

**General Certificate of Education (A-level)  
January 2012**

**Mathematics**

**MFP2**

**(Specification 6360)**

**Further Pure 2**

**Final**

***Mark Scheme***

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## Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

## No Method Shown

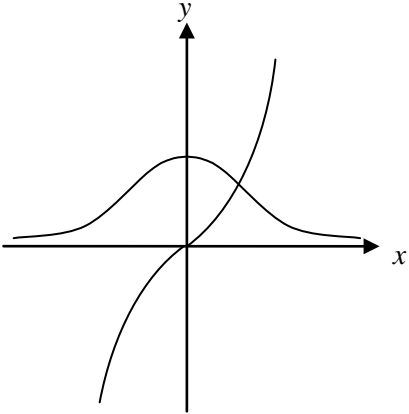
Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

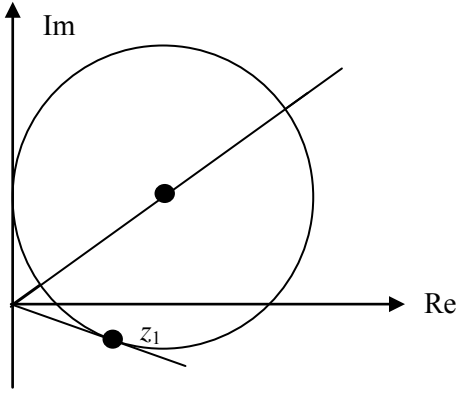
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

Q	Solution	Marks	Total	Comments
<p><b>1(a)</b></p>	<div style="text-align: center;">  </div> <p>Sketch <math>y = \sinh x</math></p> <p>Sketch <math>y = \operatorname{sech} x</math> :                      Symmetry about <math>x = 0</math> with max point                      Asymptote <math>y = 0</math>                      Point <math>(0, 1)</math> marked or implied</p>	<p>B1</p> <p>B1 B1 B1</p>	<p>4</p>	<p>gradient <math>&gt; 0</math> at <math>(0, 0)</math>; no asymptotes</p> <p>must not cross <math>x</math>-axis</p>
<p><b>(b)</b></p>	<p><math>\sinh x = \frac{1}{\cosh x}</math></p> <p><math>\sinh 2x = 2</math></p> <p>Use of <math>\ln</math></p> <p><math>x = \frac{1}{2} \ln(2 + \sqrt{5})</math></p> <p><b>or</b></p> <p><math>\frac{1}{2}(e^{2x} - e^{-2x}) = 2</math> OE</p> <p><math>e^{4x} - 4e^{2x} - 1 = 0</math></p> <p>Correct use of formula</p> <p>Result</p>	<p>M1</p> <p>M1 m1</p> <p>A1</p> <p>(M1)</p> <p>(M1) (m1) (A1)</p>	<p>4</p>	<p>use of double angle formula dependent on previous M2</p> <p>incorrect <math>\sinh x, \cosh x</math> M0 (no marks)</p> <p>ie multiply by <math>e^{2x}</math> and rewrite</p>
<b>Total</b>			<b>8</b>	

Q	Solution	Marks	Total	Comments
<p>2(a)</p> 	<p>Half-line with gradient &lt; 1</p>	<p>B1</p>	<p>1</p>	<p>condone a short line, ie it stops at or inside circle</p>
(b)(i)	<p>Circle centre on <math>L</math>, <math>x</math>-coord 6 indicated touching <math>\text{Re } z = 0</math> not at <math>(0, 0)</math></p>	<p>B1 B1</p>	<p>2</p>	<p>not touching <math>\text{Re}</math> axis</p>
(ii)	<p><math>y</math>-coord of centre is <math>2\sqrt{3}</math> or <math>\frac{6}{\sqrt{3}}</math> <math>z_0 = 6 + 2\sqrt{3}i</math>, <math>k = 6</math></p>	<p>B1 B1F, B1</p>	<p>3</p>	<p>OE; PI ft error in coords of centre</p>
(iii)	<p>Point <math>z_1</math> shown <math>\arg z_1 = -\frac{1}{6}</math></p>	<p>B1 B1</p>	<p>2</p>	<p>PI</p>
<b>Total</b>			<b>8</b>	
<p>3(a)</p>	$\frac{dy}{dx} = \frac{1}{2 \tanh x} \times \text{sech}^2 x$ $= \frac{1}{2 \sinh x \cosh x}$ $= \frac{1}{\sinh 2x}$	<p>B1 B1 M1 A1</p>	<p>4</p>	<p>for expressing in terms of <math>\sinh x</math> and <math>\cosh x</math> AG; PI by previous line</p>
(b)	$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + \frac{1}{\sinh^2 2x}}$ $= \sqrt{\frac{\cosh^2 2x}{\sinh^2 2x}}$ $= \frac{\cosh 2x}{\sinh 2x}$ <p>Integral is <math>\frac{1}{2} \ln \sinh 2x</math></p> $\sinh(2 \ln 4) = \frac{255}{32} \quad \sinh(2 \ln 2) = \frac{15}{8}$ $s = \frac{1}{2} \ln \left(\frac{17}{4}\right)$	<p>M1 m1 A1 M1A1 B1B1 A1F</p>	<p>8</p>	<p>use of formula; accept <math>\sqrt{\quad}</math> inserted at any stage relevant use of <math>\cosh^2 - \sinh^2 = 1</math> OE M1 for <math>\ln \sinh</math> PI ft error in <math>\frac{1}{2}</math></p>
<b>Total</b>			<b>12</b>	

