Solutions

1. (a) Adds 32 to $AB + BA$ (ACB) B1

47 to $AE + EA$ ($ACDE$) B1

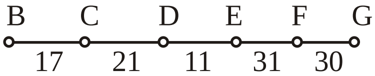
32 to $CE + EC$ (CDE) B1

53 to $DG + GD$ (DCG) B1 4

(b) $A \quad C \quad B \quad D \quad E \quad F \quad G \quad A$ M1A1

15 + 17 + 38 + 11 + 31 + 30 + 23 = 165 miles A1 3

(c) e.g. BC, CD, DE, EF, FG M1



weight of RSMT = 110 miles A1

Lower bound = 110 + 15 + 23 M1

= 148 miles A1ft 4

[11]

2. (a)

$$\begin{array}{c} \\ \left[\begin{array}{ccc} 2 & -1 & 3 \\ -3 & 4 & -4 \end{array}\right] \\ \text{col max} \quad 2 \uparrow \quad 4 \quad 3 \end{array} \quad \begin{array}{c} \text{Row min} \\ -1 \leftarrow \\ -4 \end{array}$$

M1A1

$2 \neq -1 \therefore$ not stable A1 3

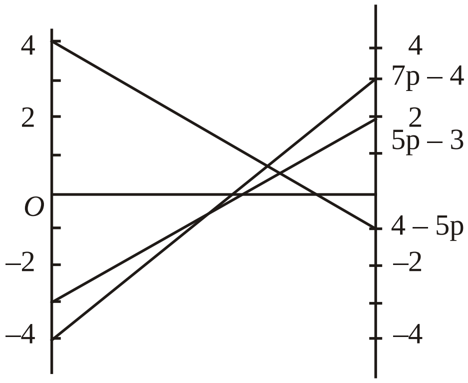
(b) Let Denis play 1 with probability p

So he'll play 2 with probability $1 - p$

If Hilary plays 1 Denis wins: $2p - 3(1 - p) = 5p - 3$ M1

If Hilary plays 2 Denis wins: $-p + 4(1 - p) = 4 - 5p$ A2,1,0

If Hilary plays 3 Denis wins: $3p - 4(1 - p) = 7p - 4$



M1A2,1,0

$5p - 3 = 4 - 5p$

$10p = 7$

$p = \frac{7}{10}$ M1A1ft

Denis should play 1 with probability $\frac{7}{10}$

2 with probability $\frac{3}{10}$

the value of the game is $\frac{1}{2}$ B1ftB1 10

[13]

3. (a)

$$\begin{bmatrix} 66 & 101 & 85 & 36 \\ 66 & 98 & 74 & 38 \\ 63 & 97 & 71 & 34 \\ 67 & 102 & 78 & 35 \end{bmatrix}$$

reducing rows first then columns

$$\begin{bmatrix} 30 & 65 & 49 & 0 \\ 28 & 60 & 36 & 0 \\ 29 & 63 & 37 & 0 \\ 32 & 67 & 43 & 0 \end{bmatrix} \quad \begin{bmatrix} 2 & 5 & 13 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 3 & 1 & 0 \\ 4 & 7 & 7 & 0 \end{bmatrix}$$

M1A1

$$\begin{bmatrix} 1 & 4 & 12 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 3 & 6 & 6 & 0 \end{bmatrix}$$

M1A1ftA1ft

$$\begin{bmatrix} 0 & 3 & 11 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 2 & 0 & 1 \\ 2 & 5 & 5 & 0 \end{bmatrix}$$

M1A1ftA1ft

A – cutting
B – stitching
C – filling
D – dressing A1 9

(b) 66 + 98 + 71 + 35 =270 seconds B1 1

(c) 20 × 98 + 66 +71 + 35 = 2132 seconds M1A1ft
= 35 minutes 32 seconds A1 3

[13]

4. (a) B2,1,0 2

	A	S	D	Seats
1			0	94
2			0	65
3			0	80
	18	200	21	

(b) total supply > total demand B1 1

(c)(d)

