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General Certificate of Education

Statistics 6380

SS03 Statistics 3

Mark Scheme

2009 examination – January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

SS03

Q	Solution	Marks	Total	Comments
1	H_0: pop median/η = 76 H_1: pop median/η > 76 1 tail 10%	B1		
	signs – + + + + + + + – –	M1		Signs (allow differences)
	n = 10 test stat = 7^+ / 3^- Model B(10 , 0.5)	 A1 M1		 test stat correct Bin model seen to be used
	$P(\leq 3^-) = P(\geq 7^+) = 0.172 > 0.10$	M1		Comparison of correct B(10, 0.5) prob with 0.10 or 0.344 with 0.20 Or use of identified cv with probability: $\{8,9,10\}$ and $0.0547 < 0.10$
	Accept H_0 There is no significant evidence to suggest that her median heart rate increases.	 A1	6	 In context
	Total		**6**	
2(a)	H_0: pop median/mean diff η_d = 0 H_1: pop median/mean diff $\eta_d \neq 0$ 2 tail 5%	B1		
	diff (1 – 2): 38, 11, 9, –4, 7 **rank**: 9, 8, 7, –3, 5	M1 m1		For differences, +/– signs can be interchanged For ranks (either way)
	6 . 8 3 –1 4 . 6 2 –1			
	$T_+ = 9 + 8 + 7 + 5 + 4 + 6 + 2 = 41$ $T_- = 3 + 1 = 4$ Test stat T = 4 n = 9 cv = 6 $T < 6$	m1 A1 B1 M1		For total attempted For one correct total For cv Comparison cv/ts
	Significant evidence at 5% level to reject H_0 Significant evidence to suggests that the number of lesions differ for preparations A and B.	A1 E1	9	In context – can be one tail conclusion that preparation A leads to more lesions.
(b)	Wilcoxon takes into account the rank order of the differences so is a more powerful test.	E1	1	or comment referring to taking size of differences into account
	Total		**10**	

SS03 (cont)

Q	Solution	Marks	Total	Comments
3	H_0: Samples are taken from identical populations	B1		Hypotheses referring to population averages also acceptable but require 1 tail alternative
	H_1: Samples are not taken from identical populations – population average ratios differ	B1		B1, B0 if 1 tail but not precise wording
	1 tail 5%			
	Ranks **Non-drinkers** 6 9 10 12 13 14 15 16 17 18 **Heavy drinkers** 1 2 3 4 5 7 8 11	M1 A1		For ranks as one group – at least 10 correct All correct Other alternative methods acceptable (eg ranks reversed)
	$T_{non} = 6 + 9 + \ldots + 18 = 130$ $T_{heavy} = 1 + 2 + \ldots + 11 = 41$	M1		For total of ranks attempted
	$U_{non} = 130 - \frac{10\times11}{2} = 75$	M1		For U attempted
	$U_{heavy} = 41 - \frac{8\times9}{2} = 5$	A1		For U correct – either
	Test stat $U = 5$			
	$n = 10$, $m = 8$ cv = 21	B1		For cv consistent with U (Upper tail cv = 59)
	$U = 5 < 21$	m1		For comparison U /cv Not negative U
	Significant evidence to reject H_0	A1		
	Evidence to suggest that heavy drinkers have a smaller average ratio of brain volume to skull size	E1	11	In context
	Total		**11**	

SS03 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	H_0: Result independent of treatment H_1: Result not independent of treatment 1 tail 1%	B1		or equivalent
	Expected frequencies			
	Much imp 17.73 17.41 13.86 **Imp** 17.73 17.41 13.86 **Slight imp** 10.49 10.30 8.20 **No change** 9.05 8.88 7.07 (Hydro / Tai Chi / Conv exercise)	M1 m1 A1		E method for 3 correct 7 correct For all E correct
	ts = $\sum \frac{(O-E)^2}{E}$			
	$= \frac{1.27^2}{17.73} + \frac{5.59^2}{17.41} + ... + \frac{6.93^2}{7.07}$	m1 m1		ts sum with correct denominators numerator method OK
	= 19.42	A1		For ts in range 19.00 ~ 20.00
	df = 6 1% cv = 16.812 ts > 16.812	B1 m1		For cv or p = 0.003511 For comparison ts/cv
	Reject H_0 Sig evidence to suggest that the outcome is not independent of the treatment.	A1	10	**SC** pooled max 4/10 M1, m1, m1, m1 only
(a)(ii)	Main sources of association: Far fewer than expected adults doing conventional classes reported that they were much improved and far more of these than expected reported no change.	E1 E1	2	For identification of any two main sources in context.
(b)(i)	**All** 55 54 43 **Not all** 9 10 13 (Hydro / Tai Chi / Conv land-based)	B1 M1 A1	3	Categories correct for table (allow correct 3×2) 4 correct All correct
(ii)	H_0: Attendance for full six months is independent of treatment H_1: Attendance for full six months is not independent of treatment 1 tail 5%	B1		
	ts = $\sum \frac{(O-E)^2}{E}$ = 1.95			
	df = 2 5% cv = 5.991 ts < 5.991	B1 m1		For cv or p = 0.37638 For comparison ts/cv
	Accept H_0 No sig evidence to doubt that attendance for the full six months is independent of treatment.	A1	4	Must be in context
	Total		**19**	

