



General Certificate of Education

Mathematics 6360

MPC3 Pure Core 3

Mark Scheme

2008 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

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М	mark is for method					
m or dM	mark is dependent on one or more M marks	and is for method	od			
А	mark is dependent on M or m marks and is	for accuracy				
В	mark is independent of M or m marks and is	s for method and	accuracy			
E	mark is for explanation					
$\sqrt{0}$ or ft or F	follow through from previous					
	incorrect result	MC	mis-copy			
CAO	correct answer only	MR	mis-read			
CSO	correct solution only	RA	required accuracy			
AWFW	anything which falls within	FW	further work			
AWRT	anything which rounds to	ISW	ignore subsequent work			
ACF	any correct form	FIW	from incorrect work			
AG	answer given	BOD	given benefit of doubt			
SC	special case	WR	work replaced by candidate			
OE	or equivalent	FB	formulae book			
A2,1	2 or 1 (or 0) accuracy marks NOS not on scheme					
–x EE	deduct x marks for each error G graph					
NMS	no method shown c candidate					
PI	possibly implied	sf	significant figure(s)			
SCA	substantially correct approach	dp	decimal place(s)			

Key to mark scheme and abbreviations used in marking

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC3				
Q	Solution	Marks	Total	Comments
1 (a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 5(3x+1)^4 \times 3$	M1		$k(3x+1)^4$
	$=15(3x+1)^4$	A1	2	with no further errors (w.n.f.e)
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{3x+1}$	M1		$\frac{k}{3x+1}$
		A1	2	w.n.f.e
(c)	$\frac{\mathrm{d}y}{\mathrm{d}x} =$	M1		product rule $uv' + u'v$ (from (a) and (b))
	$(3x+1)^5 \times \frac{3}{3x+1} + \ln(3x+1) \times 15(3x+1)^4$	A1 A1	3	either term correct CSO with no further errors
	$ \begin{pmatrix} = (3x+1)^4 [3+15\ln(3x+1)] \\ = 3(3x+1)^4 [1+5\ln(3x+1)] \end{pmatrix} $			
	$\left(=3(3x+1)^{4}\left[1+5\ln(3x+1)\right]\right)$			
	Total		7	
2(a)	$x = \cos^{-1}\frac{1}{3}$	M1		РІ
	$= 1.23, 5.05$ (0.39 π , 1.61 π)	A1,A1	3	AWRT (-1 for each error in range) SC 70.53, 289.47 B1
(b)	$\sec^2 x - 1 = 2 \sec x + 2$	M1		use of $\sec^2 x = 1 + \tan^2 x$
	$\sec^2 x - 2\sec x - 3 = 0$	A1	2	AG; CSO
(c)	$\sec^2 x - 2\sec x - 3 = 0$			
	$(\sec x - 3)(\sec x + 1) = 0$	M1		attempt to solve
	$\cos x = \frac{1}{3} \text{ or } -1 \qquad \text{o.e}$	A1		
	<i>x</i> = 1.23, 5.05,	B1f		(2 answers in range from (a)) AWRT
	3.14 (π)	B1	4	all correct and no extras in range SC 70.53, 289.47, 180 B1
	Total		9	

(Extra +c penalised once throughout paper)

MPC3	(cont)
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Q	Solution	Marks	Total	Comments
3 (a)	$\frac{dy}{dx} = -x2\sin 2x + \cos 2x$	M1		product rule $kx \sin 2x \pm \cos 2x$
J(a)	$dx = \frac{1}{2} x^2 \sin 2x + \cos 2x$	A1	2	no further incorrect working
(b)(i)	$-2\alpha\sin 2\alpha + \cos 2\alpha = 0$	M1		replacing $x = \alpha$ and writing equation
				equal to zero (at any line)
	$2\alpha \sin 2\alpha = \cos 2\alpha$ either			
	$2\alpha \tan 2\alpha = 1$		2	
	$2\alpha \tan 2\alpha - 1 = 0$	A1	2	AG; CSO
	f(0, 4) = 0, 2			
(ii)	$ \begin{array}{c} f(0.4) = 0.2 \\ f(0.5) = -0.6 \end{array} $ awrt o.e.	M1		(0.9's unsubstantiated scores M0)
	Change of sign $\therefore 0.4 < \alpha < 0.5$	A 1	2	
	Change of sign $0.4 < \alpha < 0.5$	A1	2	
(iii)	$2x \tan 2x = 1$			
	$\tan 2x = 1$			
	$\tan 2x - \frac{1}{2x}$ (either			
	$\tan 2x = \frac{1}{2x}$ $2x = \tan^{-1}\left(\frac{1}{2x}\right)$ either			
	(2x)			
	$x = \frac{1}{2} \tan^{-1} \left(\frac{1}{2x} \right)$	B1	1	AG; CSO
	2^{2} (2x)	<i>D</i> 1	1	
(iv)	$x_1 = 0.4$ $x_2 = 0.4480$			25.7
	$x_2 = 0.4480$	M1		$x_2 = 25.7$
	$x_3 = 0.4200$			
	= 0.42	A1	2	
(c)	$y = x \cos 2x$			
(-)	u = x $du = 1$			
		M1		differentiate one term must be $k \sin 2x$
	$dv = \cos 2x v = \frac{\sin 2x}{2}$			integrate one term $\int \Pi dst \delta c k \sin 2x$
		m1		correct substitution of their values into
				parts formula using $u = x$
	$\int = \frac{x \sin 2x}{2} - \int \frac{\sin 2x}{2} (\mathrm{d}x)$			
	$= \left[\frac{x\sin 2x}{2} + \frac{\cos 2x}{4}\right]_{(0)}^{(0.5)}$	A1		
				compative substituting assistant from
	$=\left(\frac{\sin 1}{4} + \frac{\cos 1}{4}\right) - \left(\frac{\cos 0}{4}\right)$	m1		correctly substituting values from previous 2 method marks
	= 0.0954	A1	5	AWRT
	Total		14	

