

General Certificate of Education  
June 2006  
Advanced Level Examination



**MATHEMATICS**  
**Unit Further Pure 3**

**MFP3**

Monday 19 June 2006 9.00 am to 10.30 am

**For this paper you must have:**

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

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1 It is given that  $y$  satisfies the differential equation

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 4y = 8x - 10 - 10\cos 2x$$

- (a) Show that  $y = 2x + \sin 2x$  is a particular integral of the given differential equation. *(3 marks)*
- (b) Find the general solution of the differential equation. *(4 marks)*
- (c) Hence express  $y$  in terms of  $x$ , given that  $y = 2$  and  $\frac{dy}{dx} = 0$  when  $x = 0$ . *(4 marks)*

2 The function  $y(x)$  satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where 
$$f(x, y) = \frac{x^2 + y^2}{xy}$$

and 
$$y(1) = 2$$

(a) Use the Euler formula

$$y_{r+1} = y_r + hf(x_r, y_r)$$

with  $h = 0.1$ , to obtain an approximation to  $y(1.1)$ . *(3 marks)*

(b) Use the improved Euler formula

$$y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$$

where  $k_1 = hf(x_r, y_r)$  and  $k_2 = hf(x_r + h, y_r + k_1)$  and  $h = 0.1$ , to obtain an approximation to  $y(1.1)$ , giving your answer to four decimal places. *(6 marks)*

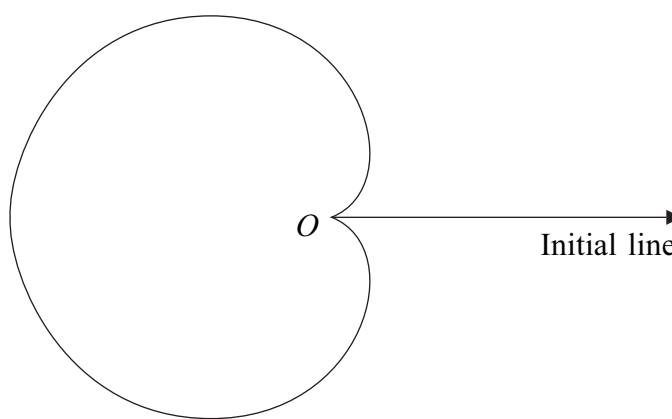
- 3 (a) Show that  $\sin x$  is an integrating factor for the differential equation

$$\frac{dy}{dx} + (\cot x)y = 2 \cos x \quad (3 \text{ marks})$$

- (b) Solve this differential equation, given that  $y = 2$  when  $x = \frac{\pi}{2}$ . (6 marks)

- 4 The diagram shows the curve  $C$  with polar equation

$$r = 6(1 - \cos \theta), \quad 0 \leq \theta < 2\pi$$



- (a) Find the area of the region bounded by the curve  $C$ . (6 marks)

- (b) The circle with cartesian equation  $x^2 + y^2 = 9$  intersects the curve  $C$  at the points  $A$  and  $B$ .

- (i) Find the polar coordinates of  $A$  and  $B$ . (4 marks)

- (ii) Find, in surd form, the length of  $AB$ . (2 marks)

- 5 (a) Show that  $\lim_{a \rightarrow \infty} \left( \frac{3a+2}{2a+3} \right) = \frac{3}{2}$ . (2 marks)

- (b) Evaluate  $\int_1^{\infty} \left( \frac{3}{3x+2} - \frac{2}{2x+3} \right) dx$ , giving your answer in the form  $\ln k$ , where  $k$  is a rational number. (5 marks)

6 (a) Show that the substitution

$$u = \frac{dy}{dx} + 2y$$

transforms the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = e^{-2x}$$

into

$$\frac{du}{dx} + 2u = e^{-2x} \quad (4 \text{ marks})$$

(b) By using an integrating factor, or otherwise, find the general solution of

$$\frac{du}{dx} + 2u = e^{-2x}$$

giving your answer in the form  $u = f(x)$ . (5 marks)

(c) Hence find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = e^{-2x}$$

giving your answer in the form  $y = g(x)$ . (5 marks)

- 7 (a) (i) Write down the first three terms of the binomial expansion of  $(1 + y)^{-1}$ , in ascending powers of  $y$ . (1 mark)

- (ii) By using the expansion

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

and your answer to part (a)(i), or otherwise, show that the first three non-zero terms in the expansion, in ascending powers of  $x$ , of  $\sec x$  are

$$1 + \frac{x^2}{2} + \frac{5x^4}{24} \quad (5 \text{ marks})$$

- (b) By using Maclaurin's theorem, or otherwise, show that the first two non-zero terms in the expansion, in ascending powers of  $x$ , of  $\tan x$  are

$$x + \frac{x^3}{3} \quad (3 \text{ marks})$$

- (c) Hence find  $\lim_{x \rightarrow 0} \left( \frac{x \tan 2x}{\sec x - 1} \right)$ . (4 marks)

**END OF QUESTIONS**

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