General Certificate of Education
Advanced Level Examination
January 2012

Mathematics MPC4

Unit Pure Core 4

Monday 23 January 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 (a) Express $\dfrac{2x+3}{4x^2-1}$ in the form $\dfrac{A}{2x-1}+\dfrac{B}{2x+1}$, where A and B are integers. *(3 marks)*

(b) Express $\dfrac{12x^3-7x-6}{4x^2-1}$ in the form $Cx+\dfrac{D(2x+3)}{4x^2-1}$, where C and D are integers. *(3 marks)*

(c) Evaluate $\displaystyle\int_1^2 \frac{12x^3-7x-6}{4x^2-1}\,\mathrm{d}x$, giving your answer in the form $p+\ln q$, where p and q are rational numbers. *(5 marks)*

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2 Angle α is acute and $\cos\alpha=\frac{3}{5}$. Angle β is **obtuse** and $\sin\beta=\frac{1}{2}$.

(a) (i) Find the value of $\tan\alpha$ as a fraction. *(1 mark)*

(ii) Find the value of $\tan\beta$ in surd form. *(2 marks)*

(b) Hence show that $\tan(\alpha+\beta)=\dfrac{m\sqrt{3}-n}{n\sqrt{3}+m}$, where m and n are integers. *(3 marks)*

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3 (a) Find the binomial expansion of $(1+6x)^{\frac{2}{3}}$ up to and including the term in x^2. *(2 marks)*

(b) Find the binomial expansion of $(8+6x)^{\frac{2}{3}}$ up to and including the term in x^2. *(3 marks)*

(c) Use your answer from part **(b)** to find an estimate for $\sqrt[3]{100}$ in the form $\dfrac{a}{b}$, where a and b are integers. *(2 marks)*

4 A scientist is testing models for the growth and decay of colonies of bacteria.

For a particular colony, which is growing, the model is $P = Ae^{\frac{1}{8}t}$, where P is the number of bacteria after a time t minutes and A is a constant.

(a) This growing colony consists initially of 500 bacteria. Calculate the number of bacteria, according to the model, after one hour. Give your answer to the nearest thousand. *(2 marks)*

(b) For a second colony, which is decaying, the model is $Q = 500\,000e^{-\frac{1}{8}t}$, where Q is the number of bacteria after a time t minutes.

Initially, the growing colony has 500 bacteria and, at the same time, the decaying colony has 500 000 bacteria.

(i) Find the time at which the populations of the two colonies will be equal, giving your answer to the nearest 0.1 of a minute. *(3 marks)*

(ii) The population of the growing colony will exceed that of the decaying colony by 45 000 bacteria at time T minutes.

Show that

$$\left(e^{\frac{1}{8}T}\right)^2 - 90e^{\frac{1}{8}T} - 1000 = 0$$

and hence find the value of T, giving your answer to one decimal place. *(4 marks)*

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5 A curve is defined by the parametric equations

$$x = 8t^2 - t\,, \qquad y = \frac{3}{t}$$

(a) Show that the cartesian equation of the curve can be written as $xy^2 + 3y = k$, stating the value of the integer k. *(2 marks)*

(b) (i) Find an equation of the tangent to the curve at the point P, where $t = \frac{1}{4}$. *(7 marks)*

(ii) Verify that the tangent at P intersects the curve when $x = \frac{3}{2}$. *(2 marks)*

6 (a) Use the Factor Theorem to show that $4x - 3$ is a factor of

$$16x^3 + 11x - 15$$ *(2 marks)*

(b) Given that $x = \cos\theta$, show that the equation

$$27\cos\theta\cos 2\theta + 19\sin\theta\sin 2\theta - 15 = 0$$

can be written in the form

$$16x^3 + 11x - 15 = 0$$ *(4 marks)*

(c) Hence show that the only solutions of the equation

$$27\cos\theta\cos 2\theta + 19\sin\theta\sin 2\theta - 15 = 0$$

are given by $\cos\theta = \frac{3}{4}$. *(4 marks)*

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7 Solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = y^2 x \sin 3x$$

given that $y = 1$ when $x = \frac{\pi}{6}$. Give your answer in the form $y = \frac{9}{\mathrm{f}(x)}$. *(9 marks)*

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8 The points A and B have coordinates $(4, -2, 3)$ and $(2, 0, -1)$ respectively.

The line l passes through A and has equation $\mathbf{r} = \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix}$.

(a) (i) Find the vector $\overrightarrow{AB}$. *(2 marks)*

(ii) Find the acute angle between AB and the line l, giving your answer to the nearest degree. *(4 marks)*

(b) The point C lies on the line l such that the angle ABC is a right angle. Given that $ABCD$ is a rectangle, find the coordinates of the point D. *(6 marks)*