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MPC3	(cont)

Q	Solution	Marks	Total	Comments
4 (a)	$f(x) \ge 0$	B1	1	allow $f \ge 0, y \ge 0, \ge 0$
(b)(i)	$y = \frac{1}{2x - 3}$ $x = \frac{1}{2y - 3}$ $x(2y - 3) = 1$	M1		swap x and y
	x(2y-3) = 1 2xy-3x = 1 2xy = 1+3x $y = \frac{1+3x}{2x} = g^{-1}(x)$ o.e.	M1 A1	3	attempt to isolate w.n.f.e
(ii)	$\left(g^{-1}(x)\right) \neq \frac{3}{2}$	B1	1	
(c)	$\left(\frac{1}{2x-3}\right)^2 = 9$	B1		
	2xy = 1 + 3x $y = \frac{1 + 3x}{2x} = g^{-1}(x)$ o.e. $\left(g^{-1}(x)\right) \neq \frac{3}{2}$ $\left(\frac{1}{2x - 3}\right)^2 = 9$ $2x - 3 = \pm \frac{1}{3}$	M1		square root and invert (condone missing \pm) alternative: attempt to solve a quadratic that comes from $4x^2 - 12 + 9 = \frac{1}{9}$ o.e.
	$x = \frac{5}{3}, \frac{4}{3}$ o.e.	A1	3	
	Total		8	

Alternative

4(b)(i)	$x \to \boxed{\times 2} \to \boxed{-3} \to \boxed{\text{divide into } 1} \to y$		
	$\frac{1}{2y} + \frac{3}{2} \leftarrow \boxed{\div 2} \leftarrow \boxed{+3} \leftarrow \boxed{\text{divide into1}} \leftarrow y$		
	$\frac{1}{y}+3$		
	M1		

MPC3 (cont					
Q	Solution	Marks	Total	C	omments
5(a)(i)		B1		shape	
		D 1	2		
	(0,b) $(a,0)$ x	B1	2	coordinates	
(ii)	<i>v</i> (<i>a</i> ,0) <i>x</i>				
		B1		shape	
	(0,-2b)	B1	2	coordinates	
(b)(i)	Translation	M1			OR I stretch M1 I +
	$\begin{bmatrix} -1 \\ 0 \end{bmatrix}$	A1			(II or III) II SF 4 III ∥ y-axis A1 (I + II + III)
	Stretch I	M1		I + (II or III)	Translation M1
	SF 4 II // y-axis III	A1		I + II + III	$\begin{bmatrix} -1 \\ -2 \end{bmatrix} \qquad \begin{array}{c} A1 \\ B1 \end{array}$
	Translation $\begin{bmatrix} 0\\ -2 \end{bmatrix}$	B1		both	
	$\lfloor -2 \rfloor$ All correct and no mistakes on order etc Alternative:	A1	6		All correct A1
	$y = 4\ln(x+1) - 2 = 4\left[\ln(x+1) - \frac{1}{2}\right]$	(B1)			
	Translation 2	(M1)			
	$\begin{bmatrix} -1\\ -\frac{1}{2} \end{bmatrix}$	(A1)			
	Stretch I	(M1)		I+(II or III)	
	SF 4 II // y-axis III	(A1)		I + II + III	
	All correct and no mistakes on order etc	(A1) (A1)	(6)	1 T 11 T 111	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
5(b)(ii)	$y = 4\ln\left(x+1\right) - 2$			
	$\begin{array}{ll} x = 0 & y = -2 \\ y = 0 \end{array}$	B1		
	$y = 0$ $4\ln(x+1) = 2$			
	$\ln\left(x+1\right) = \frac{1}{2}$	M1		isolate $\ln(x+1) = $ or $(x+1)^4$
	$x+1=e^{\frac{1}{2}}$	A1		$x+1=e^k$
	$x = e^{\frac{1}{2}} - 1$ o.e.	A1	4	CSO isw
	Total		14	
6(a)	$y = \left(e^{3x} + 1\right)^{\frac{1}{2}}$			
	$\frac{dy}{dx} = \frac{1}{2} (e^{3x} + 1)^{-\frac{1}{2}} \times 3e^{3x}$	M1		$\frac{1}{2}(e^{3x}+1)^{-\frac{1}{2}}$
		A1		e^{3x}
		A1		$\frac{3}{2}$ (allow $\frac{1}{2} \times 3$) w.n.f.e
	$x = \ln 2$:			
	$\frac{dy}{dx} = \frac{3}{2} \left(e^{\ln 8} + 1 \right)^{-\frac{1}{2}} \times e^{\ln 8}$	M1		correct substitution into their $\frac{dy}{dx}$
	3 1			(must use $\ln 8 \text{ or } \ln 2^3$)
	$=\frac{3}{2} \times \frac{1}{3} \times 8$ $= 4$	A 1	~	
	= 4	A1	5	
(b)	$\begin{array}{c cc} x & y \\ \hline 0.25 & 1.765(5) \end{array}$			
	0.75 3.238(5)	B1		correct <i>x</i> values
	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	B1		3 or 4 correct <i>y</i> values 4 s.f. or better
	$\int = 0.5 \times \sum y \qquad P.I$	M1		
	= 12.7	A1	4	sc 12.7 with no working $\frac{2}{4}$
(c)	$v = \pi \int y^2 \mathrm{d}x$			
	$= (\pi) \int \left(e^{3x} + 1 \right) (dx)$	M1		
	$= (\pi) \left[\frac{1}{3} e^{3x} + x \right]_{(0)}^{(2)}$	A1		$ke^{3x} + x$
	$= (\pi) \left[\left(\frac{1}{3} e^6 + 2 \right) - \left(\frac{1}{3} e^0 + 0 \right) \right]$	m1		correct substitution into f $(\int e^{3x})$
	$=\pi\left[\frac{1}{3}e^{6}+\frac{5}{3}\right]$	A1	4	CSO
	$\left(=\frac{\pi}{3}(e^6+5)\right)$			
	Total		13	