	A	S	D
1	18	76	
2		65	
3		59	21

B1
M1A1ft

S(1) = 0 D(A) = 5

S(2) = –0.7 D(S) = 4.5
S(3) = –0.5 D(D) = 0.5

I_{1D} = 0 – 0 – 0.5 = –0.5 *
I_{2A} = 4.2 + 0.7 – 5 = –0.1
I_{2D} = 0 + 0.7 – 0.5 = 0.2
I_{3A} = 4.6 + 0.5 – 5 = 0.1 A1

	A	S	D			A	S	D		
1	18	76 – θ	θ	Entering 1D	1	18	55	21	M1A1ft	
2		65		Exiting 3D	2		65		A1	7
3		59 + θ	21 – θ	θ = 21	3		80			

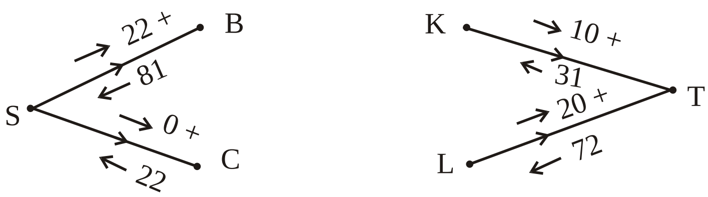
(e) S(1) = 0 D(A) = 4.9 M1
S(2) = –0.7 D(B) = 4.5 A1
S(3) = –0.5 D(B) = 0

I_{1A}= 5 – 0 – 4.9 = 0.1
I_{2D} = 0 + 0.7 – 0 = 0.7
I_{3A} = 4.6 + 0.5 – 4.9 = 0.2
I_{3D} = 0 + 0.5 – 0 = 0.5 A1

Optimal since all II's ≥ 0 A1
cost £902.70 M1A1 6

[16]

5. (a)

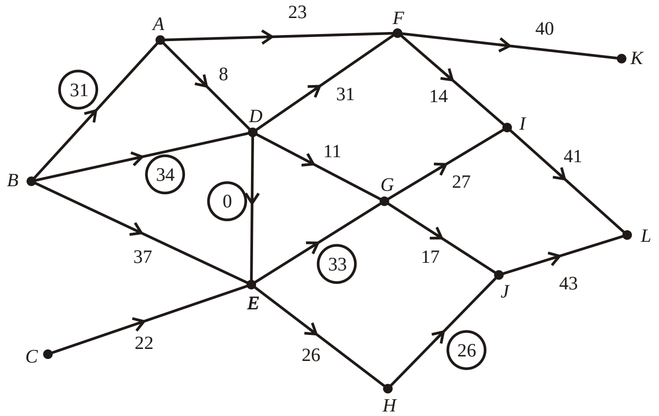


M1A1
A1 3

(b) <u>103</u> B1 1

(c) e.g. S B E G I L T – 3 M1
S B E D F K T – 5 A4,3,2,1,0 5
S B E H J G D F K T – 4
S B E G D F I L T – 9

(d) e.g.



M1A1
A1 3

Flow value <u>124</u> (given)

(e) Max flow = min cut
cut through <u>AB, BD, DE, EG, HJ</u> M1A1 2

[14]

6. <u>Alt 1</u>

Game from R's point of view.

	A1	A2	A3
R_1	–6	3	–5
R_2	2	–1	–4
R_3	3	–2	1

Add 7

	A1	A2	A3
R_1	1	10	2
R_2	9	6	3
R_3	10	5	8

B1, B1

Let R play 1 with probability P_1
2 with probability P_2
3 with probability P_3 B1
V = value of the game

Maximise P = V B1

Subject to $V - P_1 - 9P_2 - 10P_3 \leq 0$ M1A1ft
$V - 10P_1 - 6P_2 - 5P_3 \leq 0$ A1ft
$V - 2P_1 - 3P_2 - 8P_3 \leq 0$ A1ft
$P_1 + P_2 + P_3 \leq 1$ accept= A1 8
$V, P_1, P_2, P_3 \geq 0$

<u>Alt 2</u>

Add 4 to all entries B1

	R_1	R_2	R_3
A1	10	2	1
A2	1	5	6
A3	9	8	3

Let R play 1 with probability P_1
2 with probability P_2
3 with probability P_3 let V = value of game. B1

