

# 4736 Decision Mathematics 1

1	(i)	A	B	C	D	M1	A, B and C correct for first pass	
		614	416	1	198 (A=198)	A1	D = 198 on first pass	
		198	891	2	693 (A=693)	M1	sca at second <u>and</u> third passes	
		693	396	3	297	A1	Second and third passes correct	
<b>Total = 6</b>								
	(ii)	0				B1	0	[1]
	(iii)	To make the algorithm terminate				B1	So that it does not get stuck in a loop	[1]

2	(i)	eg		M1	Graph need not be simple or planar	
				A1	A graph with five vertices and at least three correct vertex orders	
				A1	A graph with five vertices of orders 1, 2, 2, 3, 4	
<b>Total = 6</b>						
	(ii)	Semi-Eulerian		M1	Unless their graph was not connected, in which case the answer is 'neither'	
		It has <u>exactly</u> two odd nodes		A1	(Unless their graph was not connected, in which case follow this through)	
	(iii)	A tree with five vertices would only have four arcs, but this graph has six <u>Or</u> A tree must have at least two vertices of order 1		B2	Give B1 for an incomplete reason, eg 'too many arcs' or 'it has a cycle'	[2]
<b>Total = 6</b>						

## ANSWERED ON INSERT

3	(i)	$AB = 9$ $DF = 14$ $BD = 16$ $CD = 18$ $FG = 20$ <del><math>CF = 22</math></del> $EG = 23$ <del><math>EF = 26</math></del> <del><math>AC = 27</math></del> <del><math>DE = 28</math></del> <del><math>AD = 29</math></del> <del><math>DG = 34</math></del> <del><math>BE = 37</math></del>	<p>Total weight = 100</p>	M1	Not selecting CF (working seen on list)	
				A1	Selecting correct arcs (working seen on list)	
				M1	A spanning tree drawn	
				A1	Correct (minimum) spanning tree drawn	
<b>Total = 5</b>						
				B1	100 cao	

