

Mark Scheme (Results)

January 2008

GCE

GCE Mathematics (6683/01)



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6683 Statistics S1
Mark Scheme

Question Number	Scheme	Marks
1. (a)	$\sum x = 773, \sum y = 724$ $r = \frac{10 \times 56076 - 773 \times 724}{\sqrt{(10 \times 60475 - 773^2)(10 \times 53122 - 724^2)}} \quad \text{o.e.}$ $r = 0.155357 \dots$	B1, B1 M1 A1ft A1 (5)
(b)	Both weak correlation Neither score is a good indication of future performance Interview test is slightly better since correlation is positive	B1g B1h (2) Total 7 marks
NB (a)	$S_{xx} = 60475 - \frac{(773)^2}{10} = 722.1, \quad S_{yy} = 53122 - \frac{(724)^2}{10} = 704.4, \quad S_{xy} = 56076 - \frac{773 \times 724}{10} = 110.8$	
(b)	1st B1 for $\sum x$ and 2nd B1 for $\sum y$, should be seen or implied. M1 for at least one correct attempt at one of S_{xx}, S_{yy} or S_{xy} and then using in the correct formula 1st A1ft for a fully correct expression. (ft their $\sum x$ and their $\sum y$) or 3 correct expressions for S_{xx}, S_{xy}, and S_{yy} but possibly incorrect values for these placed correctly in r. 2nd A1 for awrt 0.155 If $r > 0.5$ they can score B1g in (b) for saying that it (skills test) is not a good guide to performance but B0h since a second acceptable comment about both tests is not possible. Give B1 for one correct line, B1B1 for any 2. If the only comment is the test(s) <u>are</u> a good guide: scores B0B0 If the only comment is the tests are not good: scores B1B0 (second line) The third line is for a comment that suggests that the interview test is OK but the skills test is not since one is positive and the other is negative. Treat 1st B1 as B1g and 2nd as B1h An answer of “no” alone scores B0B0	

Question Number	Scheme	Marks
2.		
(a)	mean is $\frac{2757}{12}, = 229.75$ AWRT 230	M1, A1
	sd is $\sqrt{\frac{724961}{12} - (229.75)^2}, = 87.34045$ AWRT 87.3	M1, A1
	[Accept $s =$ AWRT 91.2]	
(b)	Ordered list is: 125, 160, 169, 171, 175, 186, 210, 243, 250, 258, 390, 420 $Q_2 = \frac{1}{2}(186 + 210) = 198$ $Q_1 = \frac{1}{2}(169 + 171) = 170$ $Q_3 = \frac{1}{2}(250 + 258) = 254$	B1 B1 B1
(c)	$Q_3 + 1.5(Q_3 - Q_1) = 254 + 1.5(254 - 170), = 380$ Accept AWRT (370-392) Patients F (420) and B (390) are outliers.	M1, A1 B1ft B1ft
(d)	$\frac{Q_1 - 2Q_2 + Q_3}{Q_3 - Q_1} = \frac{170 - 2 \times 198 + 254}{254 - 170}, = 0.3$ AWRT 0.33 Positive skew.	M1, A1 A1ft
		(3) Total 14 marks
(a)	1 st M1 for using $\frac{\sum x}{n}$ with a credible numerator and $n = 12$. 2 nd M1 for using a correct formula, root required but can ft their mean NB Use of $s = \sqrt{8321.84...} = 91.22...$ is OK for M1A1 here. Answers only from a calculator in (a) can score full marks	
(b)	1 st B1 for median= 198 only, 2 nd B1 for lower quartile 3 rd B1 for upper quartile	
S.C.	If all Q_1 and Q_3 are incorrect but an ordered list (with ≥ 6 correctly placed) is seen and used then award B0B1 as a special case for these last two marks.	
(c)	M1 for a clear attempt using their quartiles in given formula, A1 for any value in the range 370 - 392 1 st B1ft for any one correct decision about B or F - ft their limit in range (258, 420) 2 nd B1ft for correct decision about both F and B - ft their limit in range (258, 420) If more points are given score B0 here for the second B mark. (Can score M0A0B1B1 here)	
(d)	M1 for an attempt to use their figures in the correct formula – must be seen (≥ 2 correct substitutions) 1 st A1 for AWRT 0.33 2 nd A1ft for positive skew. Follow through their value/sign of skewness . Ignore any further calculations. “positive correlation” scores A0	

3.

