



A-LEVEL

Mathematics

Pure Core 4 – MPC4
Mark scheme

6360
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Version 1.1: Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MARK SCHEME – A-LEVEL MATHEMATICS – MPC4 – JUNE 2015

Q1	Solution	Mark	Total	Comment
(a)	$19x - 2 = A(1 + 6x) + B(5 - x)$ $A = 3$ $B = -1$	M1 A1 A1	3	Correct equation and attempt to find a value for A or B . NMS or cover up rule; A or B correct SC2 A and B correct SC3 .
(b)	$\int \frac{3}{5-x} - \frac{1}{1+6x} dx$ $= p \ln(5-x) + q \ln(1+6x)$ $= -3 \ln(5-x)$ $\quad - \frac{1}{6} \ln(1+6x)$ $\int_0^4 = [-3 \ln 1 - \frac{1}{6} \ln 25] - [-3 \ln 5 - \frac{1}{6} \ln 1]$ $= -\frac{1}{6} \ln 25 + 3 \ln 5$ $= \frac{8}{3} \ln 5$	M1 A1ft A1ft m1 A1 A1	6	Condone missing brackets OE Either term in a correct form ft on their A ft on their B Substitute limits correctly in their integral; $F(4) - F(0)$ ACF. $\ln 1 = 0$ PI CSO Condone equivalent fractions or recurring decimal
Total			9	

Q2	Solution	Mark	Total	Comment
(a)	$R = \sqrt{29}$ $\sqrt{29} \cos \alpha = 2, \sqrt{29} \sin \alpha = 5$ or $\tan \alpha = \frac{5}{2}$ $\alpha = 1.19$	B1 M1 A1	3	Allow 5.4 or better Their $\sqrt{29}$ Note $\cos \alpha = 2$ or $\sin \alpha = 5$ is M0 Must be exactly this
(b)(i)	$R \cos(x + \alpha) = R$ or $\cos(x + \alpha) = 1$ or $x + \alpha = 2\pi$ or $x + \alpha = 0$ or $x = -\alpha$ $(x =) 5.09$	M1 A1	2	Candidate's R and α Must be exactly this
(ii)	$\cos(x + \alpha) = -\frac{1}{R}$ $(x + \alpha =) 1.75757... \text{ and } 4.52560...$ $x = 0.567$ and $x = 3.34$	M1 A1 A1	3	Candidate's R and α ; PI Rounded or truncated to at least 2 dp; Ignore 'extra' solutions Condone $x = 0.568$; $x = 3.34$ must be correct NMS is 0/3 A0 if extra values in interval $0 < x < 2\pi$
Total			8	

MARK SCHEME – A-LEVEL MATHEMATICS – MPC4 – JUNE 2015

Q3	Solution	Mark	Total	Comment
(a)	$f\left(-\frac{1}{2}\right) = -1 - 3 + 1 + d = -2$ $d = 1$	M1 A1	2	Attempt to evaluate $f\left(-\frac{1}{2}\right)$ and equated to -2 NMS is 0/2
(b)(i)	$(2x+1)$ is a factor $g(x) = (2x+1)(4x^2 + bx + 3)$ $g(x) = (2x+1)(4x^2 - 8x + 3)$ $g(x) = (2x+1)(2x-1)(2x-3)$	B1 M1 A1	3	OE $\left(x + \frac{1}{2}\right)$ Attempt to find quadratic factor or a second linear factor using Factor Theorem OE if $\left(x + \frac{1}{2}\right)$ is used OE ; must be a product NMS : SC3 if product is correct SC1 if one or two factors are correct
(ii)	$\frac{4x^2 - 1}{g(x)} = \frac{1}{2x - 3}$ $\frac{d}{dx}\left(\frac{1}{2x - 3}\right) = \frac{k}{(2x - 3)^2}$ $= -\frac{2}{(2x - 3)^2}$ (Derivative is) negative, or < 0 hence decreasing	B1 M1 A1 E1	4	Attempt to differentiate simplified h Correct derivative Explanation and conclusion required Derivative must be correct
Total			9	
(b)(ii)	Special case $h(x) = \frac{1}{2x-3}$ $2x - 3$ is an increasing function, so $\frac{1}{2x-3}$ is a decreasing function	B1 E1	2	Award only if $h(x) = \frac{1}{2x-3}$ is correct