MPC3	(cont)

$7(a) y = \frac{\sin\theta}{\cos\theta}$ $\frac{dy}{d\theta} = \frac{\cos\theta\cos\theta - \sin\theta(-\sin\theta)}{\cos^2\theta} \qquad M1$ $\frac{dy}{d\theta} = \frac{\cos^2\theta\cos\theta - \sin\theta(-\sin\theta)}{\cos^2\theta} \qquad M1$ $\frac{dy}{A1} \qquad \frac{\pm\cos^2\theta \pm \sin^2\theta}{\cos^2\theta}$ $(1 + \tan^2\theta)$ $(1 + \tan^2\theta)$ $(a + \sin^2\theta)$ $\frac{\sin\theta}{\sqrt{1 - \sin^2\theta}} \qquad A1$ $(b) x = \sin\theta$ $x^2 = \sin^2\theta \qquad \frac{\sin\theta}{\sqrt{1 - \sin^2\theta}} \qquad H1$ $use of \cos^2\theta + x^2 = 1$ $use of \cos^2\theta + x^2 = 1$ $use of \cos^2\theta + x^2 = 1$ $(c) \int \frac{1}{(1 - x^2)^{\frac{1}{2}}} dx$ $x = \sin\theta$ $dx = \cos\theta d\theta \qquad o.e.$ $M1$ $\int = \int \frac{\cos\theta}{(1 - \sin^2\theta)^{\frac{3}{2}}} dx$ $dx = \cos\theta d\theta \qquad o.e.$ $M1$ $\int = \int \frac{\cos\theta}{(\cos^2\theta)^{\frac{3}{2}}} (d\theta)$ $= \int \frac{\cos\theta}{(\cos^2\theta)^{\frac{3}{2}}} (d\theta)$ $= \int \frac{\cos\theta}{(\cos^2\theta)^{\frac{3}{2}}} (d\theta)$ $= \int \frac{1}{\sqrt{1 - x^2}} (+c)$ $A1$ $f = tan\theta$ $= \frac{x}{\sqrt{1 - x^2}} (+c)$ $A1$ $f = tan\theta$ $= \frac{x}{\sqrt{1 - x^2}} (+c)$ $A1$	Q Q	Solu	ition	Marks	Total	Comments
$\frac{dy}{d\theta} = \frac{\cos\theta\cos\theta\cos\theta - \sin\theta(-\sin\theta)}{\cos^2\theta} \qquad MI \\ AI \qquad \frac{\pm\cos^2\theta\pm\sin^2\theta}{\cos^2\theta} \\ = \frac{1}{\cos^2\theta} \\ = \sec^2\theta \qquad o.e. \qquad (1+\tan^2\theta) \\ AI \qquad 3 \qquad AG; CSO \\ (b) \qquad x = \sin\theta \\ x^2 = \sin^2\theta \\ \cos^2\theta = 1 - x^2 \\ \cos^2\theta = 1 - x^2 \\ \cos^2\theta = 1 - x^2 \\ = \frac{\sin\theta}{\cos\theta} \\ = \frac{x}{\sqrt{1-x^2}} \\ = \tan\theta \qquad AG \qquad AI \qquad 2 \qquad AG; CSO \\ (c) \qquad \int \frac{1}{(1-x^2)^2} \\ \frac{1}{2}x \\ x = \sin\theta \\ dx = \cos\theta \ d\theta \qquad o.e. \\ MI \\ \int = \int \frac{\cos\theta \ (d\theta)}{(1-\sin^2\theta)^2} \\ mI \\ = \frac{1}{2} \\ \frac{1}{$	~	$\sin \theta$				
$\frac{d\theta}{d\theta} = \frac{1}{\cos^2 \theta} \qquad A1 \qquad \frac{1}{\cos^2 \theta} \qquad (1 + \tan^2 \theta)$ $= \sec^2 \theta \qquad A1 \qquad 3 \qquad AG; CSO$ (b) $x = \sin \theta$ $x^2 = \sin^2 \theta$ $\cos^2 \theta = 1 - x^2$ $\tan \theta = \frac{\sin \theta}{\cos \theta}$ $= \frac{\sin \theta}{\cos \theta}$ $= \frac{\sin \theta}{\cos \theta}$ $= \tan \theta \qquad AG \qquad A1 \qquad 2 \qquad AG; CSO$ (c) $\int \frac{1}{(1 - x^2)^{\frac{3}{2}}} dx$ $x = \sin \theta$ $dx = \cos \theta \ d\theta \qquad o.c. \qquad M1$ $\int = \int \frac{\cos \theta}{(1 - \sin^2 \theta)^{\frac{3}{2}}} dx$ $= \tan \theta \qquad AG \qquad A1 \qquad 2 \qquad AG; CSO$ (d) $\int \frac{1}{(1 - x^2)^{\frac{3}{2}}} dx$ $x = \sin \theta$ $dx = \cos \theta \ d\theta \qquad o.c. \qquad M1$ $\int = \int \frac{\cos \theta}{(\cos^2 \theta)^{\frac{3}{2}}} (d\theta)$ $= \int \frac{\cos^2 \theta}{(\cos^2 \theta)^{\frac{3}{2}}} (d\theta)$ $A1$ $= \int \sec^2 \theta \ A1$ $= \tan \theta$ $= \frac{x}{\sqrt{1 - x^2}} (+c)$ $A1 \qquad 5$ $CSO including \ d\theta s$	7(a)	$y = \frac{\sin \theta}{\cos \theta}$				
$d\theta^{-} \cos^{2}\theta$ $= \frac{1}{\cos^{2}\theta} \cos^{2}\theta$ $= \frac{1}{\cos^{2}\theta} \cos^{2}\theta$ $(1 + \tan^{2}\theta)$ $AG; CSO$ $(b) x = \sin\theta$ $x^{2} = \sin^{2}\theta$ $(cs^{2}\theta = 1 - x^{2})$ $= \frac{\sin\theta}{\cos\theta}$ $= \frac{x}{\sqrt{1 - x^{2}}}$ $= \tan\theta$ AI AI AI AI $AG; CSO$ $(c) \int \frac{1}{(1 - x^{2})^{\frac{1}{2}}} dx$ $x = \sin\theta$ $dx = \cos\theta d\theta$ $o.e.$ MI $dx = \cos\theta d\theta$ dx $dx = \frac{1}{\cos\theta}$ dx dx $dx = \frac{1}{\cos\theta}$ dx dx $dx = \frac{1}{\cos\theta}$ dx dx dx dx $dx = \frac{1}{\cos\theta}$ dx dx dx dx dx dx dx dx		dy $\cos\theta\cos\theta - \sin\theta$	$\theta(-\sin\theta)$			$\pm\cos^2\theta\pm\sin^2\theta$
$\begin{vmatrix} = \sec^{2}\theta & \text{A1} & 3 & \text{AG; CSO} \\ (b) & x = \sin\theta & \frac{\sin\theta}{\sqrt{1-\sin^{2}\theta}} \\ \cos^{2}\theta = 1-x^{2} & = \frac{\sin\theta}{\cos\theta} & \text{M1} & \text{use of } \cos^{2}\theta + x^{2} = 1 \\ \tan\theta = \frac{\sin\theta}{\cos\theta} & = \frac{x}{\sqrt{1-x^{2}}} & = \tan\theta & \text{AG} & \text{A1} & 2 & \text{AG; CSO} \\ \begin{pmatrix} c) & \int \frac{1}{(1-x^{2})^{\frac{3}{2}}} dx & & \\ x = \sin\theta & \\ dx = \cos\theta & d\theta & \text{o.e.} & \text{M1} & \\ dx = \cos\theta & d\theta & \text{o.e.} & \\ \int = \int \frac{\cos\theta}{(1-\sin^{2}\theta)^{\frac{3}{2}}} & \text{m1} & \\ all in terms of \theta & \\ = \int \frac{\cos\theta}{(\cos^{2}\theta)^{\frac{3}{2}}} (d\theta) & \text{A1} & \\ all in terms of \theta & \\ = \frac{x}{\sqrt{1-x^{2}}} (+c) & \text{A1} & 5 & \text{CSO including } d\theta \\ \end{vmatrix}$		$\frac{\mathrm{d}\theta}{\mathrm{d}\theta} = \frac{1}{\cos^2\theta}$	9	A1		$\frac{1}{\cos^2\theta}$
$\begin{vmatrix} = \sec^{2}\theta & \text{A1} & 3 & \text{AG; CSO} \\ (b) & x = \sin\theta & \frac{\sin\theta}{\sqrt{1-\sin^{2}\theta}} \\ \cos^{2}\theta = 1-x^{2} & = \frac{\sin\theta}{\cos\theta} & \text{M1} & \text{use of } \cos^{2}\theta + x^{2} = 1 \\ \tan\theta = \frac{\sin\theta}{\cos\theta} & = \frac{x}{\sqrt{1-x^{2}}} & = \tan\theta & \text{AG} & \text{A1} & 2 & \text{AG; CSO} \\ \begin{pmatrix} c) & \int \frac{1}{(1-x^{2})^{\frac{3}{2}}} dx & & \\ x = \sin\theta & \\ dx = \cos\theta & d\theta & \text{o.e.} & \text{M1} & \\ dx = \cos\theta & d\theta & \text{o.e.} & \\ \int = \int \frac{\cos\theta}{(1-\sin^{2}\theta)^{\frac{3}{2}}} & \text{m1} & \\ all in terms of \theta & \\ = \int \frac{\cos\theta}{(\cos^{2}\theta)^{\frac{3}{2}}} (d\theta) & \text{A1} & \\ all in terms of \theta & \\ = \frac{x}{\sqrt{1-x^{2}}} (+c) & \text{A1} & 5 & \text{CSO including } d\theta \\ \end{vmatrix}$		1	0.8			$(1 \pm \tan^2 \theta)$
