General Certificate of Education (A-level)
January 2012

Mathematics MPC3

(Specification 6360)

Pure Core 3

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC3

Q	Solution	Marks	Total	Comments
1(a)	$\begin{array}{c\|c} x & y \\ \hline 0 & 1 \\ \frac{1}{2} & 2 \\ 1 & 4 \\ 1\frac{1}{2} & 8 \\ 2 & 16 \\ 2\frac{1}{2} & 32 \\ 3 & 64 \end{array}$	B1 B1		all 7 x values correct (and no extra) (PI by 7 correct y values) 5 or more correct y values, exact $\left(4^{\frac{1}{2}}, 4^{1} \ldots\right)$ or evaluated (in table or in formula)
	$A = \frac{1}{3} \times \frac{1}{2} [\ 65 + 4 \times 42 + 2 \times 20\ ]$	M1		correct substitution of their 7 y-values into Simpson's rule
	$= \frac{91}{2}$ or 45.5 or $\frac{273}{6}$	A1	4	CAO
(b)(i)	$f(x) = 4^x + 2x - 8$ or $g(x) = 8 - 2x - 4^x$ $\left.\begin{array}{l} f(1.2) = -0.3 \quad \text{or} \quad g(1.2) = 0.3 \\ f(1.3) = 0.7 \quad \text{or} \quad g(1.3) = -0.7 \end{array}\right\}$ AWRT ± 0.3 and ± 0.7 condone $f(1.2) < 0$, $f(1.3) > 0$ if f is defined	M1		attempt at evaluating f(1.2) and f(1.3) **alternative method** $\left.\begin{array}{l} 4^{1.2} = 5.3,\ 8 - 2 \times 1.2 = 5.6 \\ 4^{1.3} = 6.1,\ 8 - 2 \times 1.3 = 5.4 \end{array}\right\}$ M1
	change of sign $\therefore\ 1.2 < \alpha < 1.3$ (f(x) must be defined and all working correct)	A1	2	at 1.2 LHS < RHS at 1.3 LHS > RHS $\therefore\ 1.2 < \alpha < 1.3$ A1
(ii)	$(x_2 =)\ 1.243$ $(x_3 =)\ 1.232$	B1 B1	2	 these values only
	Total		**8**	

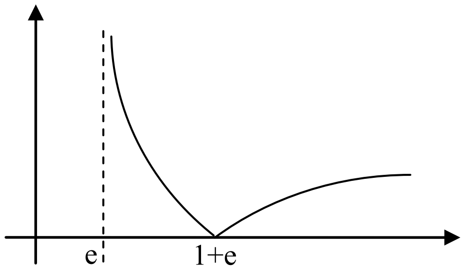
MPC3 (cont)