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Q4	Solution	Mark	Total	Comment
(a)	$1 + \frac{1}{5} \times 5x + kx^2$ $1 + x - 2x^2$	<p>M1</p> <p>A1</p>	2	<p>k any non-zero numerical expression</p> <p>Simplified to this</p>
(b) (i)	$(8 + 3x)^{-\frac{2}{3}} = 8^{-\frac{2}{3}} \left(1 + \frac{3}{8}x\right)^{-\frac{2}{3}}$ $\left(1 + \frac{3}{8}x\right)^{-\frac{2}{3}}$ $= 1 + \left(-\frac{2}{3}\right)\left(\frac{3}{8}x\right) + \frac{1}{2}\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)\left(\frac{3}{8}x\right)^2$ $\frac{1}{4} - \frac{1}{16}x + \frac{5}{256}x^2$	<p>B1</p> <p>M1</p> <p>A1</p>	3	<p>ACF for $8^{-\frac{2}{3}} = \frac{1}{4}$</p> <p>Expand correctly using their $\frac{3}{8}x$</p> <p>Condone poor use of or missing brackets</p> <p>Accept $= \frac{1}{4} \left(1 - \frac{1}{4}x + \frac{5}{64}x^2\right)$</p>
(ii)	$x = \frac{1}{3}$ <p>0.2313 (4dp)</p>	<p>M1</p> <p>A1</p>	2	<p>$x = \frac{1}{3}$ used in their expansion from (b)(i)</p> <p>Note 3 in 4th decimal place</p>
Total			7	

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Q5	Solution	Mark	Total	Comment
(a)	$\left(\frac{dx}{dt} =\right) -2 \sin 2t \quad \left(\frac{dy}{dt} =\right) \cos t$ $\left(\frac{dy}{dx} =\right) \frac{\cos t}{-2 \sin 2t}$ <p>At $t = \frac{\pi}{6}$ gradient $m_T = -\frac{1}{2}$</p>	<p>B1</p> <p>M1</p> <p>A1</p>	3	<p>Both correct</p> <p>Correct use of chain rule with their derivatives of form $a \sin 2t$, $b \cos t$</p>
(b)	<p>Gradient of normal $m_N = 2$</p> $\left(y - \cos\left(\frac{2\pi}{6}\right)\right) = m_N \left(x - \sin\left(\frac{\pi}{6}\right)\right)$ $y = 2x - \frac{1}{2}$ <p>Alternative for M1</p> $\sin\left(\frac{\pi}{6}\right) = 2 \cos\left(\frac{2\pi}{6}\right) + c$	<p>B1ft</p> <p>M1</p> <p>A1</p>	3	<p>ft gradient of tangent; $m_N = \frac{-1}{m_T}$</p> <p>For m_N, allow their m_T with a change of sign or the reciprocal at $(\sin \frac{\pi}{6}, \cos \frac{2\pi}{6})$ or $(\frac{1}{2}, \frac{1}{2})$</p> <p>Must be in this $y = mx + c$ form</p> <p>Use $y = mx + c$ to find c with their gradient m_N at $(\sin \frac{\pi}{6}, \cos \frac{2\pi}{6})$ or $(\frac{1}{2}, \frac{1}{2})$</p>
(c)	$\cos 2q = 1 - 2 \sin^2 q$ $\sin q = 2\left(1 - 2 \sin^2 q\right) - \frac{1}{2}$ $8 \sin^2 q + 2 \sin q - 3 = 0 \quad \text{OE}$ $\left(\sin q = \frac{1}{2}\right) \quad \sin q = -\frac{3}{4}$ $(x =) -\frac{1}{8}$	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p>	5	<p>Seen or used in this form</p> <p>Use parametric equations and candidate's $\cos 2q$ in the form $\pm 1 + k \sin^2 q$</p> <p>Collect like terms; must be a quadratic equation</p> <p>Must come from a correct quadratic equation with the previous 3 marks awarded</p> <p>Previous 4 marks must have been awarded</p>
Total			11	

