January 2008

# **4736 Decision Mathematics 1**

1	(i)	5 2 4 3 8         Bin 1:       5 2 3         Bin 2:       4         Bin 3:       8	M1 A1	First bin correct All correct in three bins	[2]
	(ii)	8 5 4 3 2         Bin 1:       8 2         Bin 2:       5 4         Bin 3:       3	M1 A1	First bin correct All correct in three bins	[2]
	(iii)	The heaviest box is originally at the bottom of the stack	B1	Referring to the physical act of sorting the weights into decreasing order	[1]
	(iv)	Bins in any order and boxes in any orderBin 1:8Bin 2:5352Bin 3:424	B1	Any valid packing into three bins of capacity 8 kg.	[1]
				Total =	6

2	(i)		M1 A1	A connected graph with nine vertices labelled 1 to 9 Correct graph	
		7 8 9 4 moves	B1	Stating 4	[3]
	(ii)	Neither	M1	'Neither', together with an attempt at a reason	
		It has four odd nodes The nodes 2, 4, 6, 8 each have three arcs joined to them whereas an Eulerian graph has no odd nodes and a semi- Eulerian graph has exactly two odd nodes	A1	A correct reference to the number of odd nodes for this graph. Be careful about whether 'odd' refers to the parity or the value. However, just defining Eulerian and semi- Eulerian, without reference to this graph, is not enough	[2]
				Total =	5

#### Mark Scheme

			AN	ISWERED ON INSERT	
3	(i)	AD = 16 $CD = 18$ $CF = 21$ $AC = 23$ $DF = 34$ $BE = 35$ $C$ $F$	M1 A1	Using Kruskal: Not selecting $AC$ and $DF$ Selecting correct arcs in list, or implied (16+18+21+35+46+50, in this order with no others, can imply M1, A1)	
		BG = 46 $AB = 50$	M1	Drawing a spanning tree for these six vertices	
		$\frac{EG}{EG} = \frac{55}{58}$ Total weight = 186	A1	Correct (minimum) spanning tree drawn	
		$\frac{AE}{AE} = \frac{80}{100}$	D1	186 (cao)	[5]
		<del>//// = 100</del>	DI		[3]
	( <b>ii</b> )			Correct working for wrong vertex deleted can score B1, M1, A0	
		Delete <i>BG</i> from spanning tree $186 - 46 = 140$	<b>B</b> 1	Weight of MST on reduced network (ft from part (i)	
		Two shortest arcs from $G$ are $BG$ and $EG$	1.1		
		140 + 46 + 55 = 241 Lower bound = 241	MI A1	Adding two shortest arcs to MST 241 (cao)	[3]
_	(iii)	A-D-C-F-G- or $16+18+21+58+$	M1	Using nearest neighbour	
	(111)	A - D - C - F - G - B - E - A	A1	Correct closed tour listed, not just weights	
		Upper bound = 274	B1	274 (cao)	[3]
	[		l	Total =	11

#### Mark Scheme

			ANSWERED ON INSERT	
4 (i)		B1	Times for flying route, JA = 120 $AG = 80GU = 60$ $UM = 15$ $GM = 80$	
	$F \bullet B \bullet T$ $30  400  300  80$	B1	Times for train route correct JT = 15 $JB = 5$ $BT = 20TP = 300$ $PU = 20$ $PM = 30$	
	W = V = V = P = G $40 = 15 = 10 30$ $80 = 60$ $M = 15 = U$	B1	Times for coach route and driving route correct BV = 400 $VU = 10$ $VM = 15JF = 240$ $FW = 30$ $WU = 20$ $WM = 40$	[3]
	Strictly, these are directed arcs, but they are shown as undirected arcs		Follow through their arc weights if reasonable	
	$J \begin{bmatrix} 1 & 0 \\ \hline & & A \end{bmatrix} A \begin{bmatrix} 4 & 120 \\ \hline & 120 \end{bmatrix}$	M1	<u>Permanent</u> values correct at A, F, B, T A = 120, F = 240, B = 5, T = 15	
	F     6     240     B     2     5     T     3     15       240     5     5     15	M1 d	Both 280 and 275 seen at $M$ (updating at $M$ )	
	V P 315	A1 ft	<u>All temporary</u> labels correct (or implied) and <u>no extras</u>	
	W     8       270     G       5     200       200	B1 ft	<u>All permanent</u> labels correct (or implied) (condone labelling past <i>M</i> )	
	M     9     275     U     7     260       280     275     260	B1 ft	Order of labelling correct (condone labelling past <i>M</i> )	
	Alternatively, if treating as undirected: $J, A, F, B$ and $T$ are unchanged, then			
	Or $V = 8^{\text{th}}$ and $W = 9^{\text{th}}$ $W \boxed{8 \ 270}$ 270 270 Q = 270 Q = 270		Marked as above	
	M     10     275     U     7     260       280     275     260			
	Route: <i>J</i> - <i>A</i> - <i>G</i> - <i>U</i> - <i>M</i>	B1	Correct answer only	[6]