Let $x_1 = \frac{P_1}{V}, x_2 = \frac{P_2}{V}, x_3 = \frac{P_3}{V}$ B1

Maximise $P = x_1 + x_2 + x_3$ B1

Subject to $10x_1 + 2x_2 + x_3 \leq 1$ M1A1ft

$x_1 + 5x_2 + 6x_3 \le 1$ A1ft

$9x_1 + 8x_2 + 3x_3 \le 1$ A1

$x_1, x_2, x_3 \ge 0$ accept $P_i \ge 0$

[8]

7. (a)

Stage	State	Action	Destination	Value	
	J	JY	Y	98*	
1	K	KY	Y	94*	B1
	L	LY	Y	86*	
	G	GJ	J	max(79, 98) = 98*	M1
		GK	K	max(98, 94) = 98*	
2	H	HK	K	max(95, 94) = 95	A1A1
		HL	L	max(72, 86) = 86*	
	I	IL	L	max(56, 86) = 86*	
	C	CG	G	max(50, 98) = 98*	
	D	DG	G	max(92, 98) = 98	M1
3		DH	H	max((81, 86) = 86*	A1A1ft
	E	EH	H	max(89, 86) = 89*	
	F	FH	H	max(84, 86) = 86*	
		FI	I	max(72, 86) = 86*	
	A	AC	C	max(95, 98) = 98	M1
		AD	D	max(86, 86) = 86*	A1ft
4		AE	E	max(63, 89) = 89	
	B	BE	E	max(88, 89) = 89	
		BF	F	max(87, 86) = 87*	
5	X	XA	A	max(55, 86) = 86*	A1ft
		XB	B	max(85, 87) = 87	

X A D H L Y (minimax = 86) M1A1ft 12

(b) X B F $\begin{smallmatrix} \text{H} \\ \text{I} \end{smallmatrix}$ L Y (minimax = 87) one M1A1 2

[14]

8. (a) $P - 2x - 4y - 3z = 0$ (o.e.) B2,0 2

(b) $12x + 4y + 5z \le 246$ B1

$9x + 6y + 3z \le 153$ B1

$5x + 2y - 2z \le 171$ B1 3

(c)

basic variable	x	y	z	r	s	t	Value
r	12	4	5	1	0	0	246
s	9	6	3	0	1	0	153
t	5	2	–2	0	0	1	171
P	–2	–4	–3	0	0	0	0

b.v.	x	y	z	r	s	t	Value	Row operations	
r	6	0	3	1	$-\frac{2}{3}$	0	144	$R_1 - 4R_2$	M1A1
y	$\frac{3}{2}$	1	½	0	$\frac{1}{6}$	0	25.5	$R_2 \div 6$	M1A1ft
t	2	0	–3	0	$-\frac{1}{3}$	1	120	$R_3 - 2R_2$	B1ft
P	4	0	–1	0	$\frac{2}{3}$	0	102	$R_4 + 4R_2$	

b.v.	x	y	z	r	s	t	Value	Row operations	
z	2	0	1	$\frac{1}{3}$	$-\frac{2}{9}$	0	48	$R_1 \div 3$	M1A1
y	½	1	0	$-\frac{1}{6}$	$\frac{5}{18}$	0	1.5	$R_2 - ½R_1$	M1A1
t	8	0	0	1	–1	1	264	$R_3 + 3R_1$	
P	6	0	0	$\frac{1}{3}$	$\frac{4}{9}$	0	150	$R_4 + R_1$	

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(d) $P = 150$ $x = 0$ $y = 1.5$ $z = 48$ M1A1ft
$r = 0$ $s = 0$ $t = 264$ A1ft 3

(e) (The third constraint) $t \neq 0$ B1ft 1

[18]

9. (a) 85 B1 1

(b) $c_1 = 140$, $c_2 = 104$ B1, B1 2

(c) e.g.

S	*B*	*D*	*F*	*H*	*J*	*T*		– 4	M1A1
S	*B*	*D*	*F*	*G*	*T*			– 1	
S	*B*	*D*	*F*	*C*	*H*	*I*	*T*	– 2	A1
S	*B*	*D*	*F*	*C*	*H*	*J*	*T*	– 2	A1
S	*B*	*D*	*E*	*G*	*T*			– 10	A1 5

(d) Max flow – min cut theorem, flow is 104, min cut is c_2 M1A1 2

[10]