(b) $\begin{aligned} x = \sin \theta & \text{OR LHS} = \\ x^2 = \sin^2 \theta & \frac{\sin \theta}{\sqrt{1 - \sin^2 \theta}} \\ \cos^2 \theta = 1 - x^2 & = \frac{\sin \theta}{\cos \theta} & \text{M1} \\ & = \frac{\sin \theta}{\cos \theta} & = \frac{\sin \theta}{\cos \theta} & \text{M1} \\ & = \frac{x}{\sqrt{1 - x^2}} & = \tan \theta & \text{AG} & \text{A1} & 2 & \text{AG; CSO} \end{aligned}$ (c) $\begin{aligned} \int \frac{1}{(1 - x^2)^{\frac{3}{2}}} dx & & \\ x = \sin \theta & \\ dx = \cos \theta d\theta & \text{o.e.} & \text{M1} & \frac{dx}{d\theta} = \pm \cos \theta \\ & \int = \int \frac{\cos \theta (d\theta)}{(1 - \sin^2 \theta)^{\frac{3}{2}}} & \text{m1} & \text{all in terms of } \theta \\ & = \int \frac{\cos \theta}{(\cos^2 \theta)^{\frac{3}{2}}} (d\theta) & \text{A1} \\ & = \int \sec^2 \theta (d\theta) & \text{A1} \\ & = \tan \theta & \text{A1} & \text{A1} & 5 & \text{CSO including } d\theta \end{aligned}$		$-\cos^2\theta$	0.0.			
$\begin{vmatrix} x^{2} = \sin^{2}\theta & \frac{\sin\theta}{\sqrt{1-\sin^{2}\theta}} \\ \cos^{2}\theta = 1 - x^{2} & \frac{\sin\theta}{\cos\theta} & \\ \sin\theta = \frac{\sin\theta}{\cos\theta} & \\ = \frac{x}{\sqrt{1-x^{2}}} & = \tan\theta \text{AG} \text{A1} 2 \text{AG; CSO} \end{vmatrix}$ $(c) \int \frac{1}{(1-x^{2})^{\frac{3}{2}}} dx & \\ x = \sin\theta & \\ dx = \cos\theta d\theta & \text{o.e.} \text{M1} & \\ \int \frac{dx}{d\theta} = \pm \cos\theta & \\ dx = \cos\theta d\theta & \text{o.e.} \text{M1} & \\ \int \frac{dx}{d\theta} = \pm \cos\theta & \\ 1 - \sin^{2}\theta \right)^{\frac{3}{2}} & \\ = \int \frac{\cos\theta}{(\cos^{2}\theta)^{\frac{3}{2}}} (d\theta) & \\ A1 & \\ = \int \sec^{2}\theta (d\theta) & \\ = \frac{x}{\sqrt{1-x^{2}}} (+c) & \\ \text{A1} & \\ 5 & \\ \text{CSO including } d\theta \text{S} \end{vmatrix}$		$=\sec^2\theta$		A1	3	AG; CSO
$x^{2} = \sin^{2}\theta \qquad \boxed{\sqrt{1 - \sin^{2}\theta}} = \frac{\sin\theta}{\cos\theta} \qquad M1 \qquad use of \cos^{2}\theta + x^{2} = 1$ $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta AG A1 2 AG; CSO$ (c) $\int \frac{1}{(1 - x^{2})^{\frac{3}{2}}} dx = x = \sin\theta AG A1 2 AG; CSO$ $\int \frac{1}{(1 - x^{2})^{\frac{3}{2}}} dx = x = \sin\theta AG M1 \frac{dx}{d\theta} = \pm \cos\theta$ $\int = \int \frac{\cos\theta}{(1 - \sin^{2}\theta)^{\frac{3}{2}}} m1 = \tan\theta$ $= \int \frac{\cos\theta}{(\cos^{2}\theta)^{\frac{3}{2}}} (d\theta) = A1 = \sin\theta$ $= \frac{x}{\sqrt{1 - x^{2}}} (+c) A1 5 CSO \text{ including } d\theta \text{ s}$	(b)	$x = \sin \theta$	OR LHS =			
