General Certificate of Education
Advanced Level Examination
June 2013

Mathematics MS2B

Unit Statistics 2B

Thursday 13 June 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 Gemma, a biologist, studies guillemots, which are a species of seabird. She has found that the weight of an adult guillemot may be modelled by a normal distribution with mean μ grams. During 2012, she measured the weight, x grams, of each of a random sample of 9 adult guillemots and obtained the following results.

$$\sum x = 8532 \qquad \text{and} \qquad \sum (x - \bar{x})^2 = 38\,538$$

(a) Construct a 98% confidence interval for μ based on these data. *(5 marks)*

(b) The corresponding confidence interval for μ obtained by Gemma based on a random sample of 9 adult guillemots measured during 2011 was (927, 1063), correct to the nearest gram.

(i) Find the mean weight of guillemots in this sample. *(1 mark)*

(ii) Studies of some other species of seabird have suggested that their mean weights were less in 2012 than in 2011. Comment on whether Gemma's two confidence intervals provide evidence that the mean weight of guillemots was less in 2012 than in 2011. *(2 marks)*

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2 A town council wanted residents to apply for grants that were available for home insulation. In a trial, a random sample of 200 residents was encouraged, either in a letter or by a phone call, to apply for the grants. The outcomes are shown in the table.

	Applied for grant	**Did not apply for grant**	**Total**
Letter	30	130	160
Phone call	14	26	40
Total	44	156	200

(a) The council believed that a phone call was more effective than a letter in encouraging people to apply for a grant. Use a χ^2-test to investigate this belief at the 5% significance level. *(8 marks)*

(b) After the trial, all the residents in the town were encouraged, either in a letter or by a phone call, to apply for the grants. It was found that there was no association between the method of encouragement and the outcome. State, with a reason, whether a Type I error, a Type II error or neither occurred in carrying out the test in part **(a)**. *(2 marks)*

3 Mehreen lives a 2-minute walk away from a tram stop. Trams run every 10 minutes into the city centre, taking 20 minutes to get there. Every morning, Mehreen leaves her house, walks to the tram stop and catches the first tram that arrives. When she arrives at the city centre, she then has a 5-minute walk to her office.

The total time, T minutes, for Mehreen's journey from house to office may be modelled by a rectangular distribution with probability density function

$$f(t) = \begin{cases} 0.1 & a \leqslant t \leqslant b \\ 0 & \text{otherwise} \end{cases}$$

(a) (i) Explain why $a = 27$.

(ii) State the value of b. *(3 marks)*

(b) Find the values of $\mathrm{E}(T)$ and $\mathrm{Var}(T)$. *(2 marks)*

(c) Find the probability that the time for Mehreen's journey is within 5 minutes of half an hour. *(2 marks)*

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4 Gamma-ray bursts (GRBs) are pulses of gamma rays lasting a few seconds, which are produced by explosions in distant galaxies. They are detected by satellites in orbit around Earth. One particular satellite detects GRBs at a constant average rate of 3.5 per week (7 days).

You may assume that the detection of GRBs by this satellite may be modelled by a Poisson distribution.

(a) Find the probability that the satellite detects:

(i) exactly 4 GRBs during one particular week; *(2 marks)*

(ii) at least 2 GRBs on one particular **day**; *(3 marks)*

(iii) more than 10 GRBs but fewer than 20 GRBs during the 28 days of February 2013. *(3 marks)*

(b) Give **one** reason, apart from the constant average rate, why it is likely that the detection of GRBs by this satellite may be modelled by a Poisson distribution. *(1 mark)*

Turn over ▶

5 In a computer game, players try to collect five treasures. The number of treasures that Isaac collects in one play of the game is represented by the discrete random variable X.

The probability distribution of X is defined by

$$P(X = x) = \begin{cases} \dfrac{1}{x+2} & x = 1, 2, 3, 4 \\ k & x = 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) (i) Show that $k = \frac{1}{20}$. *(2 marks)*

(ii) Calculate the value of $E(X)$. *(2 marks)*

(iii) Show that $\mathrm{Var}(X) = 1.5275$. *(3 marks)*

(iv) Find the probability that Isaac collects more than 2 treasures. *(2 marks)*

(b) The number of points that Isaac scores for collecting treasures is Y where

$$Y = 100X - 50$$

Calculate the mean and the standard deviation of Y. *(4 marks)*

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6 A supermarket buys pears from a local supplier. The supermarket requires the mean weight of the pears to be at least 175 grams. William, the fresh-produce manager at the supermarket, suspects that the latest batch of pears delivered does not meet this requirement.

(a) William weighs a random sample of 6 pears, obtaining the following weights, in grams.

160.6 155.4 181.3 176.2 162.3 172.8

Previous batches of pears have had weights that could be modelled by a normal distribution with standard deviation 9.4 grams. Assuming that this still applies, show that a hypothesis test at the 5% level of significance supports William's suspicion. *(7 marks)*

(b) William then weighs a random sample of 20 pears. The mean of this sample is 169.4 grams and $s = 11.2$ grams, where s^2 is an unbiased estimate of the population variance.

Assuming that the population from which this sample is taken has a normal distribution but with unknown standard deviation, test William's suspicion at the **1%** level of significance. *(5 marks)*

(c) Give a reason why the probability of a Type I error occurring was smaller when conducting the test in part **(b)** than when conducting the test in part **(a)**. *(1 mark)*

7 A continuous random variable X has the probability density function defined by

$$f(x) = \begin{cases} x^2 & 0 \leqslant x \leqslant 1 \\ \frac{1}{3}(5 - 2x) & 1 \leqslant x \leqslant 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the graph of f on the axes below. *(3 marks)*

(b) (i) Find the cumulative distribution function, F, for $0 \leqslant x \leqslant 1$. *(2 marks)*

(ii) Hence, or otherwise, find the value of the lower quartile of X. *(2 marks)*

(c) (i) Show that the cumulative distribution function for $1 \leqslant x \leqslant 2$ is defined by

$$F(x) = \frac{1}{3}(5x - x^2 - 3)$$ *(4 marks)*

(ii) Hence, or otherwise, find the value of the upper quartile of X. *(4 marks)*