Q	Solution	Marks	Total	Comments
4	<p>Assume result true for <math>n = k</math></p> <p>Then <math>u_{k+1} = \frac{3}{4 - \left(\frac{3^{k+1} - 3}{3^{k+1} - 1}\right)}</math></p> $= \frac{3(3^{k+1} - 1)}{4(3^{k+1} - 1) - (3^{k+1} - 3)}$ $4 \times 3^{k+1} - 3^{k+1} = 3^{k+2}$ $u_{k+1} = \frac{3^{k+2} - 3}{3^{k+2} - 1}$ $n = 1 \quad \frac{3^2 - 3}{3^2 - 1} = \frac{3}{4} = u_1$ <p>Induction proof set out properly</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>E1</p>	<p>6</p> <p><b>6</b></p>	<p>clearly shown</p> <p>must have earned previous 5 marks</p>
<b>Total</b>			<b>6</b>	
5	<p>Numerator = <math>e^{\frac{p\pi i}{8}}</math></p> <p>Denominator = <math>e^{\frac{-q\pi i}{12}}</math></p> <p>Fraction = <math>e^{\frac{p\pi i}{8} + \frac{q\pi i}{12}}</math></p> $= e^{\frac{\pi i}{24}(3p+2q)}$ <p><math>i = e^{\frac{12\pi i}{24}}</math></p> $3p + 2q = 12$ $p = 2, q = 3$ <p><b>Alternative 1</b></p> <p>Numerator = <math>\cos \frac{p\pi}{8} + i \sin \frac{p\pi}{8}</math></p> <p>Denominator = <math>\cos \frac{-q\pi}{12} + i \sin \frac{-q\pi}{12}</math></p> <p>Fraction =</p> $\left(\cos \frac{p\pi}{8} + i \sin \frac{p\pi}{8}\right) \left(\cos \frac{q\pi}{12} + i \sin \frac{q\pi}{12}\right)$ $= \cos \frac{\pi}{24}(3p + 2q) + i \sin \frac{\pi}{24}(3p + 2q)$ $= i \text{ if } \cos \frac{\pi}{24}(3p + 2q) = 0$ <p style="text-align: right;"><math>\text{or } \sin \frac{\pi}{24}(3p + 2q) = 1</math></p> $3p + 2q = 12$ $p = 2, q = 3$ <p><b>Alternative 2</b></p> <p>LHS <math>\cos \frac{p\pi}{8} + i \sin \frac{p\pi}{8}</math></p> <p>RHS <math>i \cos \frac{q\pi}{12} + \sin \frac{q\pi}{12}</math></p> $\cos \frac{p\pi}{8} = \sin \frac{q\pi}{12} \text{ or } \sin \frac{p\pi}{8} = \cos \frac{q\pi}{12}$ <p>Introduction of <math>\frac{\pi}{2}</math></p> $\frac{p\pi}{8} = \frac{\pi}{2} - \frac{q\pi}{12}$ $3p + 2q = 12$ $p = 2, q = 3$	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>m1</p> <p>A1F</p> <p>A1</p> <p>(B1)</p> <p>(B1)</p> <p>(M1)</p> <p>(A1)</p> <p>(m1)</p> <p>(A1F)</p> <p>(A1)</p> <p>(B1)</p> <p>(B1)</p> <p>(M1)</p> <p>(m1)</p> <p>(A1)</p> <p>(A1F)</p> <p>(A1)</p>	<p>7</p> <p>(7)</p> <p>(7)</p>	<p>allow for attempt to subtract powers</p> <p>OE</p> <p>ft errors of sign or arithmetic slips</p> <p>CAO</p> <p>needs more than just <math>\cos \frac{q\pi}{12} - \sin \frac{p\pi}{12}</math></p> <p>CAO</p> <p>CAO (correct answers, insufficient working 3/7 only)</p>
<b>Total</b>			<b>7</b>	