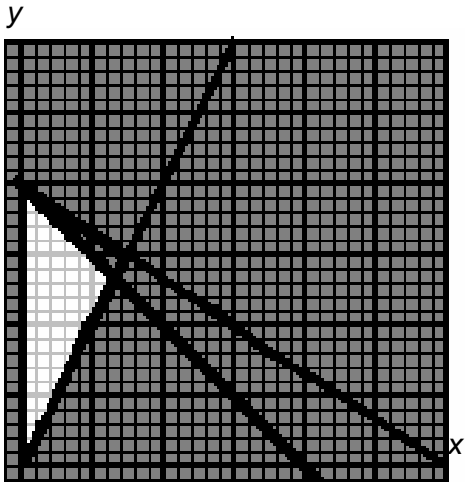
	(ii)	Delete $EG$ from spanning tree $100 - 23 = 77$ Two shortest arcs from $E$ are $EG$ and $EF$ $77 + 23 + 26 = 126$ Lower bound = 126	B1 M1 A1	Follow through from part (i) if possible Weight of MST on reduced network  Adding two shortest arcs to MST 126 cao	[3]																																										
	(iii)	$A - B - D - F - G - E$ - stall  Misses out vertex $C$	M1 A1	$A - B - D - F - G - E$  Cannot continue because $B, D$ and $F$ have already been visited	[2]																																										
	(iv)	$B - A - C - D - F - G - E - B$  Upper bound = 148	M1 A1 B1	Tour starts $B - A - C - D - F -$ Correct tour, starting and ending at $B$ 148 cao	[3]																																										
	(v)	<table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: center;"><math>B</math></td> <td style="border: 1px solid black; padding: 2px;">2</td> <td style="border: 1px solid black; padding: 2px;">9</td> <td style="text-align: center;"><math>E</math></td> <td style="border: 1px solid black; padding: 2px;">6</td> <td style="border: 1px solid black; padding: 2px;">46</td> </tr> <tr> <td></td> <td style="border: 1px solid black; padding: 2px;">9</td> <td></td> <td></td> <td style="border: 1px solid black; padding: 2px;">46</td> <td></td> </tr> </table> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: center;"><math>A</math></td> <td style="border: 1px solid black; padding: 2px;">1</td> <td style="border: 1px solid black; padding: 2px;">0</td> <td style="text-align: center;"><math>D</math></td> <td style="border: 1px solid black; padding: 2px;">3</td> <td style="border: 1px solid black; padding: 2px;">25</td> <td style="text-align: center;"><math>G</math></td> <td style="border: 1px solid black; padding: 2px;">7</td> <td style="border: 1px solid black; padding: 2px;">56</td> </tr> <tr> <td></td> <td style="border: 1px solid black; padding: 2px;"></td> <td style="border: 1px solid black; padding: 2px;"></td> <td></td> <td style="border: 1px solid black; padding: 2px;">29</td> <td style="border: 1px solid black; padding: 2px;">25</td> <td></td> <td style="border: 1px solid black; padding: 2px;">56</td> <td style="border: 1px solid black; padding: 2px;"></td> </tr> </table> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: center;"><math>C</math></td> <td style="border: 1px solid black; padding: 2px;">4</td> <td style="border: 1px solid black; padding: 2px;">27</td> <td style="text-align: center;"><math>F</math></td> <td style="border: 1px solid black; padding: 2px;">5</td> <td style="border: 1px solid black; padding: 2px;">39</td> </tr> <tr> <td></td> <td style="border: 1px solid black; padding: 2px;">27</td> <td></td> <td></td> <td style="border: 1px solid black; padding: 2px;">39</td> <td></td> </tr> </table> Weight = 56 Route = $A - B - D - G$	$B$	2	9	$E$	6	46		9			46		$A$	1	0	$D$	3	25	$G$	7	56					29	25		56		$C$	4	27	$F$	5	39		27			39		M1 A1 B1 B1 B1 B1	(Accept correct working starting from $G$ , if seen) At least three sets of temporary labels correct, with no extras Temporary labels all correct, with no extras Permanent labels correct  Order of labelling (correct or follow through their permanent labels)  56 cao $A - B - D - G$ cao	[4]      [2]
$B$	2	9	$E$	6	46																																										
	9			46																																											
$A$	1	0	$D$	3	25	$G$	7	56																																							
				29	25		56																																								
$C$	4	27	$F$	5	39																																										
	27			39																																											
	(vi)	$A, B, C$ and $G$ are odd  $AB = 9$ $AC = 27$ $AG = 56$ $CG = \underline{42}$ $BG = \underline{47}$ $BC = \underline{34}$ 51                74                90 Repeat $AB$ and $CG$ ( $C - F - G$ ) = 51  Weight = $300 + 51 = 351$	B1 M1 A1 B1	Identifying or using $A, B, C, G$ (seen)  At least one correct pairing seen or total seen (not just six weights) All three totals correct, or explanation of how it is known that other pairings are too long 351 cao	[4]																																										
<b>Total =</b>					<b>23</b>																																										