Width	1	1	4	2	3	5	3	12
Freq. Density	6	7	2	6	5.5	2	1.5	0.5

 0.5×12 or 6Total area is $(1 \times 6) + (1 \times 7) + (4 \times 2) + \dots = 70$

$$(90.5 - 78.5) \times \frac{1}{2} \times \frac{140}{\text{their } 70}$$

“70 seen anywhere”

Number of runners is 12

M1**A1****M1****B1****A1****(5)****Total 5 marks**

1st M1 for attempt at width of the correct bar (90.5 - 78.5)
[Maybe on histogram or in table]

1st A1 for 0.5×12 or 6 (may be seen on the histogram. Must be related to the area of the bar above 78.5 - 90.5.

2nd M1 for attempting area of correct bar $\times \frac{140}{\text{their } 70}$

B1 for 70 seen anywhere in their working

2nd A1 for correct answer of 12.

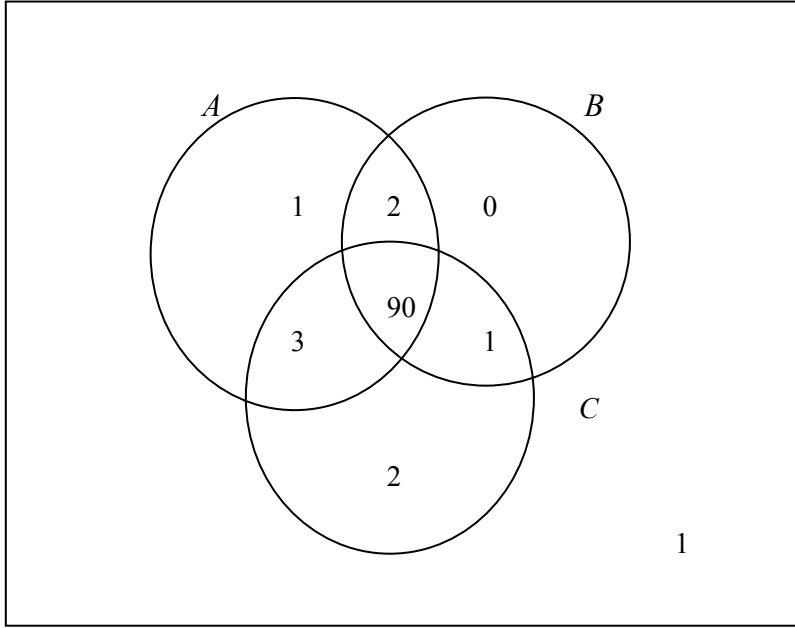
Minimum working required is $2 \times 0.5 \times 12$ where the 2 should come from $\frac{140}{70}$

Beware $90.5 - 78.5 = 12$ (this scores M1A0M0B0A0)

Common answer is $0.5 \times 12 = 6$ (this scores M1A1M0B0A0)

If unsure send to review e.g. $2 \times 0.5 \times 12 = 12$ without 70 being seen