$\begin{vmatrix} \cos^2 \theta = 1 - x^2 \\ \tan \theta = \frac{\sin \theta}{\cos \theta} \\ = \frac{x}{\sqrt{1 - x^2}} \\ = \tan \theta AG AI 2 AG; CSO \end{vmatrix}$ $(c) \int \frac{1}{(1 - x^2)^{\frac{3}{2}}} dx \\ x = \sin \theta \\ dx = \cos \theta \ d\theta o.e. MI \\ \int = \int \frac{\cos \theta \ (d\theta)}{(1 - \sin^2 \theta)^{\frac{3}{2}}} \\ = \int \frac{\cos \theta}{(\cos^2 \theta)^{\frac{3}{2}}} (d\theta) \\ = \int \frac{\cos \theta}{(\cos^2 \theta)^{\frac{3}{2}}} (d\theta) \\ = \tan \theta \\ = \frac{x}{\sqrt{1 - x^2}} (+c) \qquad AI 5 CSO \text{ including } d\theta \text{ s}$			$\sin heta$			
$\begin{vmatrix} \cos^2 \theta = 1 - x^2 \\ \tan \theta = \frac{\sin \theta}{\cos \theta} \\ = \frac{x}{\sqrt{1 - x^2}} \\ = \tan \theta AG AI 2 AG; CSO \end{vmatrix}$ $(c) \int \frac{1}{(1 - x^2)^{\frac{3}{2}}} dx \\ x = \sin \theta \\ dx = \cos \theta \ d\theta o.e. MI \\ \int = \int \frac{\cos \theta \ (d\theta)}{(1 - \sin^2 \theta)^{\frac{3}{2}}} \\ = \int \frac{\cos \theta}{(\cos^2 \theta)^{\frac{3}{2}}} (d\theta) \\ = \int \frac{\cos \theta}{(\cos^2 \theta)^{\frac{3}{2}}} (d\theta) \\ = \tan \theta \\ = \frac{x}{\sqrt{1 - x^2}} (+c) \qquad AI 5 CSO \text{ including } d\theta \text{ s}$		$x^2 = \sin^2 \theta$	$\sqrt{1-\sin^2\theta}$			
$\begin{aligned} \tan \theta &= \frac{\sin \theta}{\cos \theta} \\ &= \frac{x}{\sqrt{1 - x^2}} \\ = \tan \theta AG A1 2 AG; CSO \end{aligned}$ (c) $\int \frac{1}{(1 - x^2)^{\frac{3}{2}}} dx \\ x &= \sin \theta \\ dx &= \cos \theta \ d\theta o.e. M1 \frac{dx}{d\theta} = \pm \cos \theta \\ \int &= \int \frac{\cos \theta}{(1 - \sin^2 \theta)^{\frac{3}{2}}} \\ &= \int \frac{\cos \theta}{(\cos^2 \theta)^{\frac{3}{2}}} (d\theta) A1 \\ &= \int \sec^2 \theta \ (d\theta) \\ &= \tan \theta \\ &= \tan \theta \\ &= \frac{x}{\sqrt{1 - x^2}} \ (+c) A1 5 CSO \text{ including } d\theta \end{aligned}$				M1		5 - 2 + 2 + 1
$\begin{vmatrix} = \frac{x}{\sqrt{1-x^2}} &= \tan\theta & AG & A1 & 2 & AG; CSO \\ (c) \int \frac{1}{(1-x^2)^{\frac{3}{2}}} dx & & & \\ x = \sin\theta & & & \\ dx = \cos\theta d\theta & & o.e. & M1 & \\ dx = \cos\theta d\theta & & o.e. & M1 & \\ \int = \int \frac{\cos\theta (d\theta)}{(1-\sin^2\theta)^{\frac{3}{2}}} & & \\ m1 & & \\ all in terms of \theta & \\ = \int \frac{\cos\theta}{(\cos^2\theta)^{\frac{3}{2}}} (d\theta) & & \\ A1 & \\ = \frac{1}{\sin\theta} & & \\ = \frac{x}{\sqrt{1-x^2}} (+c) & & A1 & 5 & CSO including d\theta s \\ \end{vmatrix}$		$\cos^2\theta = 1 - x^2$	$=\frac{1}{\cos\theta}$	IVI I		use of $\cos^2\theta + x^2 = 1$
