4734 Probability & Statistics 3

Penalise 2 sf instead of 3 once only. Penalise final answer ≥ 6 sf once only.

1	(i)	$\int_0^1 \frac{2}{5}x^2\,dx + \int_1^4 \frac{2}{5}\sqrt{x}\,dx$ $= \left[\frac{2x^3}{15}\right]_0^1 + \left[\frac{4x^{3/2}}{15}\right]_1^4 = 2$	M1 A1 A1 **3**	Attempt to integrate $xf(x)$, both parts added, limits Correct indefinite integrals Correct answer
	(ii)	$\int_2^4 \frac{2}{5\sqrt{x}}\,dx = \left[\frac{4\sqrt{x}}{5}\right]_2^4 = \frac{4}{5}(2-\sqrt{2})$ or 0.4686	M1 A1 A1 **3**	Attempt correct integral, limits; needs "1 –" if $\mu < 1$ Correct indefinite integral, √ on their μ Exact aef, or in range [0.468, 0.469]
2	(i)	Po(0.5), Po(0.75) Po(0.7) and Po(0.9) $A + B \sim$ Po(1.6) P($A + B \geq 5$) = 0.0237 B(20, 0.0237) $0.9763^{20} + 20\times0.9763^{19}\times0.0237$ **= 0.9195**	M1 A1 M1 A1 M1 A1√ A1 **7**	0.5, 0.75 scaled These Sum of Poissons used, can have wrong parameters 0.0237 from tables or calculator Binomial (20, their p), soi Correct expression, their p Answer in range [0.919, 0.92]
	(ii)	Bacteria should be independent in drugs; *or* sample should be random	B1 **1**	Any valid relevant comment, must be contextualised
3	(i)	Sample mean = 6.486 $s^2 = 0.00073$ $6.486 \pm 2.776 \times \sqrt{\frac{0.00073}{5}}$ **(6.45, 6.52)**	B1 B1 M1 B1 A1A1 **6**	 0.000584 if divided by 5 Calculate sample mean ± $ts/\sqrt{5}$, allow 1.96, s^2 etc t = 2.776 seen Each answer, cwo (6.45246, 6.5195)
	(ii)	$2\pi \times$ above [= (40.5, 41.0)]	M1 **1**	
4	(i)	H_0: $p_1 = p_2$; H_1: $p_1 \neq p_2$, where p_i is the proportion of all solvers of puzzle i Common proportion 39/80 $s^2 = 0.4875 \times 0.5125 / 20$ $(\pm)\frac{0.6-0.375}{0.1117} = (\pm)2.013$ 2.013 > 1.96, or 0.022 < 0.025 Reject H_0. Significant evidence that there is a difference in standard of difficulty	B1 M1A1 B1 M1 A1√ M1 A1√ **8**	Both hypotheses correctly stated, allow eg $\hat{p}$ [= 0.4875] [= 0.01249, σ = 0.11176] (0.6 – 0.375)/s Allow 2.066√ from unpooled variance, p = 0.0195 Correct method and comparison with 1.96 or 0.025, allow unpooled, 1.645 from 1-tailed *only* Conclusion, contextualised, not too assertive
	(ii)	One-tail test used Smallest significance level 2.2(1)%	M1 A1 **2**	One-tailed test stated or implied by Φ("2.013"), OK if off-scale; allow 0.022(1)