4.				
(a)	$S_{xy} = 1818.5 - \frac{41 \times 406}{10}, = 153.9$ $S_{xx} = 188 - \frac{41^2}{10} = 19.9$	(could be seen in (b)) (could be seen in (b))	AWRT 154	M1, A1 A1 (3)
(b)	$b = \frac{153.9}{19.9}, = 7.733668....$ $a = 40.6 - b \times 4.1 (= 8.89796....)$ $y = 8.89 + 7.73x$		AWRT 7.73	M1, A1 M1 A1 (4)
(c)	A typical car will travel 7700 miles every year			B1ft (1)
(d)	$x = 5, y = 8.89 + 7.73 \times 5 (= 47.5 - 47.6)$ So mileage predicted is	AWRT 48000		M1 A1 (2)
Total 10 marks				
	Accept calculations for S_{xx} and S_{xy} in (a) or (b)			
(a)	M1 for correct attempt or expression for either 1 st A1 for one correct 2 nd A1 for both correct			
(b)	Ignore the open marks for part (b) they should be awarded as per this scheme			
	1 st M1 for $\frac{\text{their } S_{xy}}{\text{their } S_{xx}}$ 1 st A1 for AWRT 7.73 2 nd M1 for attempt at correct formula for a (minus required). Ft their b . Quoting a correct formula but making one slip in sub.eg. $\bar{y} = 406$ is OK 2 nd A1 for correct equation with 2dp accuracy. Accept $a = 8.89$, and $b = 7.73$ even if not written as final equation.			
	Correct answers only (from calc) score 4/4 if correct to 2dp or 3/4 if AWRT 2dp			
(c)	B1ft	for their $b \times 1000$ to at least 2 sf. Accept “7.7 thousand” but value is needed		
(d)	M1	for substituting $x = 5$ into their final answer to (b).		
	A1	for AWRT 48000 (Accept “48 thousands”)		

5. (a)	Diagram may be drawn with $B \subset (A \cup C)$ or with the 0 for $B \cap (A \cup C)'$ simply left blank	
	Accept decimals or probs. in Venn diagram 	3cc 90,3,2,1 1,(0),2 1 outside Box M1 A1 M1A1 A1 B1 (6)
(b)	P(none)=0.01	B1ft
(c)	P(A but not B)=0.04	M1 A1ft
(d)	P(any wine but C)=0.03	M1A1ft
(e)	P(exactly two)=0.06	M1A1ft
(f)	$P(C A) = \frac{P(C \cap A)}{P(A)} = \frac{93}{96} \text{ or } \frac{31}{32} \text{ or AWRT } 0.969$	M1A1ft,A1
		(3)
		Total 16 marks
(a)	1st M1 for 3 closed, labelled curves that overlap. A1 for the 90, 3, 2 and 1 2nd M1 for one of 1, 0 or 2 correct or a correct sum of 4 values for A, B or C 2nd A1 for all 7 values correct. Accept a blank instead of 0. NB final mark is a B1 for the box not an A mark as on EPEN In parts (b) to (f) full marks can be scored for correct answers or correct ft	
(b)	B1ft	Follow through their '1' from outside divided by 100
(c)	M1	for correct expression eg $P(A \cup B) - P(B)$ or calculation e.g. 3 + 1 or 4 on top
	A1	for a correct probability, follow through with their '3+1' from diagram
(d)	M1	for correct expression or calculation e.g. 1+2+0 or 99-96 or 3 on top
	A1	for a correct probability, follow through their '2+1+0' from diagram
(e)	M1	for a correct expression or calculation e.g. 3+2+1 or 6 on top
(f)	M1	for a correct expression upto " , " and <u>some</u> correct substitution, ft their values. One of these probabilities must be correct or correct ft. If P(C) on bottom M0
	1 st A1ft	follow through their $A \cap C$ and their A but the ratio must be in (0, 1)
	2 nd A1	for correct answer only. Answer only scores 3/3, but check working $P(A \cap C) / P(C)$ is M0