Q	Solution	Marks	Total	Comments
2(a)	$\left.\begin{matrix}\mathrm{f}(1)=21\\ \mathrm{f}(16)=1\end{matrix}\right\}$	M1		sight of 1 and 21
	$1 \le \mathrm{f}(x) \le 21$	A1	2	allow f(x) replaced by f, y
(b)(i)	$y=\dfrac{63}{4x-1}$			
	$x=\dfrac{63}{4y-1}$	M1		reverse x, y } Either order
	$x(4y-1)=63$ or better	M1		one correct step } Either order
	$\mathrm{f}^{-1}(x)=\dfrac{1}{4}\left(\dfrac{63}{x}+1\right)$ OE	A1	3	condone $y=$
(ii)	$\dfrac{1}{4}\left(\dfrac{63}{x}+1\right)=1$			
	$\dfrac{63}{x}+1=4$, or better	M1		one correct step from their (b)(i) = 1, or $x=\mathrm{f}(1)$
	$(x=)$ 21	A1	2	note: 21 scores 2/2
(c)(i)	$\left(\mathrm{fg}(x)=\right)\dfrac{63}{4x^2-1}$	B1	1	
(ii)	$\dfrac{63}{4x^2-1}=1$			
	$4x^2-1=63$ or better	M1		one correct step from their (c)(i) = 1
	$x^2=16$ OE	A1		eg $(2x+8)(2x-8)=0$, or $x=\pm 4$
	$x=-4$ **ONLY**	A1	3	
	Total		**11**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\ 12x^2-6$	B1	1	do not ISW
(b)	$\int_2^3 \frac{2x^2-1}{4x^3-6x+1}\,\mathrm{d}x$			
	$=\left[\frac{1}{6}\ln\left(4x^3-6x+1\right)\right]_{(2)}^{(3)}$	M1		$k\ln\left(4x^3-6x+1\right), k$ is a constant
		A1		$k=\frac{1}{6}$
	$=\frac{1}{6}\ln\left(4\times3^3-6\times3+1\right)$ $-\frac{1}{6}\ln\left(4\times2^3-6\times2+1\right)$	m1		correct substitution in F(3) – F(2). condone poor use or lack of brackets.
	$=\frac{1}{6}\ln 91-\frac{1}{6}\ln 21$	A1F		$k\ln 91-k\ln 21$ only follow through on their k
	$=\frac{1}{6}\ln\frac{91}{21}$ or $\left(=\frac{1}{6}\ln\frac{13}{3}\right)$	A1	5	
				or if using the substitution $u=4x^3-6x+1$ $\int=k\int\frac{\mathrm{d}u}{u}$ M1 $=\frac{1}{6}\ln u$ A1 then, either change limits to 21 and 91 m1 then A1F A1as scheme or changing back to 'x', then m1 A1F A1 as scheme
	Total		**6**	
4(a)	$\sec^2\theta-1=\ldots$	B1		correct use of $\sec^2\theta=1+\tan^2\theta$
	$\sec^2\theta+3\sec\theta-10\ (=0)$	M1		quadratic expression in $\sec\theta$ with all terms on one side
	$(\sec\theta+5)(\sec\theta-2)=0$	m1		attempt at factors of their quadratic, $(\sec\theta\pm5)(\sec\theta\pm2)$, or correct use of quadratic formula
	$\sec\theta=-5,\ 2$ $\left(\cos\theta=-\frac{1}{5},\frac{1}{2}\right)$	A1		
	$60^\circ,\ 300^\circ, 101.5^\circ,\ 258.5^\circ$ (AWRT)	B1 B1	6	3 correct, ignore answers outside interval all correct, no extras in interval
(b)	$4x-10^\circ=60^\circ, 101\cdot5^\circ, 258\cdot5^\circ, 300^\circ$	M1		$4x-10=$ any of their (60),
	$4x=70^\circ, 111\cdot5^\circ, 268\cdot5^\circ, 310^\circ$	A1F		all their answers from (a), BUT must have scored B1
	$x=17\cdot5^\circ, 27\cdot9^\circ, 67.1^\circ, 77\cdot5^\circ$ (AWRT)	A1	3	CAO, ignore answers outside interval
	Total		**9**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
5(a)	stretch I; SF 4 II; in y-direction III	M1A1		I + (II or III)
	either order			
	translate	E1		
	$\begin{pmatrix} \mathrm{e} \\ 0 \end{pmatrix}$	B1	4	accept 'e in positive x-direction'
(b)		M1		mod graph, in 2 connected sections, both in the first quadrant, touching x-axis
		A1		curve touches x-axis at 1 + e (or 3.7 or better), and labelled (ignore scale)
		A1	3	correct curvature, including at their 1+ e, approx. asymptote at x = e
(c)(i)	$\lvert 4\ln(x-\mathrm{e}) \rvert = 4$			
	$4\ln(x-\mathrm{e}) = 4$ $\quad 4\ln(x-\mathrm{e}) = -4$ or better	M1		must see 2 equations, condone omission of brackets
	(x =) 2e do not ISW	A1		accept values of AWRT 5.42, 5.43, 5.44
	$(x=)\mathrm{e}+\mathrm{e}^{-1}$ or $(x=)\mathrm{e}+\frac{1}{\mathrm{e}}$ do not ISW	A1	3	accept values of AWRT 3.08, 3.09
				if M0 then x = 2e with or without working scores SC1
(ii)	$x \geq 2\mathrm{e}$	B1		accept values of AWRT 5.42, 5.43, 5.44