Mark scheme Alternative

Q5	Solution	Mark	Total	Comment
(a)	$x = 1 - 2y^2 \quad 1 = -4y \frac{dy}{dx} \quad \text{or} \quad \frac{dx}{dy} = -4y$ $\frac{dy}{dx} = -\frac{1}{4 \sin \frac{\pi}{6}}$ At $t = \frac{\pi}{6}$ gradient $m_T = -\frac{1}{2}$	B1 M1 A1	3	Find a correct Cartesian equation and differentiate implicitly correctly Use $y = \sin \frac{\pi}{6}$ or $y = \frac{1}{2}$ in their $\frac{dy}{dx}$; PI CSO
(b)	Gradient of normal = 2 $\left(y - \cos\left(\frac{2\pi}{6}\right) \right) = m_N \left(x - \sin\left(\frac{\pi}{6}\right) \right)$ $y = 2x - \frac{1}{2}$ Alternative for M1 $\sin\left(\frac{\pi}{6}\right) = 2 \cos\left(\frac{2\pi}{6}\right) + c$	B1ft M1 A1	3	ft gradient of tangent, $m_N = \frac{-1}{m_T}$ For m_N , allow their m_T with a change of sign or the reciprocal at $\left(\sin \frac{\pi}{6}, \cos \frac{2\pi}{6}\right)$ or $\left(\frac{1}{2}, \frac{1}{2}\right)$ CSO Use $y = mx + c$ to find c with candidate's gradient m_N at $\left(\sin \frac{\pi}{6}, \cos \frac{2\pi}{6}\right)$ or $\left(\frac{1}{2}, \frac{1}{2}\right)$
(c)	$x = 1 - 2y^2$ $1 - 2y^2 = \frac{y + \frac{1}{2}}{2}$ $4y^2 + y - \frac{3}{2} = 0 \Rightarrow$ $8 \sin^2 q + 2 \sin q - 3 = 0$ $\left(\sin q = \frac{1}{2} \right) \quad \sin q = -\frac{3}{4}$ $(x =) -\frac{1}{8}$	B1 M1 A1 A1 A1	5	PI by $x = 1 - 2\left(2x - \frac{1}{2}\right)^2$ Use their Cartesian equation and normal to eliminate x Collect like terms; must be a quadratic equation Must come from a correct quadratic equation with the previous 3 marks awarded Previous 4 marks must have been awarded
	Total		11	