5 (i)	Numbers of men and women should have normal dists; with equal variance; distributions should be independent	B1 B1 B1 **3**	Context & 3 points: 2 of these, B1; 3, B2; 4, B3. [Summary data: 14.73 49.06 52.57 16.24 62.18 66.07]
(ii)	H_0: $\mu_M = \mu_W$; H_1: $\mu_M \neq \mu_W$ $3992 - \frac{221^2}{15} + 5538 - \frac{276^2}{17}$ [≈ 1793] 1793/(14 + 16) = 59.766 $(\pm)\frac{221/15 - 276/17}{\sqrt{59.766(\frac{1}{15}+\frac{1}{17})}} = (-)0.548$ Critical region: $\lvert t \rvert \geq 2.042$ Do not reject H_0. Insufficient evidence of a difference in mean number of days	B1 M1 A1 A1 M1 A1√ A1 B1 M1 A1√ **10**	Both hypotheses correctly stated Attempt at this expression (see above) Either 1793 or 30 Variance estimate in range [59.7, 59.8] (or √ = 7.73) Standardise, allow wrong (but not missing) $1/n$ Correct formula, allow $s^2(\frac{1}{15}+\frac{1}{17})$ or $(\frac{s_1^2}{15}+\frac{s_2^2}{17})$, allow 14 & 16 in place of 15, 17; 0.548 or –0.548 2.042 seen Correct method and comparison type, must be *t*, allow 1-tail; conclusion, in context, not too assertive
(iii)	Eg Samples not indep't so test invalid	B1 **1**	Any relevant valid comment, eg "not representative"

6	(i)	F(0) = 0, F(π/2) = 1 Increasing	B1 B1 **2**	Consider both end-points Consider F between end-points, can be asserted
	(ii)	$\sin^4(Q_1) = ¼$ $\sin(Q_1) = 1/\sqrt{2}$ $Q_1 = \pi/4$	M1 A1 A1 **3**	Can be implied. Allow decimal approximations Or 0.785(4)
	(iii)	$G(y) = P(Y \le y) = P(T \le \sin^{-1} y)$ $= F(\sin^{-1} y)$ $= y^4$ $g(y) = \begin{cases} 4y^3 & 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$	M1 A1 A1 M1 A1 **5**	Ignore other ranges Differentiate G(y) Function and range stated, allow if range given in G
	(iv)	$\int_0^1 \frac{4}{1+2y}\,dy = \left[2\ln(1+2y)\right]_0^1$ = **2 ln 3**	M1 A1 A1 **3**	Attempt $\int \frac{g(y)}{y^3+2y^4}dy$; $\int_0^1 \frac{4}{1+2y}dy$ Or 2.2, 2.197 or better
7	(i) α	$\Phi\left(\frac{8.084-8.592}{0.7534}\right) = \Phi(-0.674) = 0.25$ Φ(0) – Φ(above) = 0.25 P(8.592 ≤ X ≤ 9.1) = same by symmetry	M1 A1 A1 A1 **4**	Standardise once, allow √ confusions, ignore sign Obtain 0.25 for one interval For a second interval, justified, eg using Φ(0) = 0.5 For a third, justified, eg "by symmetry"
	or β	$\frac{x-8.592}{0.7534} = 0.674$ $x = 8.592 \pm 0.674\times0.7534$ = (8.084, 9.100)	[from probabilities to ranges] M1A1 A1A1	A1 for art 0.674
	(ii)	H_0: normal distribution fits data All E values 50/4 = 12.5 $X^2 = \frac{4.5^2+9.5^2+1.5^2+3.5^2}{12.5} = 10$ 10 > 7.8794 Reject H_0. Significant evidence that normal distribution is not a good fit.	B1 B1 M1 A1 B1 M1 A1√ **7**	*Not* N(8.592, 0.7534). Allow "it's normally distributed" [Yates: 8.56: A0] CV 7.8794 seen Correct method, incl. formula for χ^2 and comparison, allow wrong ν Conclusion, in context, not too assertive
	(iv)	$8.592 \pm 2.576 \times \frac{0.7534}{\sqrt{49}}$ (8.315, 8.869)	M1 A1 A1 **3**	Allow √ errors, wrong σ or z, allow 50 Correct, including z = 2.576 or t_{49} = 2.680, *not* 50 In range [8.31, 8.32] and in range (8.86, 8.87], even from 50, or (8.306, 8.878) from t_{49}