Mark Scheme (Results)

June 2013

GCE Further Pure Mathematics FP1 (6667/01)
Original Paper

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General Marking Guidance

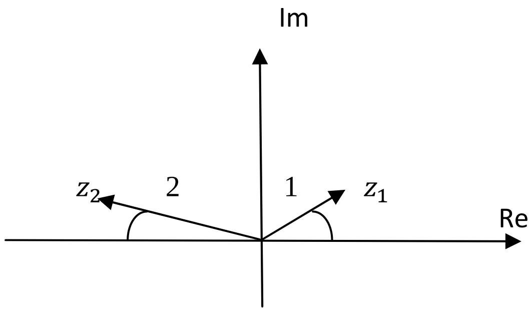
- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

Question Number	Scheme	Marks
1(a)	det $\mathbf{M} = a(2-a)-1$	M1A1
		(2)
1(b)	det $\mathbf{M} = 0$	M1
	$a^2 - 2a + 1 = 0$	
	$(a-1)^2 = 0$	M1
	$a = 1$	A1
		(3)
		[5]
	Notes	
(a)	M for " $ad - bc$ "	
(b)	First M for their det $\mathbf{M} = 0$	
	Second M for attempt to solve their 3 term quadratic	
	Method mark for solving 3 term quadratic:	
	1. Factorisation	
	$(x^2 + bx + c) = (x + p)(x + q)$, where $\lvert pq\rvert = \lvert c\rvert$, leading to x =	
	$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $\lvert pq\rvert = \lvert c\rvert$ and $\lvert mn\rvert = \lvert a\rvert$,leading to x =	
	2. Formula	
	Attempt to use correct formula (with values for a, b and c).	
	3. Completing the square	
	Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c$, $q \neq 0$, leading to $x = \ldots$	

Question Number	Scheme	Marks
2	$z = -2i - 1$ is also a root	B1
	$\left(z - (2i - 1)\right)\left(z - (-2i - 1)\right) = z^2 + 2z + 5$	M1A1
	$(z + 3)\left(z^2 + 2z + 5\right) = 0$	M1
	$z = -3$	A1
		(5)
		[5]
	Alternative	
	f(-3)=0 so $z = -3$ is also a root	M1A1
	$(z + 3)\left(z^2 + 2z + 5\right) = 0$	
	$\left(z - (2i - 1)\right)\left(z - (-2i - 1)\right) = 0$	M1A1
	$z = -2i - 1$ is also a root	B1
	Notes	
	First M for expanding their $(z - \alpha)(z - \beta)$	
	Second M for inspection or long division.	

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>3(a)</td><td>$z_1 = \frac{1}{2} + i\frac{\sqrt{3}}{2}$
$r = \sqrt{\frac{1}{4}+\frac{3}{4}} = 1$, $\tan\theta = \sqrt{3}$ so $\theta = \dfrac{\pi}{3}$, both r values

$z_2 = -\sqrt{3} + i$
$r = \sqrt{3+1} = 2$, $\tan\theta = \dfrac{-1}{\sqrt{3}}$ so $\theta = \dfrac{5\pi}{6}$</td><td>M1A1

M1A1
(4)</td></tr>
<tr><td>3(b)</td><td>$|z_1 z_2| = |z_1||z_2| = 2$</td><td>M1A1
(2)</td></tr>
<tr><td>3(c)</td><td></td><td>M1
A1ft
(2)
[8]</td></tr>
<tr><td>
(a)

(b)
(c)</td><td>Notes
First M for use of Pythagoras, A1 for $r = 1$ and 2
Second M for use of $\tan$ or $\tan^{-1}$, A1 for $\theta = \dfrac{\pi}{3}$ and $\dfrac{5\pi}{6}$
M for their $r_1 r_2$
M for either of their numbers plotted correctly on a single diagram.
A for both their numbers plotted correctly on a single diagram</td><td></td></tr>
</table>