ANSWERED ON INSERT

4	(i)	8	B1	cao	[1]																					
	(ii)	1 comparison and 1 swap	B1	1 and 1	[1]																					
	(iii)	76 65 21 13 88 62 67 28 34  2 comparisons and 1 swap	B1 B1	Correct list (complete)  2 and 1	[2]																					
	(iv)	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 80%;"></td> <td style="text-align: right;"><math>C</math></td> <td style="text-align: right;"><math>S</math></td> </tr> <tr> <td><u>76 65 21 13</u> 88 62 67 28 34</td> <td style="text-align: right;">1</td> <td style="text-align: right;">0</td> </tr> <tr> <td>88 <u>76 65 21 13</u> 62 67 28 34</td> <td style="text-align: right;">4</td> <td style="text-align: right;">4</td> </tr> <tr> <td>88 76 65 <u>62 21 13</u> 67 28 34</td> <td style="text-align: right;">3</td> <td style="text-align: right;">2</td> </tr> <tr> <td><u>88 76 67 65 62 21 13</u> 28 34</td> <td style="text-align: right;">5</td> <td style="text-align: right;">4</td> </tr> <tr> <td>88 76 67 65 62 28 21 13 34</td> <td style="text-align: right;">3</td> <td style="text-align: right;">2</td> </tr> <tr> <td><u>88 76 67 65 62 34 28 21 13</u></td> <td style="text-align: right;">4</td> <td style="text-align: right;">3</td> </tr> </table>		$C$	$S$	<u>76 65 21 13</u> 88 62 67 28 34	1	0	88 <u>76 65 21 13</u> 62 67 28 34	4	4	88 76 65 <u>62 21 13</u> 67 28 34	3	2	<u>88 76 67 65 62 21 13</u> 28 34	5	4	88 76 67 65 62 28 21 13 34	3	2	<u>88 76 67 65 62 34 28 21 13</u>	4	3	M1 M1 A1 M1 A1 A1	Underlined values correct in 3 <sup>rd</sup> and 4 <sup>th</sup> passes, values not underlined may be left blank Similarly for 5 <sup>th</sup> and 6 <sup>th</sup> passes, follow through slips in previous passes Similarly for 7 <sup>th</sup> and 8 <sup>th</sup> passes, but cao (Dependent on both M marks) Reasonable attempt at Comp and Swap 1 4 3 5 3 4 cao in figures 0 4 2 4 2 3 cao in figures	[3]      [3]
	$C$	$S$																								
<u>76 65 21 13</u> 88 62 67 28 34	1	0																								
88 <u>76 65 21 13</u> 62 67 28 34	4	4																								
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88 76 67 65 62 28 21 13 34	3	2																								
<u>88 76 67 65 62 34 28 21 13</u>	4	3																								

	(v)	Shuttle sort uses 23 comparisons and 17 swaps Shuttle sort is more efficient because although it uses the same number of swaps as bubble sort it uses fewer comparisons	M1	Follow through their totals if possible	[2]
			A1	Choosing shuttle sort with a reason or with totals seen (here) Correct reason stated (comparisons and swaps both compared, in words)	
<b>Total =</b>					<b>12</b>