	$\mathrm{e} < x \leq \mathrm{e}+\frac{1}{\mathrm{e}}$	B2	3	accept values of AWRT 2.72, 3.08, 3.09
				if B2 not earned, then SC1for any of $\mathrm{e} \leq x \leq \mathrm{e}+\frac{1}{\mathrm{e}}$, $\mathrm{e} < x < \mathrm{e}+\frac{1}{\mathrm{e}}$, $\mathrm{e} \leq x < \mathrm{e}+\frac{1}{\mathrm{e}}$
	Total		**13**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\left(\dfrac{dx}{d\theta}=\right)\dfrac{(\sin\theta\times 0)-1\times\cos\theta}{\sin^2\theta}$	M1		quotient rule $\dfrac{\pm\sin\theta\times k\pm 1\times\cos\theta}{\sin^2\theta}$ where $k=0$ or 1
		A1		must see the '0' either in the quotient or in eg $\dfrac{du}{d\theta}=0$ etc
	$=-\dfrac{\cos\theta}{\sin^2\theta}$ or $=-\dfrac{\cos\theta}{\sin\theta\sin\theta}$			or equivalent
	$=-\operatorname{cosec}\theta\cot\theta$	A1	3	CSO, AG must see one of the previous expressions
(b)	$x=\operatorname{cosec}\theta$ $\dfrac{dx}{d\theta}=-\operatorname{cosec}\theta\cot\theta$	B1		OE, eg $dx=-\operatorname{cosec}\theta\cot\theta\,d\theta$
	Replacing $\sqrt{(\operatorname{cosec}^2\theta-1)}$ by $\sqrt{\cot^2\theta}$, or better	B1		at any stage of solution
	$\int=\int\dfrac{-\operatorname{cosec}\theta\cot\theta}{\operatorname{cosec}^2\theta\sqrt{(\operatorname{cosec}^2\theta-1)}}d\theta$	M1		all in terms of θ, and including their attempt at dx, but condone omission of $d\theta$
		A1		fully correct and must include $d\theta$ (at some stage in solution)
	$\int\dfrac{-\operatorname{cosec}\theta\cot\theta}{\operatorname{cosec}^2\theta\cot\theta}(d\theta)$ $=\int\dfrac{-1}{\operatorname{cosec}\theta}(d\theta)$	A1		OE eg $\int-\sin\theta(d\theta)$
	$=\cos\theta$	A1		
	$x=2,\ \theta=0.524$ AWRT $x=\sqrt{2},\ \theta=0.785$ AWRT	B1		correct change of limits
				or $(\pm)\cos\theta=(\pm)\left[\sqrt{\left(1-\dfrac{1}{x^2}\right)}\right]_{\sqrt{2}}^{2}$ OE
	$0.8660-0.7071$	m1		c's $F(0.52)-F(0.79)$ substitution into $\pm\cos\theta$ only or $\left(\dfrac{\sqrt{3}}{2}-\dfrac{1}{\sqrt{2}}\right)$
	$=0.159$	A1	9	
	Total		**12**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\left[\dfrac{dy}{dx}=\right] p\,e^{-\frac{1}{4}x}x^2+qxe^{-\frac{1}{4}x}$	M1 A1		p, q constants $p=-\dfrac{1}{4}$ **and** $q=2$
	$\left[\Rightarrow e^{-\frac{1}{4}x}\left(-\dfrac{1}{4}x^2+2x\right)=0\right]$			
	$e^{-\frac{1}{4}x}\neq 0$	E1		or $e^{-\frac{1}{4}x}=0$ impossible OE (may be seen later)
	$\left(e^{-\frac{1}{4}x}\right)(ax^2+bx)=0$	m1		or $e^{-\frac{1}{4}x}x(ax+b)=0$
	$x=0,\ 8$	A1		
	$x=0,\ y=0$	A1		
	$x=8,\ y=64e^{-2}$	B1	7	condone $y=8^2e^{-\frac{8}{4}}$ etc ignore further numerical evaluation
(b)(i)	$\int x^2e^{-\frac{1}{4}x}\,dx \qquad u=x^2 \qquad \dfrac{dv}{dx}=e^{-\frac{1}{4}x}$ $\dfrac{du}{dx}=2x \qquad v=ke^{-\frac{1}{4}x}$	M1		where k is a constant
	$k=-4$	A1		
	$-4x^2e^{-\frac{1}{4}x}-\int -4e^{-\frac{1}{4}x}\times 2x(dx)$, or better	A1F		correct substitution of their terms
	$u=mx \qquad \dfrac{dv}{dx}=ne^{-\frac{1}{4}x}$ $\dfrac{du}{dx}=m \qquad v=-4ne^{-\frac{1}{4}x}$	m1		both differentiation and integration **must** be correct
	$\int=-4x^2e^{-\frac{1}{4}x}+8\left(-4xe^{-\frac{1}{4}x}+\int 4e^{-\frac{1}{4}x}dx\right)$			
	$=\left[-4x^2e^{-\frac{1}{4}x}-32xe^{-\frac{1}{4}x}-128e^{-\frac{1}{4}x}\right]_{(0)}^{(4)}$	A1		
	$=-e^{-1}[64+256]-[-128]$	m1 (dep on M1 only)		correct substitution and attempt at subtraction in $ax^2e^{-\frac{1}{4}x}+bxe^{-\frac{1}{4}x}+ce^{-\frac{1}{4}x}$ (may be in 3 stages)
	$=128-\dfrac{320}{e}$	A1	7	or $128-320e^{-1}$ ignore further numerical evaluation
(ii)	$v=\pi\int_{(0)}^{(4)}9x^2e^{-\frac{1}{4}x}(dx)$	M1		condone omission of brackets, limits
	$=9\pi\left(128-\dfrac{320}{e}\right)$	A1F	2	$9\pi\times$ (their exact b(i))
	Total		**16**	
	TOTAL		**75**	