Question Number	Scheme	Marks
4(a)	$xy = 3$ or $y = \frac{3}{x}$	
	$x\frac{\mathrm{d}y}{\mathrm{d}x} + y = 0$	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-y}{x}$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{3}{x^2}$	M1A1
	Gradient of normal is $\frac{x}{y}$ or $\frac{x^2}{3}$	M1
	$y - 3 = \frac{1}{3}(x-1)$	M1
	$y = \frac{1}{3}x + \frac{8}{3}$	A1
		(5)
4(b)	At R, $y = \frac{3}{x}$	M1
	$\frac{9}{x} - x = 8$	
	$x^2 + 8x - 9 = 0$	A1
	$(x+9)(x-1) = 0$	M1
	$x = -9$, $y = -\frac{1}{3}$	A1,A1
		(5) **[10]**
	Notes	
(a)	First M: Use of the product rule: The sum of two terms including d*y*/d*x*, one of which is correct or $\frac{\mathrm{d}y}{\mathrm{d}x} = k\,x^{-2}$	
	First A for correct derivative $-3x^{-2}$ or $-\frac{y}{x}$	
	Second M for use of Perpendicular gradient rule $m_N m_T = -1$	
	Third M for $y - 3 =$ their $m_N\left(x-1\right)$ or $y = mx + c$ with their m_N and $(1,3)$ in an attempt to find 'c'.	
(b)	First M for substituting $y = \frac{3}{x}$ in their normal.	
	First A for correct 3 term quadratic	
	Second M for attempt to solve their 3 term quadratic	

Question Number	Scheme	Marks
5	$\text{f}(1)=3^2+7=16=8\times 2$	B1
	True for $n=1$	
	Assume true for $n=k$,	
	$\text{f}(k)=3^{2k}+7=8p$ where p is a positive integer	
	When $n=k+1$	
	$\text{f}(k+1)-\text{f}(k)=3^{2(k+1)}+7-\left(3^{2k}+7\right)$	M1
	$=9\times 3^{2k}+7-3^{2k}-7$	dM1
	$=8\times 3^{2k}$	A1
	$\text{f}(k+1)=8(3^{2k}+p)=8q$ where q is a positive integer	
	<u>True for $n=k+1$</u>	dM1
	<u>True for $n=1$</u>, if <u>true for $n=k$</u> then true for $n=k+1$	A1cso
	So <u>$3^{2n}+7$ divisible by 8 for all n</u> by Induction.	
		(6)
		[6]
	Notes	
	B for $\text{f}(1)=3^2+7=16$ seen	
	First M for substituting into $\text{f}(k+1)-\text{f}(k)$ or showing $\text{f}(k+1)=9\times 3^{2k}+7$	
	Second M for using $\text{f}(k+1)-\text{f}(k)$ or equivalent	
	First A for $\text{f}(k+1)=\text{f}(k)+8\times 3^{2k}$ or equivalent.	
	Third M for showing divisible by 8. Accept '$\text{f}(k)$ divisible by 8 and 8×3^{2k} divisible by 8'.	
	Second A for conclusion with all 4 underlined elements that can be seen anywhere in the solution	

Question Number	Scheme	Marks
6(a)	$y^2 = 4x$	
	$2y\frac{\mathrm{d}y}{\mathrm{d}x} = 4$	M1A1
	At P, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{p}$	A1
	$y - 2p = \frac{1}{p}\left(x - p^2\right)$	M1A1
		(5)
6(b)(i)	At (-1,2)	
	$2 - 2p = \frac{1}{p}\left(-1 - p^2\right)$	M1
	$p^2 - 2p - 1 = 0$	A1
	$p = 1 \pm \sqrt{2}$	M1
	$p = 1 + \sqrt{2}$, $q = 1 - \sqrt{2}$	A1
		(4)
6(b)(ii)	$PR^2 = 32 + 16\sqrt{2}$, $QR^2 = 32 - 16\sqrt{2}$	M1A1
	Area of $PQR = \frac{1}{2}PR.QR = 8\sqrt{2}$	M1A1 **(4)**
		[13]
	Notes	
(a)	First M for $x^{\frac{1}{2}} \to x^{-\frac{1}{2}}$ or $ky\frac{\mathrm{d}y}{\mathrm{d}x} = c$ or $\frac{\mathrm{d}y}{\mathrm{d}t} \times \frac{1}{\frac{\mathrm{d}x}{\mathrm{d}t}}$**;** can be a function of *p or t*. First A for accurate differentiation Second M applies $y - 2p$ = their $m\left(x - p^2\right)$ or y = (their m)$x + c$ using $x = p^2$ *and* $y = 2p$ in an attempt to find c. **Their *m* must be a function of *p* from calculus.**	
(b)i	First M substitute coordinates of the point R into their tangent Second M for solving 3 term quadratic Second A for $1 \pm \sqrt{2}$ seen	
(b)ii	First M for attempt to find distance between their P and R or Q and R using formula or sketch and Pythagoras. Second M for using $\frac{1}{2}bh$ on their PQR Second A accept awrt 11.3	

