General Certificate of Education
Advanced Level Examination
January 2013

Mathematics MPC4

Unit Pure Core 4

Friday 25 January 2013 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The polynomial $f(x)$ is defined by $f(x) = 2x^3 + x^2 - 8x - 7$.

(a) Use the Remainder Theorem to find the remainder when $f(x)$ is divided by $(2x + 1)$. *(2 marks)*

(b) The polynomial $g(x)$ is defined by $g(x) = f(x) + d$, where d is a constant.

(i) Given that $(2x + 1)$ is a factor of $g(x)$, show that $g(x) = 2x^3 + x^2 - 8x - 4$. *(1 mark)*

(ii) Given that $g(x)$ can be written as $g(x) = (2x + 1)(x^2 + a)$, where a is an integer, express $g(x)$ as a product of three linear factors. *(1 mark)*

(iii) Hence, or otherwise, show that $\dfrac{g(x)}{2x^3 - 3x^2 - 2x} = p + \dfrac{q}{x}$, where p and q are integers. *(3 marks)*

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2 It is given that $f(x) = \dfrac{7x - 1}{(1 + 3x)(3 - x)}$.

(a) Express $f(x)$ in the form $\dfrac{A}{3 - x} + \dfrac{B}{1 + 3x}$, where A and B are integers. *(3 marks)*

(b) (i) Find the first three terms of the binomial expansion of $f(x)$ in the form $a + bx + cx^2$, where a, b and c are rational numbers. *(7 marks)*

(ii) State why the binomial expansion cannot be expected to give a good approximation to $f(x)$ at $x = 0.4$. *(1 mark)*

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3 (a) (i) Express $3\cos x + 2\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving your value of α to the nearest 0.1°. *(3 marks)*

(ii) Hence find the minimum value of $3\cos x + 2\sin x$ and the value of x in the interval $0° < x < 360°$ where the minimum occurs. Give your value of x to the nearest 0.1°. *(3 marks)*

(b) (i) Show that $\cot x - \sin 2x = \cot x \cos 2x$ for $0° < x < 180°$. *(3 marks)*

(ii) Hence, or otherwise, solve the equation

$$\cot x - \sin 2x = 0$$

in the interval $0° < x < 180°$. *(3 marks)*

4 (a) A curve is defined by the equation $x^2 - y^2 = 8$.

(i) Show that at any point (p, q) on the curve, where $q \neq 0$, the gradient of the curve is given by $\dfrac{dy}{dx} = \dfrac{p}{q}$. *(2 marks)*

(ii) Show that the tangents at the points (p, q) and $(p, -q)$ intersect on the x-axis. *(4 marks)*

(b) Show that $x = t + \dfrac{2}{t}$, $y = t - \dfrac{2}{t}$ are parametric equations of the curve $x^2 - y^2 = 8$. *(2 marks)*

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5 (a) Find $\displaystyle\int x\sqrt{x^2 + 3}\,dx$. *(2 marks)*

(b) Solve the differential equation

$$\frac{dy}{dx} = \frac{x\sqrt{x^2+3}}{e^{2y}}$$

given that $y = 0$ when $x = 1$. Give your answer in the form $y = f(x)$. *(7 marks)*

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6 (a) The points A, B and C have coordinates $(3, 1, -6)$, $(5, -2, 0)$ and $(8, -4, -6)$ respectively.

(i) Show that the vector $\overrightarrow{AC}$ is given by $\overrightarrow{AC} = n\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, where n is an integer. *(1 mark)*

(ii) Show that the acute angle ACB is given by $\cos^{-1}\left(\dfrac{5\sqrt{2}}{14}\right)$. *(4 marks)*

(b) Find a vector equation of the line AC. *(2 marks)*

(c) The point D has coordinates $(6, -1, p)$. It is given that the lines AC and BD intersect.

(i) Find the value of p. *(4 marks)*

(ii) Show that $ABCD$ is a rhombus, and state the length of each of its sides. *(4 marks)*

Turn over ▶

7 A biologist is investigating the growth of a population of a species of rodent. The biologist proposes the model

$$N = \frac{500}{1 + 9e^{-\frac{t}{8}}}$$

for the number of rodents, N, in the population t weeks after the start of the investigation.

Use this model to answer the following questions.

(a) (i) Find the size of the population at the start of the investigation. *(1 mark)*

(ii) Find the size of the population 24 weeks after the start of the investigation. Give your answer to the nearest whole number. *(1 mark)*

(iii) Find the number of weeks that it will take the population to reach 400. Give your answer in the form $t = r \ln s$, where r and s are integers. *(3 marks)*

(b) (i) Show that the rate of growth, $\frac{dN}{dt}$, is given by

$$\frac{dN}{dt} = \frac{N}{4000}(500 - N)$$ *(4 marks)*

(ii) The maximum rate of growth occurs after T weeks. Find the value of T. *(4 marks)*