Q	Solution	Marks	Total	Comments
6(a)	$7 + 4x - 2x^2 = 9 - 2(x-1)^2$	M1A1	2	
(b)	Put $u = \sqrt{2}(x-1)$ $du = \sqrt{2} dx$ $I = \frac{1}{\sqrt{2}} \int \frac{du}{\sqrt{9-u^2}}$ $= \frac{1}{\sqrt{2}} \sin^{-1} \frac{u}{3}$ Change limits or replace $u$ $= \frac{\pi}{4\sqrt{2}}$ or $\frac{\pi\sqrt{2}}{8}$	M1 A1F A1F A1 m1 A1	6	allow $u = k(x-1)$ any $k$ ft error in (a); must have $u^2$ only, ie $\frac{1}{\sqrt{2}}$ outside integrand for $\sin^{-1} \frac{u}{p}$ provided $\sin^{-1}$ CAO
	<b>Alternative</b> – if integration is attempted without substitution: $\sin^{-1} \frac{1}{\sqrt{2}}$ $(x-1) \frac{\sqrt{2}}{3}$ Substitution of limits $\frac{\pi}{4\sqrt{2}}$	(M1) (A1F) (A1) (A1F) (m1) (A1)	(6)	CAO
<b>Total</b>			<b>8</b>	
7(a)	Use of $(\sum \alpha)^2 = \sum \alpha^2 + 2\sum \alpha\beta$	M1 A1	2	AG
(b)	$p = 0, q = 5 + 6i$	B1,B1	2	
(c)(i)	Substitute $3i$ for $z$ <b>or</b> use $3i\beta\gamma = -r$ $-27i + 15i - 18 + r = 0$ <b>or</b> $\beta\gamma = 5 + 6i + \alpha^2$ $r = 18 + 12i$	M1 A1 A1F	3	allow for $3i\beta\gamma = r$ any form one error
(ii)	Cubic is $(z-3i)(z^2 + 3iz - 4 + 6i)$ or use of $\beta\gamma$ and $\beta + \gamma$	M1A1	2	clearly shown
(iii)	$f(-2) = 0$ or equate imaginary parts $\beta = -2, \gamma = 2 - 3i$	M1 A1,A1F	3	correct answers no working and no check B1 only
<b>Total</b>			<b>12</b>	

Q	Solution	Marks	Total	Comments
8(a)	$1, e^{\frac{2\pi i}{5}}, e^{\frac{4\pi i}{5}}, e^{\frac{-2\pi i}{5}}, e^{\frac{-4\pi i}{5}}$	B1	1	accept $e^0$
(b)	$\frac{z^5 - 1}{z - 1} = z^4 + z^3 + z^2 + z + 1$ $= \left(z - e^{\frac{2\pi i}{5}}\right) \left(z - e^{\frac{4\pi i}{5}}\right) \left(z - e^{\frac{-2\pi i}{5}}\right) \left(z - e^{\frac{-4\pi i}{5}}\right)$	B1 M1A1	3	B0 if assumed accept if $e^{\frac{6\pi i}{5}}, e^{\frac{8\pi i}{5}}$ used here
(c)	<p>Correct grouping of linear factors</p> $e^{\frac{2\pi i}{5}} + e^{\frac{-2\pi i}{5}} = 2 \cos \frac{2\pi}{5}$ $\left(z^2 - 2 \cos \frac{2\pi}{5} z + 1\right) \left(z^2 - 2 \cos \frac{4\pi}{5} z + 1\right)$ <p><math>\div z^2</math> to give answer</p>	M1 A1 A1 A1	4	clearly shown AG
(d)	<p>Substitute into LHS to give <math>w^2 + w - 1</math></p> <p>RHS <math>\left(w - 2 \cos \frac{2\pi}{5}\right) \left(w - 2 \cos \frac{4\pi}{5}\right)</math></p> <p>Solve <math>w^2 + w - 1 = 0</math></p> $w = \frac{-1 \pm \sqrt{5}}{2}$ $\cos \frac{2\pi}{5} = \frac{\sqrt{5} - 1}{4}$ <p>with reasons for choice</p>	B1 B1 M1 A1 A1 E1	6	
	<b>Total</b>		<b>14</b>	
	<b>TOTAL</b>		<b>75</b>	