



Friday 18 January 2013 – Afternoon

A2 GCE MATHEMATICS

4733/01 Probability & Statistics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4733/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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2

- 1 A random variable has the distribution $B(n, p)$. It is required to test $H_0: p = \frac{2}{3}$ against $H_1: p < \frac{2}{3}$ at a significance level as close to 1% as possible, using a sample of size $n = 8, 9$ or 10 . Use tables to find which value of n gives such a test, stating the critical region for the test and the corresponding significance level. [4]

- 2 A random variable C has the distribution $N(\mu, \sigma^2)$. A random sample of 10 observations of C is obtained, and the results are summarised as

$$n = 10, \sum c = 380, \sum c^2 = 14602.$$

- (i) Calculate unbiased estimates of μ and σ^2 . [4]

- (ii) Hence calculate an estimate of the probability that $C > 40$. [2]

- 3 A factory produces 9000 music DVDs each day. A random sample of 100 such DVDs is obtained.

- (i) Explain how to obtain this sample using random numbers. [3]

- (ii) Given that 24% of the DVDs produced by the factory are classical, use a suitable approximation to find the probability that, in the sample of 100 DVDs, fewer than 20 are classical. [5]

- 4 A continuous random variable X has probability density function

$$f(x) = \begin{cases} kx & 0 \leq x \leq a, \\ 0 & \text{otherwise,} \end{cases}$$

where k and a are constants.

- (i) State what the letter x represents. [1]

- (ii) Find k in terms of a . [2]

- (iii) Find $\text{Var}(X)$ in terms of a . [6]

- 5 In a mine, a deposit of the substance *pitchblende* emits radioactive particles. The number of particles emitted has a Poisson distribution with mean 70 particles per second. The warning level is reached if the total number of particles emitted in one minute is more than 4350.

- (i) A one-minute period is chosen at random. Use a suitable approximation to show that the probability that the warning level is reached during this period is 0.01, correct to 2 decimal places. You should calculate the answer correct to 4 decimal places. [5]

- (ii) Use a suitable approximation to find the probability that in 30 one-minute periods the warning level is reached on at least 4 occasions. (You should use the given rounded value of 0.01 from part (i) in your calculation.) [3]

- 6 Gordon is a cricketer. Over a long period he knows that his population mean score, in number of runs per innings, is 28, and the population standard deviation is 12. In a new season he adopts a different batting style and he finds that in 30 innings using this style his mean score is 28.98.

- (i) Stating a necessary assumption, test at the 5% significance level whether his population mean score has increased. [8]

- (ii) Explain whether it was necessary to use the Central Limit Theorem in part (i). [2]

3

- 7 The continuous random variable X has the distribution $N(\mu, \sigma^2)$. The mean of a random sample of n observations of X is denoted by \bar{X} . It is given that $P(\bar{X} < 35.0) = 0.9772$ and $P(\bar{X} < 20.0) = 0.1587$.

(i) Obtain a formula for σ in terms of n . [5]

Two students are discussing this question. Aidan says “If you were told another probability, for instance $P(\bar{X} > 32) = 0.1$, you could work out the value of σ .” Binya says, “No, the value of $P(\bar{X} > 32)$ is fixed by the information you know already.”

(ii) State which of Aidan and Binya is right. If you think that Aidan is right, calculate the value of σ given that $P(\bar{X} > 32) = 0.1$. If you think that Binya is right, calculate the value of $P(\bar{X} > 32)$. [4]

- 8 In a large city the number of traffic lights that fail in one day of 24 hours is denoted by Y . It may be assumed that failures occur randomly.

(i) Explain what the statement “failures occur randomly” means. [1]

(ii) State, in context, two different conditions that must be satisfied if Y is to be modelled by a Poisson distribution, and for each condition explain whether you think it is likely to be met in this context. [4]

(iii) For this part you may assume that Y is well modelled by the distribution $Po(\lambda)$. It is given that $P(Y = 7) = P(Y = 8)$. Use an algebraic method to calculate the value of λ and hence calculate the corresponding value of $P(Y = 7)$. [5]

- 9 The random variable A has the distribution $B(30, p)$. A test is carried out of the hypotheses $H_0: p = 0.6$ against $H_1: p < 0.6$. The critical region is $A \leq 13$.

(i) State the probability that H_0 is rejected when $p = 0.6$. [1]

(ii) Find the probability that a Type II error occurs when $p = 0.5$. [2]

(iii) It is known that on average $p = 0.5$ on one day in five, and on other days the value of p is 0.6. On each day two tests are carried out. If the result of the first test is that H_0 is rejected, the value of p is adjusted if necessary, to ensure that $p = 0.6$ for the rest of the day. Otherwise the value of p remains the same as for the first test. Calculate the probability that the result of the second test is to reject H_0 . [5]

THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE.



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