for the tension at the top or at the bottom, no need to substitute for U yet.
A1	Substitute for U and obtain one correct tension $(4mg \text{ at top or } 10mg \text{ at bottom})$
A1	for the other tension correct
M1 dep	for using tension at bottom = k x tension at the top and solving for k
A1 cso	for $k = \frac{5}{2}$ oe

Question Number	Scheme	Marks
7.		
	$T = \frac{\lambda x}{l} = \frac{\lambda \times 0.5l}{l}$	M1A1
	$\lambda = 2mg$ *	A1 (3)
(b)	$mg - T = m\ddot{x}$	M1
	$mg - \frac{2mg\left(0.5l + x\right)}{l} = m\ddot{x}$	DM1A1A1
	$\ddot{x} = -\frac{2gx}{l}$	A1
	. SHM	A1cso(B1 on e- pen) (6)
(c)	a = 0.3l	
	$\left \ddot{x}\right _{\text{max}} = 2g \times \frac{0.3l}{l} = 0.6g \ (= 5.88 \text{ or } 5.9 \text{ m s}^{-2})$	M1A1ft (2)
(d)	$x = a\cos\omega t = 0.3l\cos\left(\sqrt{\frac{2g}{l}}\right)t$	
	Time C to D: $0.15 = 0.3 \cos\left(\sqrt{\frac{2g}{l}}\right)t$	M1
	$t = \sqrt{\frac{l}{2g}} \cos^{-1} 0.5$	
	Time C to E: $t' = \text{half period} = \pi \sqrt{\frac{l}{2g}}$	B1
	Time <i>D</i> to <i>E</i> : = $(\pi - \cos^{-1} 0.5) \sqrt{\frac{l}{2g}} = \frac{2\pi}{3} \sqrt{\frac{l}{2g}}$	M1A1 (4) [15]

	Notes for Question 7
(a) M1 A1 A1	for using Hooke's Law for a correct equation for solving to get $\lambda = 2mg^*$
(b) M1	for using NL2. Weight and tension must be seen. Acceleration can be <i>a</i> here, but must be an equation at a general position
M1 dep A1 A1 A1	for using Hooke's Law for the tension. Acceleration can be <i>a</i> for a fully correct equation inc acceleration as \ddot{x} (-1 ee) for simplifying to $\ddot{x} = -\frac{2gx}{l}$ oe
A1 cso	for the conclusion $l = -\frac{l}{l}$
(c) M1	for using $ \ddot{x} _{\text{max}} = \omega^2 a$ with their ω and $a = 0.3l$. ω must be dimensionally correct
A1 ft (d)	for obtaining the max magnitude of the accel, accept 0.6g, 5.9 or 5.88 only. ft their ω
(u) M1	for using $x = a \cos \omega t$ with $x = \pm 0.15l$, $a = 0.3l$ and their ω to obtain an expression for the time from <i>C</i> to <i>D</i>
B1	for time C to $E = \text{half period} = \pi \sqrt{\frac{l}{2g}}$
M1	For any correct method for obtaining the time from D to E $2\pi \sqrt{1}$
A1 cao	for $\frac{2\pi}{3}\sqrt{\frac{l}{2g}}$ oe inc $0.473\sqrt{l}$ $0.47\sqrt{l}$
ALT for (d): (i)	
M1 M1, A1	Use $x = a \sin \omega t$ with $x = 0.15l$, $a = 0.3l$ and their ω to obtain an expression for the time from <i>B</i> to <i>D</i> as above
(ii)	Using $x = a \cos \omega t$ with $x = \pm 0.15l$, $a = 0.3l$ and their ω This gives the required time in one step. Award M2 A1 for correct substitution A1 correct answer However do not isw if further work shown. Mark according to mark scheme method and give max M1B1M0A0.



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