General Certificate of Education
Advanced Level Examination
January 2010

# Mathematics MPC4

**Unit Pure Core 4**

**Tuesday 19 January 2010 9.00 am to 10.30 am**

**For this paper you must have:**
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

**Time allowed**
- 1 hour 30 minutes

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MPC4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

**1** The polynomial $f(x)$ is defined by $f(x) = 15x^3 + 19x^2 - 4$.

(a) (i) Find $f(-1)$. *(1 mark)*

(ii) Show that $(5x - 2)$ is a factor of $f(x)$. *(2 marks)*

(b) Simplify

$$\frac{15x^2 - 6x}{f(x)}$$

giving your answer in a fully factorised form. *(5 marks)*

**2** (a) Express $\cos x + 3\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your value of $\alpha$, in radians, to three decimal places. *(3 marks)*

(b) (i) Hence write down the minimum value of $\cos x + 3\sin x$. *(1 mark)*

(ii) Find the value of $x$ in the interval $0 \leqslant x \leqslant 2\pi$ at which this minimum occurs, giving your answer, in radians, to three decimal places. *(2 marks)*

(c) Solve the equation $\cos x + 3\sin x = 2$ in the interval $0 \leqslant x \leqslant 2\pi$, giving all solutions, in radians, to three decimal places. *(4 marks)*

**3** (a) (i) Find the binomial expansion of $(1 + x)^{-\frac{1}{3}}$ up to and including the term in $x^2$. *(2 marks)*

(ii) Hence find the binomial expansion of $\left(1 + \frac{3}{4}x\right)^{-\frac{1}{3}}$ up to and including the term in $x^2$. *(2 marks)*

(b) Hence show that $\sqrt[3]{\dfrac{256}{4 + 3x}} \approx a + bx + cx^2$ for small values of $x$, stating the values of the constants $a$, $b$ and $c$. *(3 marks)*

**4** The expression $\dfrac{10x^2+8}{(x+1)(5x-1)}$ can be written in the form $2+\dfrac{A}{x+1}+\dfrac{B}{5x-1}$, where $A$ and $B$ are constants.

(a) Find the values of $A$ and $B$. *(4 marks)*

(b) Hence find $\displaystyle\int \frac{10x^2+8}{(x+1)(5x-1)}\,\mathrm{d}x$. *(4 marks)*

**5** A curve is defined by the equation

$$x^2+xy=\mathrm{e}^y$$

Find the gradient at the point $(-1,\ 0)$ on this curve. *(5 marks)*

**6** (a) (i) Express $\sin 2\theta$ and $\cos 2\theta$ in terms of $\sin\theta$ and $\cos\theta$. *(2 marks)*

(ii) Given that $0<\theta<\dfrac{\pi}{2}$ and $\cos\theta=\dfrac{3}{5}$, show that $\sin 2\theta=\dfrac{24}{25}$ and find the value of $\cos 2\theta$. *(2 marks)*

(b) A curve has parametric equations

$$x=3\sin 2\theta,\quad y=4\cos 2\theta$$

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $\theta$. *(3 marks)*

(ii) At the point $P$ on the curve, $\cos\theta=\dfrac{3}{5}$ and $0<\theta<\dfrac{\pi}{2}$. Find an equation of the tangent to the curve at the point $P$. *(3 marks)*

**7** Solve the differential equation $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{y}\cos\left(\dfrac{x}{3}\right)$, given that $y=1$ when $x=\dfrac{\pi}{2}$.

Write your answer in the form $y^2=\mathrm{f}(x)$. *(6 marks)*

**Turn over for the next question**

**8** The points $A$, $B$ and $C$ have coordinates $(2, -1, -5)$, $(0, 5, -9)$ and $(9, 2, 3)$ respectively.

The line $l$ has equation $\mathbf{r} = \begin{bmatrix} 2 \\ -1 \\ -5 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$.

(a) Verify that the point $B$ lies on the line $l$. *(2 marks)*

(b) Find the vector $\overrightarrow{BC}$. *(2 marks)*

(c) The point $D$ is such that $\overrightarrow{AD} = 2\overrightarrow{BC}$.

(i) Show that $D$ has coordinates $(20, -7, 19)$. *(2 marks)*

(ii) The point $P$ lies on $l$ where $\lambda = p$. The line $PD$ is perpendicular to $l$. Find the value of $p$. *(5 marks)*

**9** A botanist is investigating the rate of growth of a certain species of toadstool. She observes that a particular toadstool of this type has a height of 57 millimetres at a time 12 hours after it begins to grow.

She proposes the model $h = A\left(1 - e^{-\frac{1}{4}t}\right)$, where $A$ is a constant, for the height $h$ millimetres of the toadstool, $t$ hours after it begins to grow.

(a) Use this model to:

(i) find the height of the toadstool when $t = 0$; *(1 mark)*

(ii) show that $A = 60$, correct to two significant figures. *(2 marks)*

(b) Use the model $h = 60\left(1 - e^{-\frac{1}{4}t}\right)$ to:

(i) show that the time $T$ hours for the toadstool to grow to a height of 48 millimetres is given by

$$T = a \ln b$$

where $a$ and $b$ are integers; *(3 marks)*

(ii) show that $\dfrac{dh}{dt} = 15 - \dfrac{h}{4}$; *(3 marks)*

(iii) find the height of the toadstool when it is growing at a rate of 13 millimetres per hour. *(1 mark)*

**END OF QUESTIONS**