OCR RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS **4733**

Probability & Statistics 2

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Wednesday 17 June 2009
Morning

Duration: 1 hour 30 minutes

4733

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

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1   The random variable H has the distribution $N(\mu, \sigma^2)$. It is given that $P(H < 105.0) = 0.2420$ and $P(H > 110.0) = 0.6915$. Find the values of μ and σ, giving your answers to a suitable degree of accuracy. **[6]**

2   The random variable D has the distribution Po(20). Using an appropriate approximation, which should be justified, calculate $P(D \geqslant 25)$. **[6]**

3   An electronics company is developing a new sound system. The company claims that 60% of potential buyers think that the system would be good value for money. In a random sample of 12 potential buyers, 4 thought that it would be good value for money. Test, at the 5% significance level, whether the proportion claimed by the company is too high. **[7]**

4   A survey is to be carried out to draw conclusions about the proportion p of residents of a town who support the building of a new supermarket. It is proposed to carry out the survey by interviewing a large number of people in the high street of the town, which attracts a large number of tourists.

   (i) Give two different reasons why this proposed method is inappropriate. **[2]**

   (ii) Suggest a good method of carrying out the survey. **[3]**

   (iii) State two statistical properties of your survey method that would enable reliable conclusions about p to be drawn. **[2]**

5   In a large region of derelict land, bricks are found scattered in the earth.

   (i) State two conditions needed for the number of bricks per cubic metre to be modelled by a Poisson distribution. **[2]**

   Assume now that the number of bricks in 1 cubic metre of earth can be modelled by the distribution Po(3).

   (ii) Find the probability that the number of bricks in 4 cubic metres of earth is between 8 and 14 inclusive. **[3]**

   (iii) Find the size of the largest volume of earth for which the probability that no bricks are found is at least 0.4. **[4]**

6   The continuous random variable R has the distribution $N(\mu, \sigma^2)$. The results of 100 observations of R are summarised by

$$\Sigma r = 3360.0, \quad \Sigma r^2 = 115\,782.84.$$

   (i) Calculate an unbiased estimate of μ and an unbiased estimate of σ^2. **[4]**

   (ii) The mean of 9 observations of R is denoted by $\overline{R}$. Calculate an estimate of $P(\overline{R} > 32.0)$. **[4]**

   (iii) Explain whether you need to use the Central Limit Theorem in your answer to part **(ii)**. **[2]**

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7   The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{2}{9}x(3-x) & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

   (i) Find the variance of X. **[5]**

   (ii) Show that the probability that a single observation of X lies between 0.0 and 0.5 is $\frac{2}{27}$. **[2]**

   (iii) 108 observations of X are obtained. Using a suitable approximation, find the probability that at least 10 of the observations lie between 0.0 and 0.5. **[6]**

   (iv) The mean of 108 observations of X is denoted by $\overline{X}$. Write down the approximate distribution of $\overline{X}$, giving the value(s) of any parameter(s). **[3]**

8   In a large company the time taken for an employee to carry out a certain task is a normally distributed random variable with mean 78.0 s and unknown variance. A new training scheme is introduced and after its introduction the times taken by a random sample of 120 employees are recorded. The mean time for the sample is 76.4 s and an unbiased estimate of the population variance is $68.9\,\text{s}^2$.

   (i) Test, at the 1% significance level, whether the mean time taken for the task has changed. **[7]**

   (ii) It is required to redesign the test so that the probability of making a Type I error is less than 0.01 when the sample mean is 77.0 s. Calculate an estimate of the smallest sample size needed, and explain why your answer is only an estimate. **[4]**