$\begin{vmatrix} = \frac{x}{\sqrt{1-x^2}} &= \tan\theta & AG & A1 & 2 & AG; CSO \\ (c) \int \frac{1}{(1-x^2)^{\frac{3}{2}}} dx & & & \\ x = \sin\theta & & & \\ dx = \cos\theta d\theta & & o.e. & M1 & \\ dx = \cos\theta d\theta & & o.e. & M1 & \\ \int = \int \frac{\cos\theta (d\theta)}{(1-\sin^2\theta)^{\frac{3}{2}}} & & \\ m1 & & \\ all in terms of \theta & \\ = \int \frac{\cos\theta}{(\cos^2\theta)^{\frac{3}{2}}} (d\theta) & & \\ A1 & \\ = \frac{1}{\sin\theta} & & \\ = \frac{x}{\sqrt{1-x^2}} (+c) & & A1 & 5 & CSO including d\theta s \\ \end{vmatrix}$		$\tan \theta = \frac{\sin \theta}{\sin \theta}$				
(c) $\int \frac{1}{(1-x^2)^{\frac{3}{2}}} dx$ $x = \sin \theta$ $dx = \cos \theta d\theta$ o.e. M1 $\frac{dx}{d\theta} = \pm \cos \theta$ all in terms of θ $= \int \frac{\cos \theta}{(1-\sin^2 \theta)^{\frac{3}{2}}} (d\theta)$ $= \int \frac{\cos \theta}{(\cos^2 \theta)^{\frac{3}{2}}} (d\theta)$ A1 $= \int \sec^2 \theta (d\theta)$ A1 $= \tan \theta$ $= \frac{x}{\sqrt{1-x^2}} (+c)$ A1 5 CSO including $d\theta$'s		$\cos\theta$				
(c) $\int \frac{1}{(1-x^2)^{\frac{3}{2}}} dx$ $x = \sin \theta$ $dx = \cos \theta d\theta$ o.e. M1 $\frac{dx}{d\theta} = \pm \cos \theta$ all in terms of θ $= \int \frac{\cos \theta}{(1-\sin^2 \theta)^{\frac{3}{2}}} (d\theta)$ $= \int \frac{\cos \theta}{(\cos^2 \theta)^{\frac{3}{2}}} (d\theta)$ A1 $= \int \sec^2 \theta (d\theta)$ A1 $= \tan \theta$ $= \frac{x}{\sqrt{1-x^2}} (+c)$ A1 5 CSO including $d\theta$'s		$=\frac{x}{\sqrt{x}}$		A1	2	AG: CSO
$x = \sin \theta$ $dx = \cos \theta d\theta$ o.e. M1 $\int = \int \frac{\cos \theta (d\theta)}{\left(1 - \sin^2 \theta\right)^{\frac{3}{2}}}$ m1 $= \int \frac{\cos \theta}{\left(\cos^2 \theta\right)^{\frac{3}{2}}} (d\theta)$ A1 $= \int \sec^2 \theta (d\theta)$ A1 $= \tan \theta$ A $= \frac{x}{\sqrt{1 - x^2}} (+c)$ A1 5 CSO including $d\theta$'s		$\sqrt{1-x^2}$	$= \tan \theta$ AG		-	
$x = \sin \theta$ $dx = \cos \theta d\theta$ o.e. M1 $\int = \int \frac{\cos \theta (d\theta)}{\left(1 - \sin^2 \theta\right)^{\frac{3}{2}}}$ m1 $= \int \frac{\cos \theta}{\left(\cos^2 \theta\right)^{\frac{3}{2}}} (d\theta)$ A1 $= \int \sec^2 \theta (d\theta)$ A1 $= \tan \theta$ A $= \frac{x}{\sqrt{1 - x^2}} (+c)$ A1 5 CSO including $d\theta$'s		f ¹ du				
$x = \sin \theta$ $dx = \cos \theta d\theta$ o.e. M1 $\int = \int \frac{\cos \theta (d\theta)}{\left(1 - \sin^2 \theta\right)^{\frac{3}{2}}}$ m1 $= \int \frac{\cos \theta}{\left(\cos^2 \theta\right)^{\frac{3}{2}}} (d\theta)$ A1 $= \int \sec^2 \theta (d\theta)$ A1 $= \tan \theta$ A $= \frac{x}{\sqrt{1 - x^2}} (+c)$ A1 5 CSO including $d\theta$'s	(C)	$\int \frac{1}{(1-x^2)^{\frac{3}{2}}} dx$				