For M marks in (c) to (e) they must have a fraction

6.		
(a)	200 or 200g	B1
(b)	$P(190 < X < 210) = 0.6$ or $P(X < 210) = 0.8$ or $P(X > 210) = 0.2$ or diagram (o.e.) Correct use of 0.8 or 0.2 $Z = (\pm) \frac{210 - 200}{\sigma}$ $\frac{10}{\sigma} = 0.8416$ $\sigma = 11.882129....$	M1 A1 M1 B1 A1
(c)	$P(X < 180) = P\left(Z < \frac{180 - 200}{\sigma}\right)$ $= P(Z < -1.6832)$ $= 1 - 0.9535$ $= 0.0465 \quad \text{or} \quad \text{AWRT } 0.046$	M1 M1 A1
		(3)
		Total 9 marks
(a)	“mean = 200g” is B0 but “median = 200” or just “200” alone is B1 Standardization in (b) and (c). They must use σ not σ^2 or $\sqrt{\sigma}$.	
(b)	1 st M1 for a correct probability statement (as given or eg $P(200 < X < 210) = 0.3$ o.e.) or shaded diagram - must have values on z-axis and probability areas shown 1 st A1 for correct use of 0.8 or $p = 0.2$. Need a correct probability statement. May be implied by a suitable value for z seen (e.g. $z = 0.84$) 2 nd M1 for attempting to standardise. Values for x and μ used in formula. Don't need $z =$ for this M1 nor a z -value, just mark standardization. B1 for $z = 0.8416$ (or better) [$z = 0.84$ usually just loses this mark in (a)] 2 nd A1 for AWRT 11.9	
(c)	1 st M1 for attempting to Standardise with 200 and their sd(>0) e.g. $(\pm) \frac{180 - 200}{\text{their } \sigma}$ 2 nd M1 NB on open this is an A mark ignore and treat it as 2nd M1 for 1 – a probability from tables provided compatible with their probability statement. A1 for 0.0465 or AWRT 0.046 (Dependent on both Ms in part (c))	

7.(a)	$P(R = 3 \cap B = 0) = \frac{1}{4} \times \frac{1}{4}, = \frac{1}{16}$			M1, A1																									
(b)	<table><tr><td>3</td><td>0</td><td>3</td><td>6</td><td>9</td></tr><tr><td>2</td><td>0</td><td>2</td><td>4</td><td>6</td></tr><tr><td>1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td><i>B</i></td><td><i>R</i></td><td>0</td><td>1</td><td>2</td><td>3</td></tr></table>	3	0	3	6	9	2	0	2	4	6	1	0	1	2	3	0	0	0	0	0	<i>B</i>	<i>R</i>	0	1	2	3	All 0s All 1,2,3s All 4,6,9s	B1 B1 B1
3	0	3	6	9																									
2	0	2	4	6																									
1	0	1	2	3																									
0	0	0	0	0																									
<i>B</i>	<i>R</i>	0	1	2	3																								
(c)	$a = \frac{7}{16}, b = c = d = \frac{1}{16}$			B1, B1 B1																									
(d)	$E(T) = \left(1 \times \frac{1}{16}\right) + \left(2 \times \frac{1}{8}\right) + \left(3 \times \frac{1}{8}\right) + \left(4 \times \frac{1}{16}\right) + \dots$ $= 2\frac{1}{4}$ or exact equivalent e.g. 2.25, $\frac{9}{4}$			M1																									
(e)	$Var(T) = \left(1^2 \times \frac{1}{16}\right) + \left(2^2 \times \frac{1}{8}\right) + \left(3^2 \times \frac{1}{8}\right) + \left(4^2 \times \frac{1}{16}\right) + \dots - \left(\frac{9}{4}\right)^2$ $= \frac{49}{4} - \frac{81}{16} = 7\frac{3}{16}$ or $\frac{115}{16}$ (o.e.)			M1A1,M1																									
	AWRT 7.19			A1																									
				Total 14 marks																									
(a)	M1	for $\frac{1}{4} \times \frac{1}{4}$																											
(c)	1 st B1	for $\frac{7}{16}$,																											
	2 nd B1	for only one error in b, c, d ($b = c = d \neq \frac{1}{16}$ or $b = c = \frac{1}{16} \neq d$ etc), 3 rd B1 all of $b, c, d = \frac{1}{16}$																											
(d)	M1	for attempting $\sum tP(T = t)$, 3 or more terms correct or correct ft. Must Attempt to sum. NB calculating $E(T)$ and then dividing by a number other than 1 scores M0.																											
(e)	1 st M1	for attempt at $E(T^2)$, 3 or more terms correct or correct ft.																											
	1 st A1	for $\frac{49}{4}$ (o.e.) or a fully correct expression (all non-zero terms must be seen)																											
	2 nd M1	for subtracting their $[E(T)]^2$, Must be some attempt to square $-\frac{9}{4}$ is M0 but $-\frac{9}{16}$ could be M1																											
	2 nd A1	for correct fraction or AWRT 7.19 Full marks can still be scored in (d) and (e) if a is incorrect																											