### Mark Scheme

(ii)	The quickest journey time from Jenny's house to the meeting venue	B1	Quickest journey / least travel time or equivalent	[1]
(iii)	Does not allow for waiting for connections There may be delays at the airport She may not want to fly because of the 'carbon footprint' She may want to choose the cheapest route rather than the quickest route She may not like flying She may want to see her friend She may want to head the journey overpight	B1 B1	Any reasonable suggestion for why she may not want to use the drive/fly/underground route or why she may want to use a different route Any second reasonable suggestion	[2]
	She may want to break the journey overnight			[4]
			Total =	12

5	(i)	x = area of wall to be panelled (m2) y = area to be painted z = area to be covered with pinboard	B1 B1	Reference to area or $m^2$ (at least once) Identifying x as panelling, y as paint and z as pinboard, in any way	[2]
	( <b>ii</b> )	$Cost \le \pounds 150$ $\Rightarrow 8x + 4y + 10z \le 150$ $\Rightarrow 4x + 2y + 5z \le 75 \text{ (given)}$	B1 B1	Use of word 'cost' or equivalent $8x + 4y + 10z \le 150$ seen or explicitly referred to	[2]
	(iii)	(Minimise $P =$ ) $15x + 30y + 20z$	B1 ft	Any positive multiple of this eg $3x + 6y + 4z$ or $\frac{1}{4}x + \frac{1}{2}y + \frac{1}{3}z$	[1]
	(iv)	(Minimise $P = 480 + - 5x + 10y$ Subject to $x + 3y \ge 45$ $x \ge 10$ $y \ge 0$ $x + y \le 22$	B1 ft B1 B1	Any positive multiple of this, eg $2y-x(+c)$ - or maximise a negative multiple Any equivalent simplified form $x \ge 10$ may be implied $y \ge 0$ may be implied $x + y \le 22$ , any equivalent simplified form	[3]
	(v)	y 14 12 10 10 12 14 10 12 14	M1 M1 A1 x	ANSWERED ON GRAPH PAPER x = 10 drawn accurately with a sensible scale x + y = 22 drawn accurately with a sensible scale Their $x + 3y = 45$ drawn accurately with a sensible scale Shading correct or identification of the feasible region (triangle with (10, $11\frac{2}{3}$ ), (10, 12) and $(10\frac{1}{2}, 11\frac{1}{2})$ as vertices)	[4]
				Total =	12

#### Mark Scheme

6	(i)	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	B1 B1	Rows and columns may be in any order Objective row with -25, -14, 32 Constraint rows correct (condone omission of <i>P</i> column)	[2]
	(ii)	x column has a negative value in objective row	B1	'negative in top row', '-25', or similar 'most negative in top row' $\Rightarrow$ bod B1	
		entries in all the other rows	B1	Correct reason for not choosing <i>y</i> column	
		$24 \div 6 = 4$ $15 \div 5 = 3$ Least non-negative ratio is 3, so pivot on 5	B1	Both divisions seen and correct choice made (or both divisions seen and correct choice implied from pivoting)	[3]
	(iii)	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	M1 A1 B1 B1 B1 B1 ft B1 ft	Follow through their sensible tableau (with two slack variable columns) and pivot Pivot row correct (no numerical errors) Other rows correct (no numerical errors) Calculation for pivot row Calculation for objective row Calculation for objective row Calculation for other row x, y and $z$ from their tableau P from their tableau provided $P > 0$	[2] [3] [2]
	(iv)	Problem is unbounded No limit to how big $y$ (and hence $P$ ) can be Only negative in objective row is $y$ column, but all entries in this column are negative	B1	Any one of these, or equivalent. If described in terms of pivot choices, must be complete and convincing	[1]
				Total =	13

		$F = N \div B$ $G = INT(A + B \times C)$ $H = B \times C$ C = N - H N = G	3 F) 7 H					For reference only	
7	(i)	F	G	Н	С	Ν	M1	A reasonable attempt at first pass (presented in any form)	
		2.5	2	4	1	2	A1	F = 2.5 and $G = 2$	
		1	1	2	0	1	AI	H = 4 (or double their G value) and $C = 5$ – their H	
		0.5	0	0	1	0	A1	F, G, H, C and N correct for second pass	
		0.5	0	0	1	0	A1	(It their $N$ value) F, G, H, C and $N$ correct for third pass (ft their $N$ value)	[5]
	( <b>ii</b> )	F	G	Н	С	N	M1	A reasonable attempt	
		-2.5 -1.5	-3 -2	-6 -4	1 1	-3 -2	M1 d	First pass correct (or implied)	
		-1	-1	-2	0	-1			
		-0.5 -0.5	-1 -1	-2 -2	1 1	-1 -1	AI	Reaching two lines with the same value for $G$	
								If described in words only, then M1 for a correct statement; M1 d for all correct statements (sufficient to guarantee result), and A1 for convincingly correct explanation of how they know these to be true and why the result follows	
		Does not	termina	te			B1	Saying 'does not stop', or equivalent	[4]
	(iii)	F 3.7 0.3	G 3 0	H 30 0	C 7 3	N 3 0	M1 A1	First pass correct All correct	
		The first v second va the hundr	value is due is th eds digi	the units the tens dig t, and so o	digit of <i>N</i> git, the th on.	V, the ird value is	M1 A1	Outputs are digits of <i>N</i> In reverse order	[4]
	I	·						Total =	13
					-				