General Certificate of Education
January 2006
Advanced Level Examination

**MATHEMATICS** **MPC4**
**Unit Pure Core 4**

Wednesday 25 January 2006 9.00 am to 10.30 am

**For this paper you must have:**
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

## Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC4.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.

## Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

## Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

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Answer **all** questions.

**1** (a) The polynomial $f(x)$ is defined by $f(x) = 3x^3 + 2x^2 - 7x + 2$.

(i) Find $f(1)$. *(1 mark)*

(ii) Show that $f(-2) = 0$. *(1 mark)*

(iii) Hence, or otherwise, show that

$$\frac{(x-1)(x+2)}{3x^3 + 2x^2 - 7x + 2} = \frac{1}{ax + b}$$

where $a$ and $b$ are integers. *(3 marks)*

(b) The polynomial $g(x)$ is defined by $g(x) = 3x^3 + 2x^2 - 7x + d$.

When $g(x)$ is divided by $(3x - 1)$, the remainder is 2. Find the value of $d$. *(3 marks)*

**2** A curve is defined by the parametric equations

$$x = 3 - 4t \qquad y = 1 + \frac{2}{t}$$

(a) Find $\dfrac{dy}{dx}$ in terms of $t$. *(4 marks)*

(b) Find the equation of the tangent to the curve at the point where $t = 2$, giving your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers. *(4 marks)*

(c) Verify that the cartesian equation of the curve can be written as

$$(x-3)(y-1) + 8 = 0$$ *(3 marks)*

**3** It is given that $3\cos\theta - 2\sin\theta = R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$.

(a) Find the value of $R$. *(1 mark)*

(b) Show that $\alpha \approx 33.7°$. *(2 marks)*

(c) Hence write down the maximum value of $3\cos\theta - 2\sin\theta$ and find a **positive** value of $\theta$ at which this maximum value occurs. *(3 marks)*

**4** On 1 January 1900, a sculpture was valued at £80.

When the sculpture was sold on 1 January 1956, its value was £5000.

The value, £$V$, of the sculpture is modelled by the formula $V = Ak^t$, where $t$ is the time in years since 1 January 1900 and $A$ and $k$ are constants.

(a) Write down the value of $A$. *(1 mark)*

(b) Show that $k \approx 1.07664$. *(3 marks)*

(c) Use this model to:

(i) show that the value of the sculpture on 1 January 2006 will be greater than £200 000; *(2 marks)*

(ii) find the year in which the value of the sculpture will first exceed £800 000. *(3 marks)*

**5** (a) (i) Obtain the binomial expansion of $(1 - x)^{-1}$ up to and including the term in $x^2$. *(2 marks)*

(ii) Hence, or otherwise, show that

$$\frac{1}{3 - 2x} \approx \frac{1}{3} + \frac{2}{9}x + \frac{4}{27}x^2$$

for small values of $x$. *(3 marks)*

(b) Obtain the binomial expansion of $\dfrac{1}{(1 - x)^2}$ up to and including the term in $x^2$. *(2 marks)*

(c) Given that $\dfrac{2x^2 - 3}{(3 - 2x)(1 - x)^2}$ can be written in the form $\dfrac{A}{(3 - 2x)} + \dfrac{B}{(1 - x)} + \dfrac{C}{(1 - x)^2}$, find the values of $A$, $B$ and $C$. *(5 marks)*

(d) Hence find the binomial expansion of $\dfrac{2x^2 - 3}{(3 - 2x)(1 - x)^2}$ up to and including the term in $x^2$. *(3 marks)*

**Turn over for the next question**

**6** (a) Express $\cos 2x$ in the form $a\cos^2 x + b$, where $a$ and $b$ are constants. *(2 marks)*

(b) Hence show that $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx = \frac{\pi}{a}$, where $a$ is an integer. *(5 marks)*

**7** The quadrilateral $ABCD$ has vertices $A(2, 1, 3)$, $B(6, 5, 3)$, $C(6, 1, -1)$ and $D(2, -3, -1)$.

The line $l_1$ has vector equation $\mathbf{r} = \begin{bmatrix} 6 \\ 1 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

(a) (i) Find the vector $\overrightarrow{AB}$. *(2 marks)*

(ii) Show that the line $AB$ is parallel to $l_1$. *(1 mark)*

(iii) Verify that $D$ lies on $l_1$. *(2 marks)*

(b) The line $l_2$ passes through $D(2, -3, -1)$ and $M(4, 1, 1)$.

(i) Find the vector equation of $l_2$. *(2 marks)*

(ii) Find the angle between $l_2$ and $AC$. *(3 marks)*

**8** (a) Solve the differential equation

$$\frac{dx}{dt} = -2(x-6)^{\frac{1}{2}}$$

to find $t$ in terms of $x$, given that $x = 70$ when $t = 0$. *(6 marks)*

(b) Liquid fuel is stored in a tank. At time $t$ minutes, the depth of fuel in the tank is $x$ cm. Initially there is a depth of 70 cm of fuel in the tank. There is a tap 6 cm above the bottom of the tank. The flow of fuel out of the tank is modelled by the differential equation

$$\frac{dx}{dt} = -2(x-6)^{\frac{1}{2}}$$

(i) Explain what happens when $x = 6$. *(1 mark)*

(ii) Find how long it will take for the depth of fuel to fall from 70 cm to 22 cm. *(2 marks)*

**END OF QUESTIONS**