MARK SCHEME – A-LEVEL MATHEMATICS – MPC4 – JUNE 2015

Q6	Solution	Mark	Total	Comment
(a)	$(\overline{AB} =) \begin{bmatrix} 2 \\ -4 \\ -6 \end{bmatrix}$ $\overline{AB} \bullet \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix} = (2 \times 3) + (-4 \times 1) + (6 \times -2)$ $\sqrt{56}\sqrt{14} \cos BAC = 14$ <p>angle $BAC = 60^\circ$</p>	<p>B1</p> <p>M1</p> <p>m1</p> <p>A1</p>	4	<p>Or $(\overline{BA} =) \begin{bmatrix} -2 \\ 4 \\ 6 \end{bmatrix}$</p> <p>Correctly ft on “their” \overline{AB}</p> <p>Correct use of formula with consistent vectors; ACF or $\pi/3$; NMS 60° scores 0/4</p>
(b)	$(\overline{BC} =) \begin{bmatrix} 3 \\ 2 \\ 10 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix} - \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$ $\overline{AB} \bullet \overline{BC} =$ $2(3\lambda - 2) - 4(\lambda + 4) - 6(-2\lambda + 6) = 0$ $14\lambda - 56 = 0 \Rightarrow \lambda = 4$ <p>C is at (15,6,2)</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p>	4	<p>$\pm \overline{BC}$ ACF</p> <p>Correct scalar product with their \overline{AB}, their \overline{BC}, equate to 0 and solve for λ</p> <p>Accept as a column vector NMS (15,6,2) scores 0/4</p>
(c)	<p>E_1 is at (11,0,0)</p> $\overline{OD} = \overline{OC} + \overline{AB} = \begin{bmatrix} 15 \\ 6 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ -4 \\ -6 \end{bmatrix} = \begin{bmatrix} 17 \\ 2 \\ -4 \end{bmatrix}$ $\overline{OE}_2 = \begin{bmatrix} 17 \\ 2 \\ -4 \end{bmatrix} + \frac{1}{2} \times 4 \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$ <p>E_2 is at (23,4,-8)</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p>	4	<p>Accept as a column vector</p> <p>Correct vector expression with their λ and their \overline{OD}</p> <p>Accept as a column vector</p>
Total			12	
(b)	Alternative by Pythagoras			
	$(\overline{BC} =) \begin{bmatrix} 3 \\ 2 \\ 10 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix} - \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$ $(3\lambda)^2 + (\lambda)^2 + (-2\lambda)^2$ $= 56 + (-2 + 3\lambda)^2 + (4 + \lambda)^2 + (6 - 2\lambda)^2$ $112 - 28\lambda = 0 \quad \lambda = 4$ <p>C is at (15,6,2)</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p>	4	<p>$\pm \overline{BC}$ ACF</p> <p>$AC^2 = AB^2 + BC^2$ Correct Pythagoras expression, attempt to expand and solve for λ</p> <p>Accept as a column vector</p>

(b)	Alternative by $\cos 60 = \frac{1}{2}$			
	$\frac{1}{2} = \frac{ \overline{AB} }{ \overline{AC} } = \frac{\sqrt{56}}{\sqrt{(3\lambda)^2 + (\lambda)^2 + (-2\lambda)^2}}$ $\frac{1}{4} = \frac{56}{14\lambda^2}$ $\lambda^2 = 16 \Rightarrow \lambda = 4 \quad (\text{or } \lambda = -4)$ <p>C is at (15, 6, 2)</p>	B1		
		M1		Square and simplify
		A1		
		A1	4	Accept as a column vector

(c)	Alternatives			
Alt (i)				
	$\overline{OE}_1 = \overline{OB} + \frac{1}{2}\overline{AC} = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 12 \\ 4 \\ -8 \end{bmatrix}$ <p>E_1 is at (11, 0, 0)</p> $\overline{OE}_2 = \overline{OB} + 3\overline{BE}_1 = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix} + 3 \begin{bmatrix} 6 \\ 2 \\ -4 \end{bmatrix}$ <p>E_2 is at (23, 4, -8)</p>	B1		
		M1		Correct vector expression with their \overline{BE}_1
		B1		All correct
		A1	4	
Alt (ii)				
	$\overline{OD} = \overline{OB} + \overline{AC} = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix} + \begin{bmatrix} 12 \\ 4 \\ -8 \end{bmatrix}$ <p>D is at (17, 2, -4)</p> $\overline{OE}_2 = \overline{OD} + \frac{1}{2}\overline{AC} = \begin{bmatrix} 17 \\ 2 \\ -4 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 12 \\ 4 \\ -8 \end{bmatrix}$ <p>E_2 is at (23, 4, -8)</p> $\overline{OE}_1 = \overline{OB} + \frac{1}{2}\overline{AC} = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 12 \\ 4 \\ -8 \end{bmatrix}$ <p>E_1 is at (11, 0, 0)</p>	B1		
		M1		Correct vector expression with their \overline{OD} and their \overline{AC}
		A1		
		B1	4	

