

General Certificate of Education
June 2007
Advanced Level Examination



MATHEMATICS
Unit Further Pure 3

MFP3

Wednesday 20 June 2007 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 (a) Find the value of the constant k for which kx^2e^{5x} is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + 25y = 6e^{5x} \quad (6 \text{ marks})$$

- (b) Hence find the general solution of this differential equation. (4 marks)

- 2 The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where $f(x, y) = \sqrt{x^2 + y^2 + 3}$

and $y(1) = 2$

- (a) Use the Euler formula

$$y_{r+1} = y_r + hf(x_r, y_r)$$

with $h = 0.1$, to obtain an approximation to $y(1.1)$, giving your answer to four decimal places. (3 marks)

- (b) Use the improved Euler formula

$$y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$$

where $k_1 = hf(x_r, y_r)$ and $k_2 = hf(x_r + h, y_r + k_1)$ and $h = 0.1$, to obtain an approximation to $y(1.1)$, giving your answer to four decimal places. (6 marks)

- 3 By using an integrating factor, find the solution of the differential equation

$$\frac{dy}{dx} + (\tan x)y = \sec x$$

given that $y = 3$ when $x = 0$. (8 marks)

4 (a) Show that $(\cos \theta + \sin \theta)^2 = 1 + \sin 2\theta$. (1 mark)

(b) A curve has cartesian equation

$$(x^2 + y^2)^3 = (x + y)^4$$

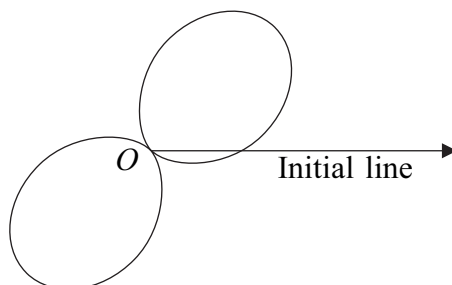
Given that $r \geq 0$, show that the polar equation of the curve is

$$r = 1 + \sin 2\theta \quad (4 \text{ marks})$$

(c) The curve with polar equation

$$r = 1 + \sin 2\theta, \quad -\pi \leq \theta \leq \pi$$

is shown in the diagram.



(i) Find the two values of θ for which $r = 0$. (3 marks)

(ii) Find the area of one of the loops. (6 marks)

Turn over for the next question

Turn over ►

5 (a) A differential equation is given by

$$(x^2 - 1) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} = x^2 + 1$$

Show that the substitution

$$u = \frac{dy}{dx} + x$$

transforms this differential equation into

$$\frac{du}{dx} = \frac{2xu}{x^2 - 1} \quad (4 \text{ marks})$$

(b) Find the general solution of

$$\frac{du}{dx} = \frac{2xu}{x^2 - 1}$$

giving your answer in the form $u = f(x)$.

(5 marks)

(c) Hence find the general solution of the differential equation

$$(x^2 - 1) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} = x^2 + 1$$

giving your answer in the form $y = g(x)$.

(3 marks)

- 6 (a) The function f is defined by

$$f(x) = \ln(1 + e^x)$$

Use Maclaurin's theorem to show that when $f(x)$ is expanded in ascending powers of x :

- (i) the first three terms are

$$\ln 2 + \frac{1}{2}x + \frac{1}{8}x^2 \quad (6 \text{ marks})$$

- (ii) the coefficient of x^3 is zero. (3 marks)

- (b) Hence write down the first two non-zero terms in the expansion, in ascending powers of x , of $\ln\left(\frac{1+e^x}{2}\right)$. (1 mark)

- (c) Use the series expansion

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \dots$$

to write down the first three terms in the expansion, in ascending powers of x , of $\ln\left(1 - \frac{x}{2}\right)$. (1 mark)

- (d) Use your answers to parts (b) and (c) to find

$$\lim_{x \rightarrow 0} \left[\frac{\ln\left(\frac{1+e^x}{2}\right) + \ln\left(1 - \frac{x}{2}\right)}{x - \sin x} \right] \quad (4 \text{ marks})$$

- 7 (a) Write down the value of

$$\lim_{x \rightarrow \infty} xe^{-x} \quad (1 \text{ mark})$$

- (b) Use the substitution $u = xe^{-x} + 1$ to find $\int \frac{e^{-x}(1-x)}{xe^{-x} + 1} dx$. (2 marks)

- (c) Hence evaluate $\int_1^{\infty} \frac{1-x}{x+e^x} dx$, showing the limiting process used. (4 marks)

END OF QUESTIONS

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