SS03 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	From calculator $r = 0.758$ **Alternative** $r = \dfrac{129553 - (\frac{358\times3088}{11})}{\sqrt{3160.73}\times\sqrt{464586.18}}$ $= \dfrac{29052.64}{56.22\times681.61}$ $= 0.758$	B3		One correct value; either $n = 11$ $\sum w = 358 \quad \sum x = 3088$ $\sum w^2 = 14812$ $\sum x^2 = 1331472$ $\sum wx = 129553$ M1 m1 0.750 ~ 0.770 A1
(ii)	From calculator $r = -0.488$ **Alternative** or $r = \dfrac{2992.8 - (\frac{358\times106.8}{11})}{\sqrt{3160.73}\times\sqrt{310.07}}$ $= \dfrac{-483.05}{56.22\times17.61}$ $= -0.488$ $r_{wx} = 0.758 \quad r_{wy} = -0.488 \quad r_{xy} = -0.853$	B2	5	Second correct value $\sum y = 106.8$ $\sum y^2 = 1347$ $\sum wy = 2992.8$ M1 –0.480 ~ –0.500 A1
(b)	$H_0 \ \rho = 0$ $H_1 \ \rho \neq 0$ 2 tail 5 % sig level Need only be stated once test stat $r_{wx} = 0.758$ $\lvert cv \rvert = 0.6021 \quad n = 11$ since $\lvert ts \rvert > 0.6021$ Reject H_0 test stat $r_{wy} = -0.488$ $\lvert cv \rvert = 0.6021 \quad n = 11$ since $\lvert ts \rvert < 0.6021$ Accept H_0 test stat $r_{xy} = -0.853$ $\lvert cv \rvert = 0.6021 \quad n = 11$ since $\lvert ts \rvert > 0.6021$ Reject H_0	B1 B1 M1 A1 M1 A1 A1	7	For any pair of hypotheses Allow 1 tail for r_{wx} $H_0 \ \rho = 0$ $H_1 \ \rho > 0$ 1 tail 5 % sig For 0.6021 (or 0.5214) Allow 1 tail for $r_{xy \text{ or } wy}$ $H_o \ \rho = 0$ $H_1 \ \rho < 0$ 1 tail 5 % sig or cv = –0.5214 for 1 tail A1A1A1 can be gained in part (c)
(c)	There is significant evidence of a (positive) correlation between maximum life span and average gestation time. The longer the max lifespan, the longer the average gestation time. There is significant evidence of a (negative) correlation between average gestation time and average daily sleep time. The longer the average gestation time, the less average daily sleep time. There is no significant evidence of a correlation between maximum lifespan and average daily sleep time.	E1 E1 E1	3	Interpretation in context E1E1E1 can be gained in part (b)
	Total		**15**	

SS03 (cont)

Q	Solution	Marks	Total	Comments
6(a)	H_0: Samples are taken from identical populations	B1		or H_0 $\eta_{20} = \eta_{30} = \eta_{40}$
	H_1: Samples are not taken from identical populations – population average times differ	B1		H_1 at least two of $\eta_{20}, \eta_{30}, \eta_{40}$ do differ
	5%			
	Ranks	M1		ranks

20	30	40
1	3	5
2	4	7
6	9	12
8	11	13
10	14	16
15	17	18

$T_{20} = 42$	$T_{30} = 58$	$T_{40} = 71$	m1	totals
$n_{20} = 6$	$n_{30} = 6$	$n_{40} = 6$	A1	any one correct

$$\sum_{i=1}^{m} \frac{T_i^2}{n_i} = \frac{42^2}{6} + \frac{58^2}{6} + \frac{71^2}{6} = 1694.83$$ m1

$$H = \frac{12}{18 \times 19} \times 1694.83 - (3 \times 19) = 2.47$$ m1 test stat $H = \frac{12}{N(N+1)} \sum_{i=1}^{m} \frac{T_i^2}{n_i} - 3(N+1)$

A1 2.2 ~ 2.7

Critical value from $\chi^2_2 = 5.991$ B1

$H < 5.991$ M1

No significant evidence to reject H_0. A1 No difference

Conclude that there is no significant evidence to doubt that samples are from identical populations and there is no difference in the average times taken to solve the anagram for the different levels of sleep deprivation. E1 12 In context

(b) A Type II error is when an incorrect null hypothesis is accepted as true. B1

Or H_0 false but test conclusion is that H_0 is true.

In context, conclusion would be that samples are from identical populations and there is no difference in average times to complete puzzle but, in fact, there is a difference between at least two of the average times, for different levels of sleep deprivation, to complete puzzle. E1 2

	Total		**14**	
	TOTAL		**75**	