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Q7	Solution	Mark	Total	Comment
(a)	$k = \left(\frac{1}{2}\right)^3 + 2e^{-3\ln 2} \times \frac{1}{2} - \ln 2$ $= \frac{1}{8} + \frac{1}{8} - \ln 2 = \frac{1}{4} - \ln 2$	B1	1	Clear use of $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$ and $e^{-3\ln 2} = \frac{1}{8}$ Accept $\frac{2}{8} - \ln 2$
(b)	$3y^2 \frac{dy}{dx}$ $pye^{-3x} + qe^{-3x} \frac{dy}{dx}$ $-6ye^{-3x} + 2e^{-3x} \frac{dy}{dx}$ $-1 = 0$ $\frac{3}{4} \frac{dy}{dx} - 6 \times \frac{1}{8} \times \frac{1}{2} + 2 \times \frac{1}{8} \frac{dy}{dx} - 1 = 0$ $\frac{dy}{dx} = \frac{11}{8} \text{ or } 1.375$	B1 M1 A1 B1 m1 A1	6	Both required -1 and no other terms Substitute $x = \ln 2$ or $e^{-3x} = \frac{1}{8}$ and $y = \frac{1}{2}$ into their expression
	Total		7	

MARK SCHEME – A-LEVEL MATHEMATICS – MPC4 – JUNE 2015

Q8	Solution	Mark	Total	Comment
(a)(i)	$\int \frac{1}{\sqrt{4+5x}} dx = \int \frac{1}{5(1+t)^2} dt$ $a(4+5x)^{\frac{1}{2}} \text{ or } b(1+t)^{-1}$ $\frac{2}{5}(4+5x)^{\frac{1}{2}}$ $-\frac{1}{5}(1+t)^{-1} \quad (+C)$ $x=0, t=0 \Rightarrow C=1$ $\frac{2}{5}(4+5x)^{\frac{1}{2}} = 1 - \frac{1}{5}(1+t)^{-1}$ $x = \frac{5}{4} \left(1 - \frac{(1+t)^{-1}}{5} \right)^2 - \frac{4}{5}$	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>m1</p> <p>A1</p> <p>A1</p>	<p>7</p>	<p>Correct separation and notation seen on a single line somewhere in their solution</p> <p>OE $a\sqrt{4+5x}$ or $b\left(\frac{1}{1+t}\right)$</p> <p>OE $\frac{2}{5}\sqrt{4+5x}$</p> <p>OE $-\frac{1}{5(1+t)}$</p> <p>Use (0,0) to find a constant</p> <p>OE</p> <p>ACF eg $x = \frac{1}{20} \left(\frac{4+5t}{1+t} \right)^2 - \frac{4}{5}$</p>
(b)(i)	$\frac{dr}{dt}$ $\frac{1}{r^2}$ $\frac{dr}{dt} = \frac{k}{r^2}$	<p>B1</p> <p>M1</p> <p>A1</p>	<p>3</p>	<p>Seen; allow R for r</p> <p>$\frac{1}{r^2}$ seen ; allow R for r</p> <p>Any constant k including $\frac{c}{\pi}$ but not including variable t</p> <p>Must use R or r consistently</p>
(ii)	$\left(\frac{dr}{dt}\right) = 4.5 = \frac{k}{1^2} \text{ or } 4.5 = \frac{c}{\pi \times 1^2}$ $0.5 = \frac{4.5}{r^2} \Rightarrow r = 3 \text{ (metres)}$	<p>M1</p> <p>A1</p>	<p>2</p>	<p>Use $\frac{dr}{dt} = 4.5$ with $r = 1$ to find a value for the constant</p>
Total			12	