Question Number	Scheme	Marks
7(a)		
	$\sum_{r=1}^{n} r^2(r-1) = \sum_{r=1}^{n} r^3 - \sum_{r=1}^{n} r^2$	M1
	$= \frac{n^2(n+1)^2}{4} - \frac{n(n+1)(2n+1)}{6}$	A1
	$= \frac{n(n+1)}{12}\left(3n\left(n+1\right)-2\left(2n+1\right)\right)$	M1
		A1
	$= \frac{n(n+1)(3n^2-n-2)}{12}$	
	$= \frac{n(n+1)(3n+2)(n-1)}{12}$	A1cso
		(5)
7(b)	$\sum_{r=10}^{r=50} r^2(r-1) = \sum_{r=1}^{50} r^2(r-1) - \sum_{r=1}^{r=9} r^2(r-1)$	M1
	$= \frac{1}{12}\left(50\times51\times152\times49\right) - \frac{1}{12}\left(9\times10\times29\times8\right)$	A1
	$= 1582700 - 1740 = 1580960$	A1
		(3)
		[8]
	Notes	
(a)	First M for expanding brackets	
	First A for correct expressions for $\sum r^3$ and $\sum r^2$	
	Second M for factorising by $n(n+1)$	
	Second A for $(3n^2-n-2)$ or equivalent factor	
(b)	First M for f(49 or 50) – f(9 or 10) and attempt to use part (a).	

Question Number	Scheme		Marks
8(a)	$(\mathrm{f}(1)=)-4(<0)$	-4	B1
	$(\mathrm{f}(2)=)1(>0)$	1	B1
	Changes sign so root (in [1,2])		B1
			(3)
8(b)	(table below)		B1M1
	Interval is [1.75,2]		A1
			(3)
8(c)	$\mathrm{f}'(x)=3x^2-2$		M1A1
	$x_1=1.8-\dfrac{1.8^3-2\times1.8-3}{3\times1.8^2-2}$		M1A1
	$x_1=1.90$ to 3sf.		A1
			(5)
			[11]

a	$\mathrm{f}(a)$	b	$\mathrm{f}(b)$	$\dfrac{a+b}{2}$	$\mathrm{f}\left(\dfrac{a+b}{2}\right)$
1	-4	*2*	1	1.5	-2.625
1.5	-2.625	2	1	1.75	-1.140625

Notes

(b) B for awrt -2.6

M for attempt to find $\mathrm{f}(1.75)$

A for $\mathrm{f}(1.75)=$ awrt -1.1 with $1.75\leqslant\alpha\leqslant2$ or $1.75<\alpha<2$ or $[1.75,\ 2]$ or $(1.75,\ 2)$.

(c) First M for at least one of the two terms differentiated correctly.

First A for correct derivative

Second M for correct application of Newton-Raphson using their values.

Second A for f(1.8)= -0.768

Third A for 1.90 cao

Question Number	Scheme	Marks
9(a)	$\begin{pmatrix}3 & 1\\ 1 & -2\end{pmatrix}\begin{pmatrix}3 & 1\\ 1 & -2\end{pmatrix}=\begin{pmatrix}10 & 1\\ 1 & 5\end{pmatrix}$	M1A1 **(2)**
(b)	$\det \mathbf{A} = -7 \neq 0$ so $\mathbf{A}$ is non-singular	M1A1 **(2)**
(c)	$\mathbf{A}^{-1} = -\frac{1}{7}\begin{pmatrix}-2 & -1\\ -1 & 3\end{pmatrix}$	M1A1 **(2)**
(d)	$-\frac{1}{7}\begin{pmatrix}-2 & -1\\ -1 & 3\end{pmatrix}\begin{pmatrix}k-1\\ 2-k\end{pmatrix}=-\frac{1}{7}\begin{pmatrix}-2(k-1)-1(2-k)\\ -1(k-1)+3(2-k)\end{pmatrix}$	M1
	$=\begin{pmatrix}\frac{1}{7}k\\ \frac{4}{7}k-1\end{pmatrix}$	A1,A1
	(p lies on $y=4x-1$)	
		(3)
		[9]
	Notes	
(d)	**Alt**	
	$\begin{pmatrix}3 & 1\\ 1 & -2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}k-1\\ 2-k\end{pmatrix}$ then multiply out and attempt to solve simultaneous equations for x or y in terms of k. M1 $x=\frac{1}{7}k$ A1 $y=\frac{4}{7}k-1$ A1	

PMT

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