

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS****Advanced Subsidiary General Certificate of Education  
Advanced General Certificate of Education****MATHEMATICS****4733**

Probability &amp; Statistics 2

Thursday

**15 JUNE 2006**

Afternoon

1 hour 30 minutes

Additional materials:

8 page answer booklet

Graph paper

List of Formulae (MF1)

**TIME** 1 hour 30 minutes**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

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**This question paper consists of 3 printed pages and 1 blank page.**

## 2

- 1 Calculate the variance of the continuous random variable with probability density function given by

$$f(x) = \begin{cases} \frac{3}{37}x^2 & 3 \leq x \leq 4, \\ 0 & \text{otherwise.} \end{cases} \quad [6]$$

- 2 (i) The random variable  $R$  has the distribution  $B(6, p)$ . A random observation of  $R$  is found to be 6. Carry out a 5% significance test of the null hypothesis  $H_0: p = 0.45$  against the alternative hypothesis  $H_1: p \neq 0.45$ , showing all necessary details of your calculation. [4]

- (ii) The random variable  $S$  has the distribution  $B(n, p)$ .  $H_0$  and  $H_1$  are as in part (i). A random observation of  $S$  is found to be 1. Use tables to find the largest value of  $n$  for which  $H_0$  is not rejected. Show the values of any relevant probabilities. [3]

- 3 The continuous random variable  $T$  has mean  $\mu$  and standard deviation  $\sigma$ . It is known that  $P(T < 140) = 0.01$  and  $P(T < 300) = 0.8$ .

- (i) Assuming that  $T$  is normally distributed, calculate the values of  $\mu$  and  $\sigma$ . [6]

In fact,  $T$  represents the time, in minutes, taken by a randomly chosen runner in a public marathon, in which about 10% of runners took longer than 400 minutes.

- (ii) State with a reason whether the mean of  $T$  would be higher than, equal to, or lower than the value calculated in part (i). [2]

- 4 (i) Explain briefly what is meant by a random sample. [1]

Random numbers are used to select, with replacement, a sample of size  $n$  from a population numbered 000, 001, 002, ..., 799.

- (ii) If  $n = 6$ , find the probability that exactly 4 of the selected sample have numbers less than 500. [3]

- (iii) If  $n = 60$ , use a suitable approximation to calculate the probability that at least 40 of the selected sample have numbers less than 500. [6]

- 5 An airline has 300 seats available on a flight to Australia. It is known from experience that on average only 99% of those who have booked seats actually arrive to take the flight, the remaining 1% being called 'no-shows'. The airline therefore sells more than 300 seats. If more than 300 passengers then arrive, the flight is over-booked. Assume that the number of no-show passengers can be modelled by a binomial distribution.

- (i) If the airline sells 303 seats, state a suitable distribution for the number of no-show passengers, and state a suitable approximation to this distribution, giving the values of any parameters. [2]

Using the distribution and approximation in part (i),

- (ii) show that the probability that the flight is over-booked is 0.4165, correct to 4 decimal places, [2]

- (iii) find the largest number of seats that can be sold for the probability that the flight is over-booked to be less than 0.2. [5]

## 3

- 6 Customers arrive at a post office at a constant average rate of 0.4 per minute.
- (i) State an assumption needed to model the number of customers arriving in a given time interval by a Poisson distribution. [1]

Assuming that the use of a Poisson distribution is justified,

- (ii) find the probability that more than 2 customers arrive in a randomly chosen 1-minute interval, [2]
- (iii) use a suitable approximation to calculate the probability that more than 55 customers arrive in a given two-hour interval, [6]
- (iv) calculate the smallest time for which the probability that no customers arrive in that time is less than 0.02, giving your answer to the nearest second. [5]
- 7 Three independent researchers, *A*, *B* and *C*, carry out significance tests on the power consumption of a manufacturer's domestic heaters. The power consumption,  $X$  watts, is a normally distributed random variable with mean  $\mu$  and standard deviation 60. Each researcher tests the null hypothesis  $H_0: \mu = 4000$  against the alternative hypothesis  $H_1: \mu > 4000$ .

Researcher *A* uses a sample of size 50 and a significance level of 5%.

- (i) Find the critical region for this test, giving your answer correct to 4 significant figures. [6]

In fact the value of  $\mu$  is 4020.

- (ii) Calculate the probability that Researcher *A* makes a Type II error. [6]
- (iii) Researcher *B* uses a sample bigger than 50 and a significance level of 5%. Explain whether the probability that Researcher *B* makes a Type II error is less than, equal to, or greater than your answer to part (ii). [2]
- (iv) Researcher *C* uses a sample of size 50 and a significance level bigger than 5%. Explain whether the probability that Researcher *C* makes a Type II error is less than, equal to, or greater than your answer to part (ii). [2]
- (v) State with a reason whether it is necessary to use the Central Limit Theorem at any point in this question. [2]

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