$dx = \cos \theta d\theta \qquad \text{o.e.} \qquad M1 \qquad \qquad \frac{dx}{d\theta} = \pm \cos \theta$ $\int = \int \frac{\cos \theta (d\theta)}{\left(1 - \sin^2 \theta\right)^{\frac{3}{2}}} \qquad m1 \qquad \qquad \text{all in terms of } \theta$ $= \int \frac{\cos \theta}{\left(\cos^2 \theta\right)^{\frac{3}{2}}} (d\theta) \qquad \qquad A1$ $= \int \sec^2 \theta (d\theta) \qquad \qquad A1$ $= \tan \theta$ $= \frac{x}{\sqrt{1 - x^2}} (+c) \qquad \qquad A1 \qquad 5 \qquad \text{CSO including } d\theta \text{s}$						
$\int = \int \frac{\cos \theta (d\theta)}{(1-\sin^2 \theta)^{\frac{3}{2}}}$ m1 all in terms of θ $= \int \frac{\cos \theta}{(\cos^2 \theta)^{\frac{3}{2}}} (d\theta)$ A1 $= \int \sec^2 \theta (d\theta)$ A1 $= \tan \theta$ $= \frac{x}{\sqrt{1-x^2}} (+c)$ A1 $= 10$			o.e.	MI		$dx = \pm \cos \theta$
$= \int \frac{\cos\theta}{\left(\cos^2\theta\right)^{\frac{3}{2}}} (d\theta) \qquad A1$ $= \int \sec^2\theta (d\theta) \qquad A1$ $= \tan\theta$ $= \frac{x}{\sqrt{1-x^2}} (+c) \qquad A1 \qquad 5 \text{CSO including } d\theta$'s $= \frac{10}{10}$				IVI I		$\frac{1}{d\theta} = 1\cos\theta$
$= \int \frac{\cos\theta}{\left(\cos^2\theta\right)^{\frac{3}{2}}} (d\theta) \qquad A1$ $= \int \sec^2\theta (d\theta) \qquad A1$ $= \tan\theta$ $= \frac{x}{\sqrt{1-x^2}} (+c) \qquad A1 \qquad 5 \text{CSO including } d\theta$'s $= \frac{10}{10}$		$\int = \int \frac{\cos\theta (\mathrm{d}\theta)}{\mathrm{d}\theta}$				
$= \int \frac{\cos\theta}{\left(\cos^2\theta\right)^{\frac{3}{2}}} (d\theta) \qquad A1$ $= \int \sec^2\theta (d\theta) \qquad A1$ $= \tan\theta$ $= \frac{x}{\sqrt{1-x^2}} (+c) \qquad A1 \qquad 5 \text{CSO including } d\theta$'s $= \frac{10}{10}$		$\int \int \int \frac{3}{(1-\sin^2\theta)^2}$		m1		all in terms of θ
$\begin{vmatrix} = \int \sec^2 \theta (d\theta) \\ = \tan \theta \\ = \frac{x}{\sqrt{1 - x^2}} (+c) \\ \hline \end{bmatrix} \qquad A1 \qquad A1 \qquad S \qquad CSO including \ d\theta's \qquad \hline \end{bmatrix}$						
$\begin{vmatrix} = \int \sec^2 \theta (d\theta) \\ = \tan \theta \\ = \frac{x}{\sqrt{1 - x^2}} (+c) \\ \hline \end{bmatrix} \qquad A1 \qquad A1 \qquad S \qquad CSO including \ d\theta's \qquad \hline \end{bmatrix}$		$=\int \frac{\cos\theta}{(1-2)^{\frac{3}{2}}} (\mathrm{d}\theta)$		A1		
$\begin{vmatrix} = \int \sec^2 \theta (d\theta) \\ = \tan \theta \\ = \frac{x}{\sqrt{1 - x^2}} (+c) \\ \hline \end{bmatrix} \qquad A1 \qquad A1 \qquad S \qquad CSO including \ d\theta's \qquad \hline \end{bmatrix}$		$(\cos^2 \theta)^2$				
$=\frac{x}{\sqrt{1-x^2}} (+c)$ A1 5 CSO including d θ 's Total 10				A1		
Total 10		$= \tan \theta$				
		$=\frac{x}{\sqrt{1-x^2}} \ (+c)$		A1	5	CSO including $d\theta$'s
TOTAL 75						
			TOTAL		75	

Alte	rnative

7(a)	$y = \frac{\tan \theta}{1}$			
	$\frac{\mathrm{d}y}{\mathrm{d}\theta} = \frac{1\mathrm{sec}^2\theta - 0}{1^2}$	M1 A1		
	$=\sec^2\theta$	A1		