

FP2 Paper 5 *adapted 2005

1. (a) Sketch the graph of $y = |x - 2a|$, given that $a > 0$. (2)

(b) Solve $|x - 2a| > 2x + a$, where $a > 0$.

(3)(Total 5 marks)

2. Find the general solution of the differential equation

$$\frac{dy}{dx} + 2y \cot 2x = \sin x, \quad 0 < x < \frac{\pi}{2},$$

giving your answer in the form $y = f(x)$.

(Total 7 marks)

3. (a) Show that the transformation $y = xv$ transforms the equation

$$x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + (2 + 9x^2)y = x^5, \quad \text{I}$$

into the equation
$$\frac{d^2v}{dx^2} + 9v = x^2. \quad \text{II}$$

(5)

- (b) Solve the differential equation II to find v as a function of x .

(6)

- (c) Hence state the general solution of the differential equation I.

(1)(Total 12 marks)

4. The curve C has polar equation $r = 6 \cos \theta$, $-\frac{\pi}{2} \leq \theta < \frac{\pi}{2}$,

and the line D has polar equation $r = 3 \sec \left(\frac{\pi}{3} - \theta \right)$, $-\frac{\pi}{6} < \theta < \frac{5\pi}{6}$.

- (a) Find a cartesian equation of C and a cartesian equation of D .

(5)

- (b) Sketch on the same diagram the graphs of C and D , indicating where each cuts the initial line.

(3)

The graphs of C and D intersect at the points P and Q .

- (c) Find the polar coordinates of P and Q .

(5)(Total 13 marks)

5. Find the general solution of the differential equation

$$(x + 1) \frac{dy}{dx} + 2y = \frac{1}{x}, \quad x > 0.$$

giving your answer in the form $y = f(x)$.

(7)(Total 7 marks)

6. (a) On the same diagram, sketch the graphs of $y = |x^2 - 4|$ and $y = |2x - 1|$, showing the coordinates of the points where the graphs meet the axes.

(4)

- (b) Solve $|x^2 - 4| = |2x - 1|$, giving your answers in surd form where appropriate.

(5)

- (c) Hence, or otherwise, find the set of values of x for which $|x^2 - 4| > |2x - 1|$.

(3)(Total 12 marks)

7. (a) Find the general solution of the differential equation

$$2 \frac{d^2 x}{dt^2} + 5 \frac{dx}{dt} + 2x = 2t + 9.$$

(6)

- (b) Find the particular solution of this differential equation for which $x = 3$ and $\frac{dx}{dt} = -1$ when $t = 0$.

(4)

The particular solution in part (b) is used to model the motion of a particle P on the x -axis. At time t seconds ($t \geq 0$), P is x metres from the origin O .

- (c) Show that the minimum distance between O and P is $\frac{1}{2}(5 + \ln 2)$ m and justify that the distance is a minimum.

(4)(Total 14 marks)

8. The curve C which passes through O has polar equation

$$r = 4a(1 + \cos \theta), \quad -\pi < \theta \leq \pi.$$

The line l has polar equation

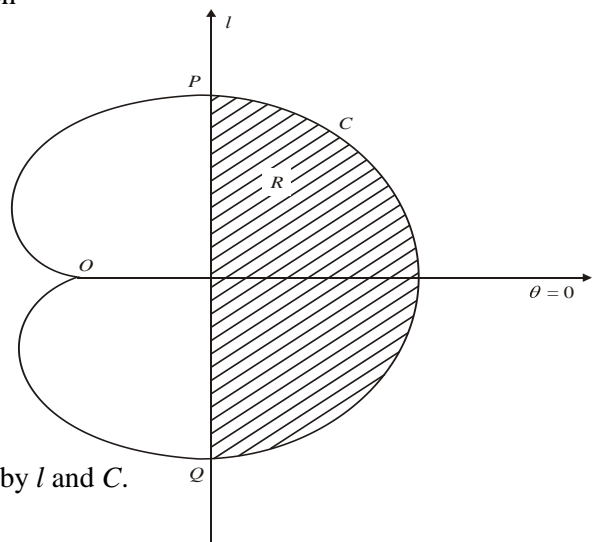
$$r = 3a \sec \theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}.$$

The line l cuts C at the points P and Q , as shown in the diagram.

- (a) Prove that $PQ = 6\sqrt{3}a$. (6)

The region R , shown shaded in the diagram, is bounded by l and C .

- (b) Use calculus to find the exact area of R .



(7)(Total 13 marks)

9. A complex number z is represented by the point P in the Argand diagram. Given that

$$|z - 3i| = 3,$$

(a) sketch the locus of P .

(2)

(b) Find the complex number z which satisfies both $|z - 3i| = 3$ and $\arg(z - 3i) = \frac{3}{4}\pi$.

(4)

The transformation T from the z -plane to the w -plane is given by

$$w = \frac{2i}{z}.$$

(c) Show that T maps $|z - 3i| = 3$ to a line in the w -plane, and give the cartesian equation of this line.

(5)(Total 11 marks)

10. (a) Given that $z = e^{i\theta}$, show that

$$z^n - \frac{1}{z^n} = 2i \sin n\theta,$$

where n is a positive integer.

(2)

(b) Show that

$$\sin^5 \theta = \frac{1}{16} (\sin 5\theta - 5 \sin 3\theta + 10 \sin \theta).$$

(5)

(c) Hence solve, in the interval $0 \leq \theta < 2\pi$,

$$\sin 5\theta - 5 \sin 3\theta + 6 \sin \theta = 0.$$

(5)(Total 12 marks)

11. The variable y satisfies the differential equation

$$4(1 + x^2) \frac{d^2 y}{dx^2} + 4x \frac{dy}{dx} = y.$$

At $x = 0$, $y = 1$ and $\frac{dy}{dx} = \frac{1}{2}$.

(a) Find the value of $\frac{d^2 y}{dx^2}$ at $x = 0$. (1) (c) Find the value of $\frac{d^3 y}{dx^3}$ at $x = 0$ (4)

(d) Express y as a series, in ascending powers of x , up to and including the term in x^3 . (2)

(e) Find the value that the series gives for y at $x = 0.1$, giving your answer to 5 decimal places.

(1)(Total 14 marks)