General Certificate of Education January 2009 Advanced Level Examination



# MATHEMATICS Unit Further Pure 3

MFP3

Wednesday 21 January 2009 1.30 pm to 3.00 pm

# For this paper you must have:

- a 12-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

#### **Instructions**

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP3.
- Answer all questions.
- Show all necessary working; otherwise marks for method may be lost.

#### **Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

#### **Advice**

• Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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## Answer all questions.

1 The function y(x) satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where

$$f(x, y) = \frac{x^2 + y^2}{x + y}$$

and

$$y(1) = 3$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h f(x_r, y_r)$$

with h = 0.2, to obtain an approximation to y(1.2).

(3 marks)

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(b) Use the improved Euler formula

$$y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$$

where  $k_1 = hf(x_r, y_r)$  and  $k_2 = hf(x_r + h, y_r + k_1)$  and h = 0.2, to obtain an approximation to y(1.2), giving your answer to four decimal places. (5 marks)

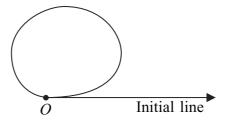
2 (a) Show that  $\frac{1}{x^2}$  is an integrating factor for the first-order differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{2}{x}y = x \tag{3 marks}$$

(b) Hence find the general solution of this differential equation, giving your answer in the form y = f(x). (4 marks)

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3 The diagram shows a sketch of a loop, the pole O and the initial line.



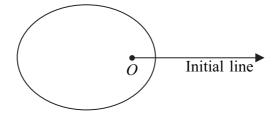
The polar equation of the loop is

$$r = (2 + \cos \theta) \sqrt{\sin \theta}, \quad 0 \le \theta \le \pi$$

Find the area enclosed by the loop.

(6 marks)

- 4 (a) Use integration by parts to show that  $\int \ln x \, dx = x \ln x x + c$ , where c is an arbitrary constant. (2 marks)
  - (b) Hence evaluate  $\int_0^1 \ln x \, dx$ , showing the limiting process used. (4 marks)
- 5 The diagram shows a sketch of a curve C, the pole O and the initial line.



The curve C has polar equation

$$r = \frac{2}{3 + 2\cos\theta}, \quad 0 \leqslant \theta \leqslant 2\pi$$

- (a) Verify that the point L with polar coordinates  $(2, \pi)$  lies on C. (1 mark)
- (b) The circle with polar equation r = 1 intersects C at the points M and N.
  - (i) Find the polar coordinates of M and N. (3 marks)
  - (ii) Find the area of triangle *LMN*. (4 marks)
- (c) Find a cartesian equation of C, giving your answer in the form  $9y^2 = f(x)$ . (5 marks)

### Turn over for the next question

4

- 6 The function f is defined by  $f(x) = e^{2x}(1+3x)^{-\frac{2}{3}}$ .
  - (a) (i) Use the series expansion for  $e^x$  to write down the first four terms in the series expansion of  $e^{2x}$ . (2 marks)
    - (ii) Use the binomial series expansion of  $(1+3x)^{-\frac{2}{3}}$  and your answer to part (a)(i) to show that the first three non-zero terms in the series expansion of f(x) are  $1+3x^2-6x^3$ . (5 marks)
  - (b) (i) Given that  $y = \ln(1 + 2\sin x)$ , find  $\frac{d^2y}{dx^2}$ . (4 marks)
    - (ii) By using Maclaurin's theorem, show that, for small values of x,

$$\ln(1+2\sin x) \approx 2x - 2x^2 \tag{2 marks}$$

(c) Find

$$\lim_{x \to 0} \frac{1 - f(x)}{x \ln(1 + 2\sin x)} \tag{3 marks}$$

7 (a) Given that  $x = e^t$  and that y is a function of x, show that

$$x^2 \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{\mathrm{d}^2 y}{\mathrm{d}t^2} - \frac{\mathrm{d}y}{\mathrm{d}t} \tag{7 marks}$$

(b) Hence show that the substitution  $x = e^t$  transforms the differential equation

$$x^2 \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 4x \frac{\mathrm{d}y}{\mathrm{d}x} = 10$$

into

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} - 5\frac{\mathrm{d}y}{\mathrm{d}t} = 10 \tag{2 marks}$$

- (c) Find the general solution of the differential equation  $\frac{d^2y}{dt^2} 5\frac{dy}{dt} = 10$ . (5 marks)
- (d) Hence solve the differential equation  $x^2 \frac{d^2y}{dx^2} 4x \frac{dy}{dx} = 10$ , given that y = 0 and  $\frac{dy}{dx} = 8$  when x = 1.

## **END OF QUESTIONS**