5	(i)	Katie must spend at least 8 minutes preparing the first batch of cookies so she has at most 52 minutes of baking time. $52 \div 12 = 4.3$ , hence at most 4 batches	M1	Identifying why there is less than 60 minutes of baking time (or seeing 52)	[2]																																																								
			A1	Explaining why 4 is the greatest possible number of batches																																																									
	(ii)	The last batch takes 12 minutes to bake, so Katie has (at most) 48 minutes of preparation time  $8x + 12y + 10z \leq 48 \Rightarrow 4x + 6y + 5z \leq 24$ as given	B1	<u>Explaining why</u> total time for preparation cannot exceed 48 minutes	[2]																																																								
			B1	$8x + 12y + 10z \leq 48$ seen or explicitly referred to																																																									
	(iii)	Must be integer valued	B1	Integers	[1]																																																								
	(iv)	$P = 5x + 4y + 3z$  Assumes that she sells all the cookies (batches) that she makes	B1	$5x + 4y + 3z$ or any positive multiple of this	[2]																																																								
			B1	Assumes she sells them all																																																									
	(v)	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>P</td><td>x</td><td>y</td><td>z</td><td>s</td><td>t</td><td></td></tr> <tr><td>1</td><td>-5</td><td>-4</td><td>-3</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>1</td><td>1</td><td>1</td><td>0</td><td>4</td></tr> <tr><td>0</td><td>4</td><td>6</td><td>5</td><td>0</td><td>1</td><td>24</td></tr> </table> <p><math>4 \div 1 = 4</math>, <math>24 \div 4 = 6</math>, <math>4 &lt; 6</math> Pivot on the 1 in the <math>x</math> column</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>P</td><td>x</td><td>y</td><td>z</td><td>s</td><td>t</td><td></td></tr> <tr><td>1</td><td>0</td><td>1</td><td>2</td><td>5</td><td>0</td><td>20</td></tr> <tr><td>0</td><td>1</td><td>1</td><td>1</td><td>1</td><td>0</td><td>4</td></tr> <tr><td>0</td><td>0</td><td>2</td><td>1</td><td>-4</td><td>1</td><td>8</td></tr> </table> <p>Row 1 = R1 + 5×R2 Row 2 = R2 ÷ 1 Row 3 = R3 - 4×R2</p> <p><math>x = 4</math>, <math>y = 0</math>, <math>z = 0</math>, <math>P = 20</math></p> <p>Katie should make 4 batches of plain cookies, and no chocolate chip or fruit cookies, to give a profit of £20.</p>	P	x	y	z	s	t		1	-5	-4	-3	0	0	0	0	1	1	1	1	0	4	0	4	6	5	0	1	24	P	x	y	z	s	t		1	0	1	2	5	0	20	0	1	1	1	1	0	4	0	0	2	1	-4	1	8	M1	Correct use of slack variable columns	[3]
P			x	y	z	s	t																																																						
1	-5	-4	-3	0	0	0																																																							
0	1	1	1	1	0	4																																																							
0	4	6	5	0	1	24																																																							
P	x	y	z	s	t																																																								
1	0	1	2	5	0	20																																																							
0	1	1	1	1	0	4																																																							
0	0	2	1	-4	1	8																																																							
	A1	Objective row correct (cao)																																																											
			A1	Constraint rows correct (cao)																																																									
			B1	Working need not be seen Correct pivot choice (row 2) (cao)																																																									
			M1	Follow through their tableau and pivot choice, if possible	[4]																																																								
			A1	sca pivoting ( $x$ , $t$ cols, $P$ not decreased) Correct tableau (final column contains no negative values)																																																									
			B1	Showing valid method, may imply row 2																																																									
				Follow through their tableau, if reasonable (non-negative variables)																																																									
			M1	Reading off values from tableau (may be implied from answer)	[3]																																																								
			A1	Interpretation: 4 batches of plain cookies (may imply none of others)																																																									
			A1	Interpretation: £20																																																									

<p>(vi)</p>  <p>Vertices of feasible region are  <math>(0, 0)</math>, <math>(0, 4)</math> and <math>(1\frac{1}{3}, 2\frac{2}{3})</math></p> <p><math>x = 0, y = 4 \Rightarrow P = 16</math>  <math>x = 1, y = 3 \Rightarrow P = 17</math>  <math>(x = 1\frac{1}{3}, y = 2\frac{2}{3} \Rightarrow P = 17\frac{1}{3})</math></p> <p>Make 1 batch of plain cookies and 3 batches of chocolate chip cookies</p>		<p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>B1</p>	<p>At least two of the lines <math>y = 2x</math>, <math>x+y = 4</math> and <math>4x + 6y = 24</math> drawn correctly</p> <p>All three lines drawn correctly and graph has both scales and labels</p> <p>Feasible region identified and correct</p> <p><u>Follow through</u> their feasible region if possible</p> <p>At least two correct</p> <p><u>All</u> (three) correct (1 dp or better)</p> <p>Or a line of constant profit <u>drawn</u> (or gradient discussed) and used correctly on <u>integer-valued</u> coordinates</p> <p>For (1, 3) or 17 chosen (cao)</p> <p>Interpretation: 1 batch of plain, 3 batches of chocolate chip (cao)</p>	<p>[3]</p> <p>[2]</p> <p>[3]</p> <p>[3]</p>
<b>Total =</b>			<b>25</b>	